Hidden Markov Regimes in Operational Loss Data

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Summary

We propose a method to consider business cycles in the computation of capital for operational risk.

We use the operational loss data of American banks to examine whether the data contain a Hidden Markov Regime switching feature for the 2001-2010 period.

We build on the scaling model of Dahen and Dionne (2010b) and show that the operational loss data of American banks are indeed characterized by a Hidden Markov Regime switching model.

Summary

The distribution of monthly losses is asymmetric, with a normal component in the low regime and a skew t type 4 component in the high regime.

Statistical tests do not allow us to reject this asymmetry.

The presence of regime affects the distribution of losses in general.

Summary

We also analyze the scaling of the data to banks of different sizes and risk exposures.

Results of the model backtesting in two different banks will also be presented.

Banks tend to allot too much capital to operational risk when the regimes are not considered in our period of analysis.

Data

We use the Algo OpData Quantitative Database for operational losses of $1 million and more sustained by US banks.

The study period is from January 2001 to December 2010.

We examine the operational losses of US Bank Holding Companies (BHC) valued at over $1 billion.

The source of information on these banks is the Federal Reserve of Chicago.

Data

Table 1: Number of BHC banks per year and their assets

	Assets (in billions $)				
Year	Median	Mean	Max	Sd	Number
2001	2.1	19.7	944.3	82.3	356
2002	2.1	19.5	1,097.2	84.8	378
2003	2.0	20.3	1,264.0	93.0	408
2004	2.0	25.4	1,484.1	122.1	421
2005	2.0	24.4	1,547.8	121.9	445
2006	2.1	26.0	1,884.3	140.5	461
2007	2.1	28.9	2,358.3	168.1	460
2008	2.0	28.5	2,251.5	182.5	470
2009	2.1	33.8	2,323.4	190.6	472
2010	2.1	34.7	2,370.6	198.3	458

Note: Sd is standard deviation.

Data

Table 2: Operational losses of BHC banks with bank assets in deciles

Asset deciles (in billions $)	Loss (in millions $)					
	Min	Max	Median	Mean	Sd	Number
2,022.7 to 2,370.6	1.0	8,045.3	26.3	265.9	1,129.5	51
1,509.6 to 2,022.7	1.0	8,400.0	14.0	268.3	1,207.5	49
1,228.3 to 1,509.6	1.0	2,580.0	7.5	94.5	357.8	53
799.3 to 1,228.3	1.0	3,782.3	24.0	199.8	610.7	48
521.9 to 799.3	1.0	8,400.0	7.4	218.9	1,156.4	53
1,247.1 to 521.9	1.1	210.2	7.2	17.0	31.1	50
98.1 to 247.1	1.0	663.0	6.0	45.3	115.4	51
33.7 to 98.1	1.0	775.0	10.2	55.2	152.8	51
8.31 to 33.7	1.1	691.2	8.6	32.2	98.6	51
0.96 to 8.31	1.0	65.0	4.3	9.9	14.5	51
All	1.0	8,400.0	8.6	120.1	680.7	508

Note: Sd is standard deviation.

Data

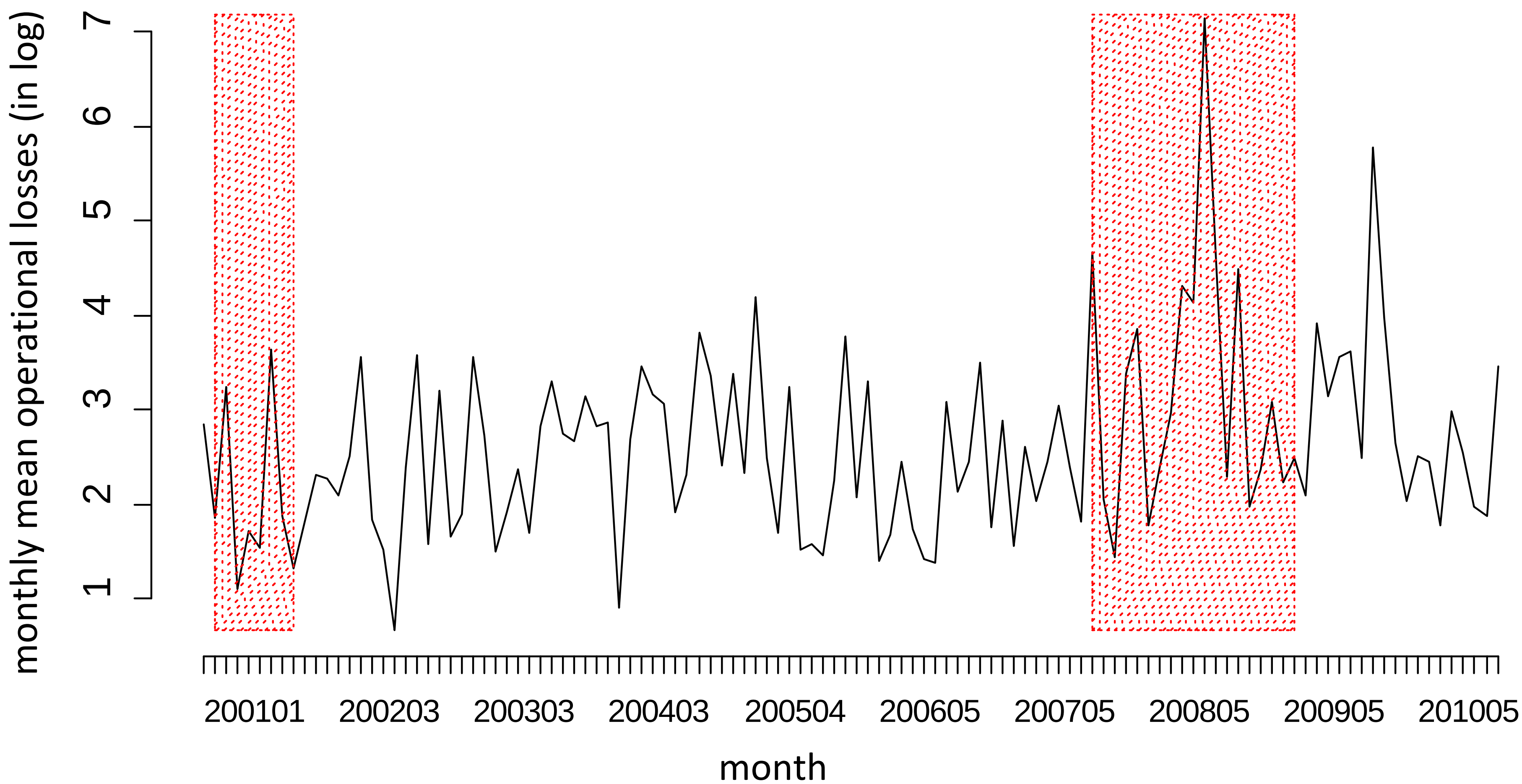

Figure 1: Changes in monthly mean operational losses

Markov Switching Regimes

We examine the case of two states $(n=2)$, f_1 being the density function of a normal law for the low-loss regime (state 1), f_2 being the density function of the skew t-distribution type 4 representing the high-loss regime (state 2).

The normal density:

$$f_1\left(x_t \middle| \mu_1, \sigma_1^2\right) = \frac{1}{\sigma_1\sqrt{2\pi}} \exp\left[\frac{\left(x_t - \mu_1\right)^2}{2\sigma_1^2}\right]$$

where $\sigma_1 > 0$ and $\mu_1 \in \mathbb{R}$.

Markov Switching Regimes

The ST4 (skew *t* type 4) density:

$$f_2\left(x_t \middle| \mu_2, \sigma_2, \nu, \tau\right) =$$

$$\frac{c}{\sigma_2}\left\{\left[1+\frac{\left(x_t-\mu_2\right)^2}{\nu\sigma_2^2}\right]^{-(\nu+1)/2} I\left(x_t<\mu_2\right)+\left[1+\frac{\left(x_t-\mu_2\right)^2}{\tau\sigma_2^2}\right]^{-(\tau+1)/2} I\left(x_t\geq\mu_2\right)\right\}$$

where

$$\sigma_2, \nu, \tau > 0, \mu_2 \in \mathbb{R}, c = 2\left[\nu^{1/2}B\left(1/2, \nu/2\right)+\tau^{1/2}B\left(1/2, \tau/2\right)\right]^{-1}.$$

B is the beta function. $B\left(a,b\right)=\Gamma(a)\Gamma(b)/\Gamma(a+b)$ where Γ is the gamma function.

Markov Switching Regimes

We also consider the skew normal distribution (SN) in state 2 while keeping the normal distribution in state 1. The skew normal density:

$$f_2\left(x_t \middle| \mu_2, \sigma_2, \gamma\right) =$$

$$c\left\{\exp\left[-\frac{1}{2}\left(\frac{x_t-\mu_2}{\sigma_2}\right)^2\gamma^2\right]I\left(x_t<\mu_2\right)+\exp\left[-\frac{1}{2}\left(\frac{x_t-\mu_2}{\sigma_2}\right)^2\frac{1}{\gamma^2}\right]I\left(x_t\geq\mu_2\right)\right\}$$

where $c=\sqrt{2/\pi}\,\dfrac{\gamma}{\sigma_2\left(1+\gamma^2\right)}$, $\mu_2 \in \mathbb{R}$, $\sigma_2 > 0$, $\gamma > 0$ and I is an indicator variable. γ is an asymmetry measure.

When $\gamma = 1$, the SN distribution becomes the normal distribution.

Results

Table 4: Estimation of the Hidden Markov Model

	Model 1 $N+N$		Model 2 $N+SN$		Model 3 $N+ST4$	
	Variable	Coefficient	Variable	Coefficient	Variable	Coefficient
Probability of transition to high regime						
	Intercept	0.2879	Intercept	0.3318	Intercept	0.9772
		(0.7392)		(0.7648)		(0.8161)
	L_1	-1.4853***	L_1	-1.5468***	L_1	-1.7371***
		(0.2001)		(0.2766)		(0.2798)
Probability of staying in high regime						
	Intercept	-25.3101***	Intercept	-28.1742***	Intercept	-25.7285***
		(4.6082)		(5.2129)		(4.5707)
	L_2	11.5681***	L_2	12.9137***	L_2	11.7434***
		(2.5745)		(3.1253)		(2.4739)

Response distributions						
Low regime	Normal law		Normal law		Normal law	
	μ_1	2.4277***	μ_1	2.4570***	μ_1	2.4172***
		(0.4366)		(0.4546)		(0.5876)
	σ_1	0.7685***	σ_1	0.7979***	σ_1	0.7653***
		(0.2214)		(0.2494)		(0.2006)
High regime	Normal law		SN		ST4	
	μ_2	4.0294***	μ_2	3.3991**	μ_2	3.7872***
		(6.5251)		(1.5123)		(0.5449)
	σ_2	1.2968***	σ_2	1.0207***	$\log(\sigma_2)$	-0.0415
		(0.1683)		(0.2370)		(0.2546)
			$\log(\gamma)$	0.5401	$\log(\nu)$	2.7734*
				(0.7609)		(1.4299)
					$\log(\tau)$	0.9492
						(0.8007)
Log likelihood	-152.566		-151.863		-148.838	
AIC criteria	323.132		323.726		319.677	
Number of observations	120		120		120	

Notes: Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

The log likelihood of the EGB2 is -148.785 and the AIC criteria is 319.570.

Results

Table 5: Skewness and kurtosis analysis with different distributions in the high regime

	Skewness	Kurtosis
Data (log)	1.192	5.974
N	0.000	3.000
SN	0.653	3.319
ST4	1.119	6.127
EGB2	1.008	5.093

Results

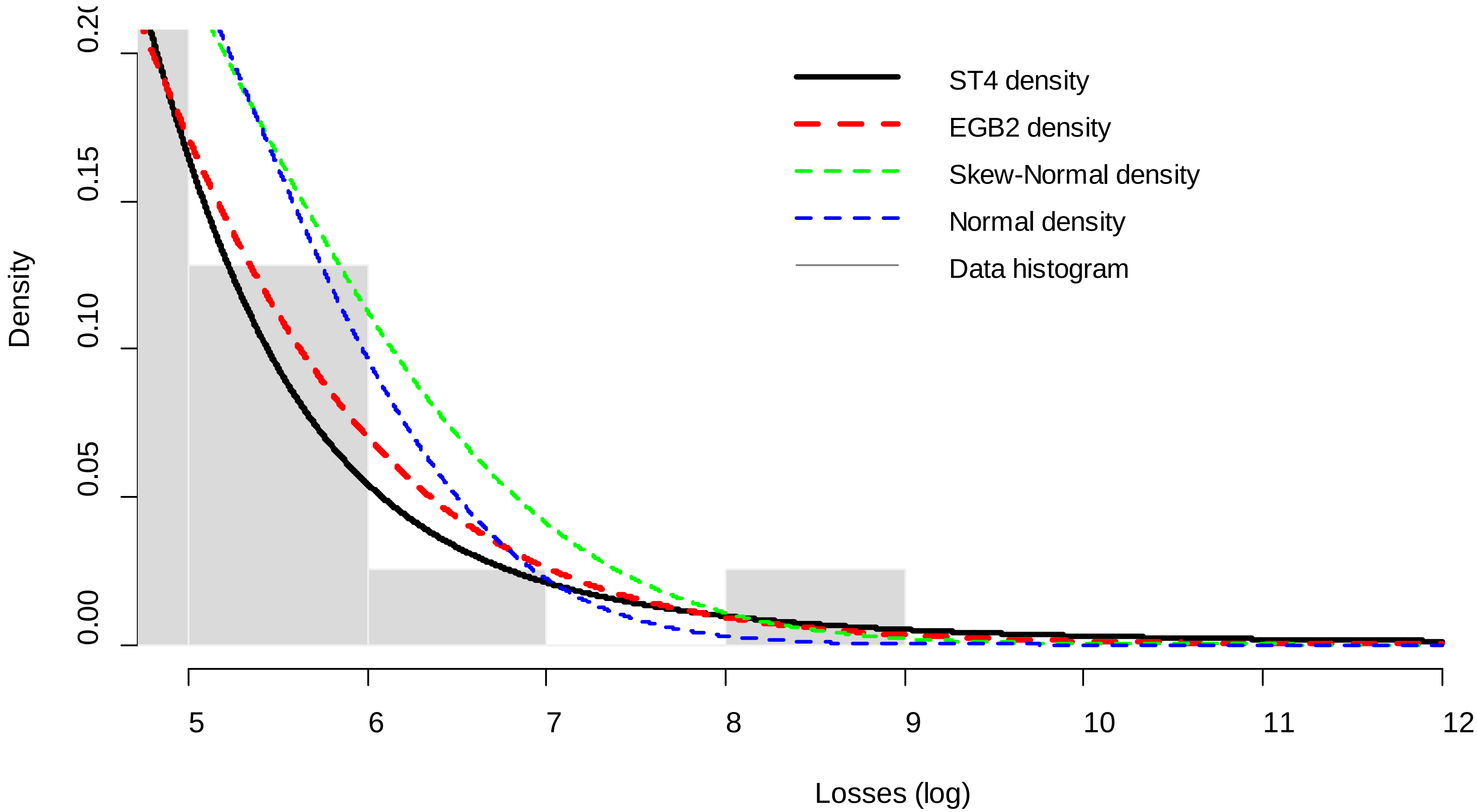

Figure 3: Data histogram and right tail density of log of losses for different distributions

Results

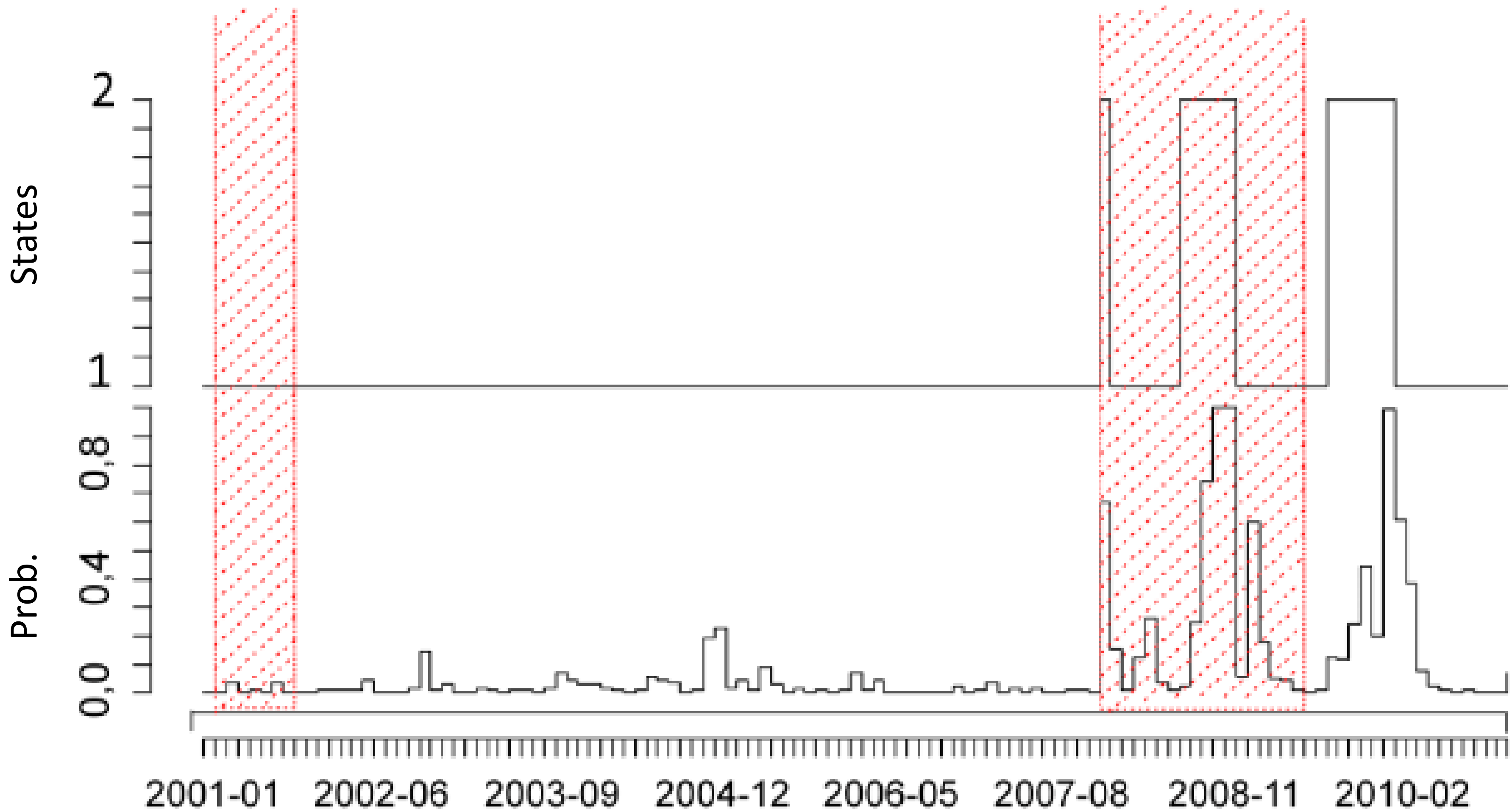

Figure 4 Markov Regimes detected from January 2001 to December 2010

Results

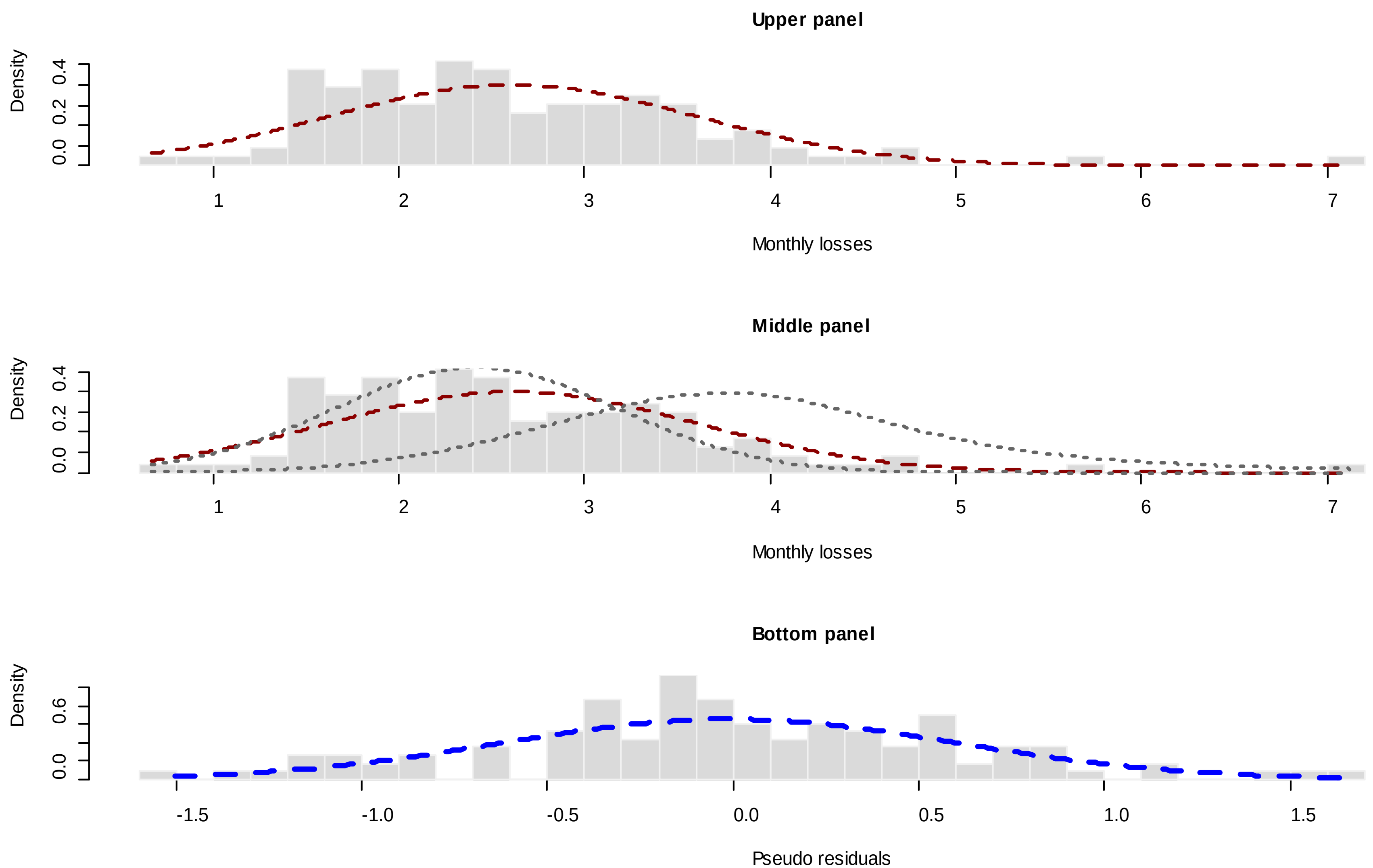

Figure 5: Histograms of monthly losses and pseudo-residuals

Effects of regimes detected

$$\log\left(Loss_{it}\right)=\alpha+\beta\log\left(Assets_{it}\right)+\sum_j \lambda_j BL_{ijt}+\sum_k \delta_k ET_{ikt}+\sum_t \gamma_t T_t+\varepsilon_{it}$$

$$\log\left(Loss_{it}\right)=\alpha+\beta\log\left(Assets_{it}\right)+\sum_j \lambda_j BL_{ijt}+\sum_k \delta_k ET_{ikt}+\sum_t \gamma_t T_t+\xi RHMM+\varepsilon_{it}^{1}$$

$$\begin{aligned}\log\left(Loss_{it}\right)=\alpha+\beta\log\left(Assets_{it}\right)+\sum_j \lambda_j BL_{ijt}+\sum_k \delta_k ET_{ikt}+\sum_t \gamma_t T_t+\xi RHMM\\ +\sum_j \lambda_j^{1} BL_{ijt}\times RHMM+\sum_k \delta_k^{1} ET_{ikt}\times RHMM+\varepsilon_{ij}^{11}\end{aligned}$$

Effects of regimes detected

Table 9: Effect of regimes detected on *log (Loss)*

Variable	(4.1) Reference model	(4.2) Adding HMM regime	(4.3) Adding HMM regime and interaction
Intercept	-0.297 (0.433)	-0.260 (0.446)	-0.160 (0.436)
Log(Assets)	0.139*** (0.037)	0.139*** (0.038)	0.126*** (0.036)
High HMM Regime		0.977*** (0.331)	1.538* (0.791)
Paymt and Settlmnt	1.261*** (0.438)	1.199*** (0.438)	1.196** (0.466)
Trading and Sales	1.104*** (0.290)	1.026*** (0.304)	0.906** (0.372)
Comm. Banking	1.182*** (0.167)	1.117*** (0.164)	1.159*** (0.172)
Retail Banking	0.930*** (0.207)	0.867*** (0.207)	0.827*** (0.171)
Agency Services	1.223*** (0.413)	1.161*** (0.435)	1.532*** (0.443)
Corp. Finance	2.056*** (0.237)	2.063*** (0.250)	1.999*** (0.294)

Asset Mngmt	1.358***	1.321***	1.307***
	(0.274)	(0.254)	(0.283)
Bus.Disrup. syst.Fail.	-1.080	-0.926	-0.878
	(0.687)	(0.569)	(0.630)
Damage Phy.Assets	-0.086	-0.044	0.047
	(1.925)	(1.923)	(1.953)
Employ.Prac.Wrkplac.Saf.	-0.676***	-0.622**	-0.476**
	(0.252)	(0.254)	(0.224)
External Fraud	-0.502***	-0.489***	-0.433**
	(0.157)	(0.161)	(0.170)
Internal Fraud	-0.593***	-0.524**	-0.304
	(0.227)	(0.226)	(0.211)
Exer. Deliv. Proc. Mnmt	-0.214	-0.217	-0.130
	(0.228)	(0.230)	(0.256)
High Regime × Employ.Prac.Wrkplac.Saf.			-2.321***
			(0.513)
High Regime × External Fraud			0.120
			(1.088)
High Regime × Internal Fraud			-3.314***
			(0.547)
High Regime × Exec. Deliv. Proc. Mngmt			0.115
			(1.228)
High Regime × Paymt and Settlmnt			-0.561
			(1.584)
High Regime × Trading and Sales			0.317
			(1.248)

High Regime × Comm. Banking			-1.511
			(1.266)
High Regime × Retail Banking			0.401
			(1.075)
High Regime × Agency Services			-4.491***
			(1.114)
High Regime × Corp. Finance			0.645
			(1.565)
High Regime × Asset Mngmt			-0.249
			(0.963)
Year fixed effects	yes	yes	yes
Adj. R^2	0.170	0.186	0.223
AIC	1993.52	1985.23	1978.04
Log Likelihood	-971.8	-966.6	-952.0
p-value Chi2		0.001 (4.2 vs 4.1)	0.002 (4.3 vs 4.2)
Num. obs.	508	508	508

Note: Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Clients, Products and Business Practices, Retail Brokerage, and Year 2001 are the omitted categories for Event Types, Business Lines, and Years respectively.

Frequency models

$$\Pr\left(Y=k\middle|\lambda,\delta\right)=f_{NB}\left(k,\lambda,\delta\right)=\frac{\Gamma\left(k+1/\delta\right)}{k!\Gamma\left(1/\delta\right)}\left[\frac{1}{1+\delta\lambda}\right]^{\frac{1}{\delta}}\left[\frac{\delta\lambda}{1+\delta\lambda}\right]^{k}$$

$$\Pr\left(Y_{ij}=k\right)=\begin{cases}q_{ij}+\left(1-q_{ij}\right)f_{NB}\left(0,\lambda_{ij},\delta\right) & k=0\\ \left(1-q_{ij}\right)f_{NB}\left(k,\lambda_{ij},\delta\right) & k=1,2,\ldots\end{cases}$$

$$\log\left(\lambda_{ij}\right)=\zeta_0+\zeta_1\log\left(Assets_{ij}\right)+\zeta_2\text{RHMM}+\zeta_3 GDP\ Growth_j$$
$$+\zeta_4 Bank\text{-}Cap_{ij}+\zeta_5 Mean\text{-}Salary_{ij}$$

$$\log\left(\frac{q_{ij}}{1-q_{ij}}\right)=\xi_0+\xi_1\log\left(Assets_{ij}\right)+\xi_2\text{RHMM}$$
$$+\xi_3 GDP\ Growth_j+\xi_4 Mean\text{-}Salary_{ij}.$$

Frequency models

Table 10: Effect of regimes on frequencies

	Model 1 Reference model	Model 2 Adding HMM regime
Count model		
Intercept	-10.969***	-11.370***
	(0.741)	(0.424)
Log(Assets)	0.885***	0.916***
	(0.053)	(0.034)
High Regime		-0.531*
		(0.291)
GDP Growth	0.018	0.011
	(0.034)	(0.039)
Bank-Cap	4.428***	4.103***
	(0.933)	(0.705)
Mean-Salary	-0.751	-1.642*
	(0.913)	(0.841)
Log(δ)	2.097***	1.085***
	(0.634)	(0.417)

Zero-inflated model		
Intercept	1.176	-4.580*
	(1.681)	(2.712)
Log(Assets)	-0.176	-0.149
	(0.120)	(0.202)
High Regime		7.888***
		(2.502)
GDP Growth	0.001	2.734***
	(0.109)	(0.787)
Mean-Salary	1.466	-48.468**
	(2.569)	(23.625)
AIC	1640.089	1597.558
Log Likelihood	-810.044	-786.779
Log-Likelihood ratio test		
- Statistic		46.530
- p.value		0.000
Number of observations	4329	4329

Note: Standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Scaling model

We want to scale a real loss from bank A to bank B.

To save space, we write $Loss_{it}$ for $Loss_{itblr}$ where *i* is for bank, *t* for time, *b* for business line, *l* for type of loss, and *r* for regime.

We first compute the estimated $Log\widehat{(\,Loss_{it})}$ for each of the two banks using the regression results.

The next equation shows how we proceed for bank *i* (A or B) at period *t* where we use the size of each bank, and the business line, the type of loss, and the regime of the observed loss in bank A.

Scaling model

The interactions between both *ET* and *BL*, and *RHMM* are also taken into account.

$$Log(\widehat{Loss_{it}}) = \hat{\alpha} + \hat{\beta} Log(Size_{it}) + \sum_j \widehat{\lambda_j}\, BL_{ijt} + \sum_k \widehat{\delta_k}\, ET_{ikt} + \sum_t \widehat{\gamma_t}\, T_t + \sum_t \hat{\xi}\, RHMM_{it}$$

$$+ \sum_j \widehat{\lambda_j^1}\, BL_{ijt} \times RHMM_{it} + \sum_k \widehat{\delta_k^1}\, ET_{ikt} \times RHMM_{it}$$

From the above equation, we can compute $\widehat{Loss_{\iota,t}}$ for each bank.

Let us write:

$$Log\left(f\left(\hat{\omega},\theta_{it}\right)\right) = \sum_j \widehat{\lambda_j} BL_{ijt} + \sum_k \widehat{\delta_k} ET_{ikt} + \sum_t \widehat{\gamma_t} T_t + \sum_t \hat{\xi} RHMM_{it}$$

$$+ \sum_j \widehat{\lambda_j^1} BL_{ijt} \times RHMM_{it} + \sum_k \widehat{\delta_k^1} ET_{ikt} \times RHMM_{it}$$

where θ_{it} is a vector of characteristics and ω a vector of parameters.

Scaling model

So the estimed loss for each bank *i* can be written as follows:

$$\widehat{Loss}_{Bt} = e^{\hat{\alpha}} Size_{Bt}^{\hat{\beta}} \times f\left(\hat{\omega}, \theta_{Bt}\right)$$

$$\widehat{Loss}_{At} = e^{\hat{\alpha}} Size_{At}^{\hat{\beta}} \times f\left(\hat{\omega}, \theta_{At}\right).$$

See Dahen and Dionne (*JBF*, 2010) for more details.

From these estimations, the scaling of a real loss from bank A to bank B becomes:

$$Loss_{Bt}^{S} = Loss_{At} \left(\frac{Size_{Bt}}{Size_{At}} \right)^{\hat{\beta}} \times \frac{f\left(\hat{\omega}, \theta_{Bt}\right)}{f\left(\hat{\omega}, \theta_{At}\right)}$$

where $Loss_{Bt}^{S}$ is the scaling of $Loss_{At}$ to bank B.

Backtesting

Table 11: Backtesting of U.S. Bancorp bank

Backtesting	Confidence level	Frequency		VaR[1]
		Theoretical	Observed	
Reference model	95%	0.050	0.043	269.7
(Model 1)	99%	0.010	0.012	842.3
	99.5%	0.005	0.008	1289.7
	99.9%	0.001	0.002	2957.4
HMM regimes and interactions				
(Model 2)	95%	0.050	0.043	209.0
	99%	0.010	0.016	619.3
	99.5%	0.005	0.004	913.6
	99.9%	0.001	0.002	2060.7

[1] In millions of US dollars.

Backtest results are presented in the paper.

Backtesting

Table 12: Backtesting of Fifth Third Bancorp bank

Backtesting	Confidence level	Frequency		VaR[1]
		Theoretical	Observed	
Reference model	95%	0.050	0.038	115.0
(Model 1)	99%	0.010	0.007	430.7
	99.5%	0.005	0.004	689.8
	99.9%	0.001	0.003	1722.6
HMM regimes and interactions				
(Model 2)	95%	0.050	0.042	94.4
	99%	0.010	0.013	338.1
	99.5%	0.005	0.007	522.6
	99.9%	0.001	0.003	1291.5

[1] In millions of US dollars.

Backtest results are presented in the paper.

Conclusion

We demonstrate that considering business cycles can reduce capital (at 99.9%) for operational risk by redistributing it between high regime and low regime states.

The variation of capital is estimated to be between 25% and 30% in our period of analysis.

Three possible extensions

- The most promising would be to test the stability of the results using different regime detection methods (Maalaoui, Chun et al., 2014).

- Another possible extension is to use a different approach than the scaling of operational losses. Some banks use the Change of Measure Approach proposed by Dutta and Babbel (2013).

- Compare the results of this model (AMA) with those of the Standardized Measurement Approach (SMA) proposed by the Basel Committee.

Appendix

Table 6: Summary of losses of BHC banks from July 2008 to November 2008

	Bank	Loss	EventType	BusLine	Date	% Loss
1	Wachovia Bank	8.4 billion	CliPBP	RBn	2008-07-21	40.73
2	CFC – Bank of America	8.4 billion	CliPBP	RBn	2008-10-06	40.73
	Other (< 80%)	3.4 billion	30 losses			
	All	20.6 billion	32 losses			

Table 7: Summary of losses of main BHC banks from August 2009 to February 2010

	Bank	Loss	EventType	BusLine	Date	% Loss
1	Citibank N.A.	840 million	ExeDPM	TraS	2010-01-19	20.77
2	Discover Financial Service	775 million	CliPBP	RBn	2010-02-12	19.16
3	JP Morgan Securities Inc.	722 million	CliPBP	CorF	2009-11-04	17.85
4	State Street Global Advis	663 million	CliPBP	AssM	2010-02-04	16.39
5	Merrill Lynch and Company	150 million	CliPBP	CorF	2010-02-22	3.71
6	Bank of America Corporation	142 million	EF	ComB	2009-09-21	3.51
	Other (< 80%)	753 million	21 losses			
	All	4.05 billion	27 losses			