Experience Rating Schemes for Fleets of Vehicles

by Denise Desjardins, Georges Dionne and Jean Pinquet

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Experience Rating Schemes for Fleets of Vehicles

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Abstract
This paper proposes bonus-malus systems for fleets of vehicles, by using the individual characteristics of both the vehicles and the carriers. Bonus-malus coefficients are computed from the history of claims or from the history of safety offences of the carriers and the drivers. The empirical results are derived from a data set obtained from the Société de l’Assurance Automobile du Québec, the public insurer for bodily injuries and the regulator of road safety.

Keywords: Stratiﬁed portfolios, credibility, vehicle, ﬂeet, accidents, safety oﬀences.

JEL Numbers: D80, C23, C35, G22.

Résumé

Mots clés : Portefeuille stratiﬁé, crédibilité, véhicule, ﬂotte, accidents, infractions au code de la sécurité routière.

Classification JEL: D80, C23, C35, G22.

1 Introduction
This paper stems from a study carried out for the Société de l’Assurance Automobile du Québec, later referred as the SAAQ (see Dionne, Desjardins, Pinquet (1999, 2000)). Its objective is to provide Bonus-Malus Systems (BMS) for ﬂeets of vehicles from the history of claims or from that of safety oﬀences.

Fleets of vehicles are owned by rms, which are commercial motor carriers in the SAAQ portfolio. A portfolio of insurance contracts subscribed by rm has a stratiﬁed structure, and the size of the stratum (the set of policies held by a given rm) is a key variable in risk analysis. The propensity to self-insurance increases with the size of
the stratum. Insurance contracts for fleets of vehicles often use stop-loss risk sharing schemes (see Marie-Jeanne (1994) for their properties as a function of the fleet size, and Teugels, Sundt (1991) for experience rating schemes on the aggregate loss). These rating structures are designed for large fleets, which is not the case on average for the portfolio analyzed in this article. Notice that, in general, fleet insurance business is offered mostly for fleets with little or medium size.

In our data set, the characteristics of each fleet are recorded by the SAAQ in real-time (see Section 2.2 for more details), and the tariff structures proposed in this article use the individual characteristics of both the vehicles and the carriers. The history of a vehicle should have a greater ability to predict the risk level of this vehicle than that of the other vehicles in the fleet. The basic issue in the statistical analysis of the portfolio is the assessment of these predictive abilities. Information on the drivers is not available in the data set, so a new vehicle can only be related to the fleet to which it belongs. Bonus-malus coefficients for the next period will then depend on an expected turnover for the vehicles of the fleet.

The experience rating schemes are based on models with hierarchical random effects (see Jewell (1975)). Two types of bonus-malus systems are analyzed. A bonus-malus system designed from the number of claims is presented in Section 3. The coverage is for bodily injuries. Bonus-malus coefficients are obtained from vehicle-specific and fleet-specific credibilities and from an expected turnover for the vehicles of the fleet. Compensations for bodily injuries are performed in Quebec within a pure no-fault framework (Devlin (1992); Boyer and Dionne (1987)), so it is difficult to use the history of claims in the rating structure, because standard BMS always have a "crime and punishment" flavor. Since 1992, the history of safety offenses is used in the tariff structure for private vehicles (see Dionne and Vanasse (1997b) for a related study). Experience rating schemes with this approach for fleets of vehicles are presented in Section 4.

The BMS designed in Section 3 is consistent with respect to the fleet-specific components, which is not the case when claims are replaced by safety offenses as in Section 4. However, the BMS based on safety offenses outperforms the one based on accidents after a year of experience with our data. The explanation of this somewhat surprising finding is the following. The frequency of offenses is fourteen times higher than that of claims with bodily injuries. Even if the BMS based on safety offenses is less efficient than the one based on accidents in the long run, the former system is closer to its limit in the short run, due to the higher frequency of safety offenses.

A short conclusion summarizes the main results and proposes some extensions to the models presented in this article.

2 Economic environment and data set

2.1 Economic environment

Let us precise first the context of the study. The Province of Quebec introduced a new Automobile Insurance Act in March, 1978 to govern accident compensation. The Government had two goals in mind in tabling this legislation - to provide a rapid and
reliable method for compensating all victims of bodily injuries, and to ensure better
control of the cost of car repairs and faster compensation for property damage.
Fault has been entirely eliminated for bodily injuries. Compensation is provided by a
compulsory and universal public plan. This plan is administered by a public corpora-
tion, the SAAQ. There is a maximum indemnity (which was estimated to compensate
the total loss of income of 85 per cent of the population in 1978) for disability and
death bene.ts. The indemnities for bodily injury are in lieu of all rights to sue for
bodily injuries or death, and no action is admitted before any court of justice.
The pricing procedure is very simple. The main sources of nancing are from drivers’
permits and automobile registration fees. Weight and type of vehicle driven are taken
into consideration for vehicles other than pleasure vehicles. Past driving experience is
taken into account since 1992 by using demerit points of the drivers.
So the SAAQ is a state insurer which provides motor insurance for bodily injuries in a
monopolistic situation. As a state company, the SAAQ is also involved in road safety
regulation. Consequently, it has a direct access to the information on individual safety
oxences. It was decided in 1992 to use such information for the pricing of private cars
insurance. Besides their ability of screening risks, experience rating schemes provide
incentives to careful driving. Indeed, the frequency of claims decreased by at least ve
per cent since the new regulation (see Dionne and Vanasse (1997) for more details).
The SAAQ also provides insurance for bodily injuries for teets of vehicles. This insur-
ance is also compulsory. Information is brought in real time for each vehicle, a situation
which is not often encountered in this market. In order to create road safety incentives,
the introduction of an experience rating scheme (as well as an a priori rating structure)
is under consideration, which motivated the present study. This type of insurance rat-
ing is easy to implement for the SAAQ since it has a direct access to all the necessary
data.

2.2 Data set

We created the data bank from the SAAQ .les. Since January 1991, the SAAQ has
been mandated to verify that commercial vehicles respect the laws and regulations
governing, for example, the vehicle load and size limits, etc. In addition, the SAAQ
was also given the mandate to verify the mechanical conformity of the vehicles.
In our working sample, the vehicles were observed during the years 1995 and 1996. The
duration of observation of a vehicle is the validity duration of its licence plate. The
weight of the vehicles has to be greater than 3000 kgs, hence teets of cars do not belong
to this sample. The portfolio contains 50746 teets and 124629 vehicles, and teets are
of small size on average. The size of the teet is measured in vehicle-years, which is the
sum of the validity durations. The other teet-speci.c rating factors are the age of the
.teet and its activity sector. The vehicle-speci.c rating factors are the weight, the type
of use, the type of fuel, the number of cylinders and the number of axles.
The initial .le is the .le of all registered motor carriers as of July 23, 1997. To be in
that .le a motor carrier must own or lease (long term) one or more eligible vehicles. For
administrative reasons, the motor carriers may commit safety violations before being
registered in the .le. A vehicle is eligible if
2 The vehicle has one of the following license plate category: public, private or school bus, general merchandise transport, bulk transport or commercial vehicles;

2 The license plate status is D for detain;

2 The type of the vehicle is a bus or a truck;

2 The vehicle status is active; and

2 The vehicle weight is greater than or equal to 3,000 kgs.

Figure 1 presents the different steps for the construction of the data set by linking SAAQ files. The MOTOR CARRIER file is continuously updated and new information erases previous ones. Over a period of time, a motor carrier may becomes inactive if:

2 All his eligible vehicles have no more valid plate;

2 He no longer detains a C.T.Q permit; or

2 Owner or renter (long term) becomes a short-term rental.

A sequential identification (ID) number, which identifies a motor carrier, has been extracted from the MOTOR CARRIER file. This number has been iterated from CLEDPA file that links old and new identification numbers. In addition, a motor carrier may be united to other motor carriers, in which case they are written in a .le name FUSION. Another reason for being included in the FUSION file is for administrative reasons when a motor carrier has more than one identification numbers. In this case, they are paired together. In order to identify motor carriers, we create a resulting identification number from all identification numbers extracted from MOTOR CARRIER, CLEDPA and FUSION files. This step has been necessary because, in 1996, a carrier could have up to 28 identification numbers.

From all the identification numbers a motor carrier may have, we matched information concerning all their eligible vehicles written in the REGISTRATION file, such as characteristics concerning their authorization to circulate written in the AUTHORIZATION file; characteristics of the vehicles written in the VEHICLE file, and characteristics of the license plate written in the PLATE file. Characteristics of safety violations committed by a motor carrier written in the MOTOR CARRIER VIOLATION file was also matched. From license plate number of all eligible vehicles, crashes and violations committed by drivers have been extracted from ACCIDENT and VIOLATION files. And, finally, from the vehicle number, characteristics concerning mechanical conformity of the vehicles has been extracted from the MECHANICAL file.

All information concerning vehicles and motor carriers were brought together to create a file for accident analyses. In the models, the unit of observation is a vehicle with at least one day with a valid license plate in 1996 and where dates of creation and dissolution of the motor carrier agreed with those of the authorization to circulate. This yields up to
143,744 vehicles (trucks and buses) associated to 52,662 motor carriers. For this article, we consider only the 124,629 trucks associated to 50,746 motor carriers. In considering the safety offenses committed in 1995 in the analyses, 24,581 trucks with no day with a valid license plate in 1995 have been dropped from the data set.

3 Bonus-malus system from the number of claims

3.1 Bonus-malus coefficients as functions of the size of the fleet: Two limit examples

On a stratified portfolio, fixed and random effects introduced to design an optimal BMS must have a hierarchical structure (Jewell (1975)). The risk distribution of each vehicle includes then a vehicle-specific effect and a fleet-specific effect. Let us compute bonus-malus coefficients in two limit situations:

1 Only the vehicle-specific effect is retained. The history of a vehicle cannot be used to predict the risk levels of the other vehicles in the fleet. If all the vehicles have the same a priori frequency risk, the credibility computed at the fleet level is the one given to each vehicle. As the variance of the ratio between the number of claims and the frequency premium decreases towards 0 when the size of the fleet goes to infinity, the same result holds for the variance of the bonus-malus coefficient.

2 Only the fleet-specific effect is included in the number of claims distribution. Denote \( m \) as the number of vehicles in a given fleet, \( n_i \) as the number of claims reported by the vehicle \( i \) and \( \gamma \) as the a priori frequency risk for all the vehicles. We then have

\[
N_i \sim P \left( \gamma, u \right) \quad \forall i = 1, \ldots, m \quad N_i \sim P \left( m, u \right);
\]

if the \( N_i \) are independent in the fixed effects model (the fixed effect common to the vehicles in the fleet is denoted as \( u \)). If we write \( E(U) = 1; \ V(U) = \frac{3}{2} \) in the random effects model, the credibility granted to the fleet in the prediction is equal to

\[
\gamma = \frac{m \gamma^2}{1 + m \gamma^2}.
\]

This credibility increases towards one when the size \( m \) goes to infinity, and the bonus-malus coefficient converges towards the fleet-specific fixed effect \( u \). The variance of the bonus-malus coefficient increases with the size of the fleets. The variance of the bonus-malus coefficients is not a monotonic function of the size of the fleets. The
increase of risk revelation with the size of the fleet is balanced by risk compensation between the vehicles.

3.2 Estimation of a model with random effects on a stratified portfolio

The hierarchical nature of the portfolio is taken into account by a double indexation. The fleets are indexed by \( f = 1; \ldots; F; \) and the vehicles are indexed by \( i = 1; \ldots; m_f; \) where \( m_f \) is the size of the fleet \( f \). If \( N_{fi} \) is the number of claims reported by the vehicle \( i \) in the fleet \( f \); we write

\[
N_{fi} > P(.; f; u_{fi}) \; ; \; f = 1; \ldots; F; \; i = 1; \ldots; m_f
\]

in the fixed effects model. The parameter \( , f; i \) is a function of rating factors observed at the fleet level or at the vehicle level. The fixed effect \( u_{fi} \) represents the residual heterogeneity in the number of claims distribution. We distinguish fleet-specific and vehicle specific effects in the regression and heterogeneity components, and write

\[
, f; i = d_f \exp(x_f^o + z_f; i); \; u_{fi} = r_f s_f;
\]  

(1)

The parameter \( , f; i \) is proportional to the duration of observation of the vehicle \( d_f \). The line-vectors \( x_f \) and \( z_f; i \) are the regression components connected to the fleet and to the vehicle. The related parameters are represented by the column-vectors \( ^o \) and \( ^e \). The fixed effect \( u_{fi} \) splits into a fleet-specific effect \( r_f \) and a vehicle-specific effect \( s_f; i \). Vehicle-specific heterogeneity components could reflect the behaviour of the drivers, if a given vehicle is used by few drivers. Fleet owners may obey (or not) to safety rules related to the mechanical check-up of vehicles, bulk trucking regulation, driving and work hour rules, etc. The nancial structure of the carrier (which is not recorded by the SAAQ) probably in uences safety activities, and hence the risk level. Economic and empirical results on the relationship between the nancial structure of air carriers and safety are given by Dionne et al. (1997).

The preceding distributions hold for real individuals, and the variables \( (N_{fi})_f = 1; \ldots; F; i = 1; \ldots; m_f \) are supposed to be independent in the fixed effects model. The random effects \( (R_f)_f = 1; \ldots; F \) and \( (S_f)_i = 1; \ldots; F; i = 1; \ldots; m_f \) are i.i.d. in each family and mutually independent. If \( R \) and \( S \) are random variables with these distributions, we suppose that

\[
E(R) = E(S) = 1; \; V(R) = V_{RR}; \; V(S) = V_{SS};
\]

The random effects model deals with generic individuals, defined conditionally on the regression components. Within a semiparametric approach, the distributions on the random effects will only be speci ed by the variances. If \( U = RS; \) we have

\[
E(U) = E(R)E(S) = 1; \; V(U) = V_{UU} = E(R^2)E(S^2) \; ; \; 1 = V_{RR} + V_{SS} + V_{RR}V_{SS};
\]

With the total variance and covariance formula, we obtain

\[
V(N_{fi}) = , f; i + , f; i^2 V(U_{fi}) = , f; i + , f; i^2 V_{UU};
\]

\[
Cov(N_{fi}; N_{fi'}) = , f; i , f; i' Cov(U_{fi}; U_{fi'}) = , f; i , f; i' V_{RR} (i \neq i');
\]

(2)
in the random effects model. As the size of the portfolio is large, we will use a frequentist approach, and will describe the data by consistent estimators.

Let $c_{fi} = d_{fi} \exp(x_{fi} b + z_{fi})$ be the frequency premium computed in the a priori rating model, where $b$ and $z_{fi}$ are the maximum likelihood estimators. If data are generated in the random effects model, we have (Pinquet (1999))

$$c_{fi} \sim E(N_{fi}) = \mu_{fi} E(U) = \mu_{fi};$$

The expectation is computed in the random effects model. From the moments computed in (2), we obtain the following limits

$$V_{RR} = \sum_{f} \sum_{i} \sum_{i \neq i^0} \left( \frac{n_{fi} c_{fi} \mu_{fi} - n_{f0i} \mu_{f0i}}{n_{fi} - c_{fi} \mu_{fi}} \right);$$

$$V_{UU} = \sum_{f} \sum_{i} \sum_{i \neq i^0} \left( \frac{n_{fi} c_{fi} \mu_{fi}^2 - n_{f0i} \mu_{f0i}^2}{n_{fi} - c_{fi} \mu_{fi}} \right);$$

Thus consistent estimators of $V(U)$ and $V(R)$ are obtained from the estimators derived in the a priori model. Since $V_{UU} = V_{RR} + V_{SS} + V_{RR} V_{SS}$;

$$V_{SS} = \frac{V_{UU} V_{RR}}{1 + V_{RR}};$$

is a consistent estimator of $V_{SS}$.

Let us interpret these results. The estimator $V_{RR}$ assesses observed contagion between the claims histories connected to different vehicles within the same fleet. If $V_{RR}$ is greater than zero, the positive observed contagion means that the history of a vehicle can reveal hidden features in the risk distributions of every vehicle in the same fleet.

The numerator of the ratio which defines the estimator $V_{RR}$ is easily derived from

$$V_{SS} > 0, \quad V_{UU} > V_{RR};$$

if we write $n_{fi} = \sum_{i} n_{fi}; c_{fi} = \sum_{i} c_{fi}$. We then have

$$\sum_{f} \sum_{i} \sum_{i \neq i^0} \left( \frac{n_{fi} c_{fi} \mu_{fi} - n_{f0i} \mu_{f0i}}{n_{fi} - c_{fi} \mu_{fi}} \right) > \sum_{f} \sum_{i} \sum_{i \neq i^0} \left( \frac{n_{fi} c_{fi} \mu_{fi}^2 - n_{f0i} \mu_{f0i}^2}{n_{fi} - c_{fi} \mu_{fi}} \right);$$

if we write $n_{fi} = \sum_{i} n_{fi}; c_{fi} = \sum_{i} c_{fi}$. We then have
The estimated variance of the vehicle-specific random effect is greater than zero if the relative overdispersion derived at the vehicle level is greater than its counterpart computed at the fleet level.

3.3 Linear credibility predictors

In this section, we compute linear credibility predictors for each vehicle. They are derived from the history of claims observed at the fleet level, whereas the credibility coefficient depends on the vehicle. Let $i_0$ be a vehicle which belongs to the fleet $f_0$. The portfolio is observed during one period, and a bonus-malus coefficient is computed for the next one. In order to allow for a turnover in the portfolio, this vehicle may appear at the second period. Predictors are obtained separately for each fleet, and the fleet index is suppressed in order to simplify the notations. The fleet is supposed to contain $m$ vehicles during the first period.

The bonus-malus coefficient for the vehicle $i_0$ is supposed to depend only on the number of claims reported on the whole fleet. It is written as $b_{a_{i_0}} + b_{b_{i_0}} \sum_{i=1}^{m} n_i$.

The estimated expectation is derived in the random effects model. Notice that no specific weight is given to the history of the vehicle. As $E(U_{i_0}) = 1$, we have $\hat{a}_{X_n} = \frac{\sum_{i=1}^{m} n_i}{m}$, $\hat{a}_{b_{i_0}} = \frac{\sum_{i=1}^{m} b_i}{m}$, $\hat{a}_{b_{i_0}} = \frac{\sum_{i=1}^{m} b_i}{m}$.

Consistent estimators for the individual moments are

$$\text{Cov}(U_{i_0}; N_i) = \frac{\sum_{i=1}^{m} \text{Cov}(U_{i_0}; U_i)}{m} \hat{b}_{i_0} \hat{b}_{i_0} \hat{b}_{i_0} \hat{b}_{i_0} \hat{b}_{i_0}$$

$$\hat{b}_{i_0} = \frac{\sum_{i=1}^{m} \text{Cov}(U_{i_0}; U_i)}{m}$$

with the estimators obtained in the preceding section. In the computation of the credibility coefficient, two situations may happen:
Either the vehicle was not observed during the first period ($i_0 \in \mathbb{N}$ with $i_0 = 1; \ldots; m$).
From the estimations obtained in (5), we have
\[
\text{cred}_{i_0} = \gamma_0 + \bar{\gamma}_0 = \frac{\sum_{i=1}^{m} b_i \text{VV}_R^i \text{PR}_m \text{PP}_m}{1 + \sum_{i=1}^{m} b_i \text{VV}_R^i} + \frac{\sum_{i=1}^{m} b_i \text{VV}_U^i \text{PP}_m}{1 + \sum_{i=1}^{m} b_i \text{VV}_U^i} \bar{\gamma}_0.
\]
This fleet-specific credibility coefficient roughly increases with the estimated variance of the fleet-specific random effect and with the frequency-premium computed at the fleet level.

Or the vehicle was observed during the first period ($1 \leq i \leq m$). Then
\[
\text{cred}_{i_0} = \gamma_0 + \bar{\gamma}_0 = \frac{\sum_{i=1}^{m} b_i \text{VV}_R^i \text{PR}_m \text{PP}_m}{1 + \sum_{i=1}^{m} b_i \text{VV}_R^i} + \frac{\sum_{i=1}^{m} b_i \text{VV}_U^i \text{PP}_m}{1 + \sum_{i=1}^{m} b_i \text{VV}_U^i} \bar{\gamma}_0.
\]

The credibility coefficient is the sum of the fleet-specific coefficient and of a vehicle-specific coefficient. It can be computed only if the estimated variance of the vehicle-specific effect $\text{VV}_S$ is greater than zero (which amounts to $\text{VV}_U > \text{VV}_R$ from (4)), a condition fulfilled in our data.

Fleets are open in most cases, which means that an endorsement is not brought to the insurance policy after each arrival or departure of a vehicle in the fleet. In this context, bonus-malus coefficients computed at the vehicle level may appear unrealistic. If $\frac{1}{2} \bar{\gamma}$ is the expected turnover for the vehicles of the fleet, a credibility equal to $\gamma_0 + \left( \bar{\gamma}_0 \right)$ can be retained at the fleet level, where $\bar{\gamma}_0$ is the average of the $\gamma_i$.

3.4 Empirical results

Table 1 presents the results of a Poisson model which explains the number of claims reported in 1996 by regression components derived from the rating factors discussed above. The only continuous rating factor is the age of the firm. We observe that the frequency of claims decreases - ceteris paribus - by 3.4% with a supplementary year of age. The other rating factors have a finite number of categories.

In Table 1, the vehicles are weighted by the risk exposure measured by the number of days the vehicle is authorized to circulate. The estimated exponential of the coefficients (written in a multiplicative way) related to the different levels of each rating factor are averaged to one (column ST. COFF., for standardized coefficient). Two advantages are obtained.

2 The coefficients do not depend on the category that must be omitted in the regression for each rating factor in order to avoid collinearity.

2 These coefficients can be compared to the relative frequency of each category, which is the frequency of claims for one category divided by the global frequency, column REL. FRE. in Table 1. Consider for instance the category “bulk transport” of the rating factor “firm's activity sector”. The relative frequency is 1.617,
whereas the standardized coefficient derived from the Poisson model equals 1.146. From the likelihood equations of the Poisson model, the number of claims equals the sum of the frequency premiums for each level. The ratio $1.617/1.146 = 1.411$ means that the vehicles belonging to this type of fleet have, with respect to other rating factors, a frequency risk level which is 41% higher than the average.

Table 1 also provides levels of significance for the coefficients estimated in the regression. The P-VALUE column is obtained from a studentized statistic (i.e. the ratio between the estimated coefficient and its estimated standard deviation). For each rating factor, the reference group is related to the level which was suppressed in order to avoid colinearity.
### TABLE 1
**RATING SCORE FOR THE FREQUENCY OF CLAIMS WITH BODILY INJURIES**

<table>
<thead>
<tr>
<th>VARIABLE: FIRM’S ACTIVITY SECTOR</th>
<th>WEIGHT (%)</th>
<th>REL.FRE.</th>
<th>ST.COFF.</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>general merchandise transport</td>
<td>13.7</td>
<td>1.508</td>
<td>1.233</td>
<td>0.011</td>
</tr>
<tr>
<td>bulk transport</td>
<td>10.9</td>
<td>1.617</td>
<td>1.146</td>
<td>0.079</td>
</tr>
<tr>
<td>short term rental</td>
<td>2.5</td>
<td>0.959</td>
<td>0.840</td>
<td>0.501</td>
</tr>
<tr>
<td>independent trucker, other sector</td>
<td>72.9</td>
<td>0.813</td>
<td>0.940</td>
<td>ref. group</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE: VEHICLES-YEARS</th>
<th>WEIGHT (%)</th>
<th>REL.FRE.</th>
<th>ST.COFF.</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 or 1 vehicle-year</td>
<td>31.8</td>
<td>0.758</td>
<td>0.803</td>
<td>ref. group</td>
</tr>
<tr>
<td>2 vehicle-years</td>
<td>11.9</td>
<td>0.887</td>
<td>0.920</td>
<td>0.145</td>
</tr>
<tr>
<td>3 vehicle-years</td>
<td>7.2</td>
<td>1.032</td>
<td>1.055</td>
<td>0.010</td>
</tr>
<tr>
<td>4 to 9 vehicle-years</td>
<td>17.1</td>
<td>1.111</td>
<td>1.083</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>10 to 20 vehicle-years</td>
<td>9.6</td>
<td>1.292</td>
<td>1.177</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>more than 20 vehicle-years</td>
<td>22.4</td>
<td>1.183</td>
<td>1.164</td>
<td>&lt;0.001</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE: TYPE OF FUEL</th>
<th>WEIGHT (%)</th>
<th>REL.FRE.</th>
<th>ST.COFF.</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gasoline</td>
<td>20.4</td>
<td>0.430</td>
<td>0.597</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>fuel oil</td>
<td>79.6</td>
<td>1.147</td>
<td>1.104</td>
<td>ref. group</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE: WEIGHT OF THE VEHICLE</th>
<th>WEIGHT (%)</th>
<th>REL.FRE.</th>
<th>ST.COFF.</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>from 3000 to 3870 kgs</td>
<td>20</td>
<td>0.624</td>
<td>0.718</td>
<td>0.014</td>
</tr>
<tr>
<td>from 3871 to 6220 kgs</td>
<td>20</td>
<td>0.674</td>
<td>0.888</td>
<td>0.025</td>
</tr>
<tr>
<td>from 6221 to 7620 kgs</td>
<td>20</td>
<td>1.174</td>
<td>1.108</td>
<td>0.982</td>
</tr>
<tr>
<td>from 7621 to 8850 kgs</td>
<td>20</td>
<td>1.428</td>
<td>1.174</td>
<td>0.479</td>
</tr>
<tr>
<td>more than 8850 kgs</td>
<td>20</td>
<td>1.099</td>
<td>1.110</td>
<td>ref. group</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE: TYPE OF USE</th>
<th>WEIGHT (%)</th>
<th>REL.FRE.</th>
<th>ST.COFF.</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>commercial use</td>
<td>75.8</td>
<td>0.809</td>
<td>0.969</td>
<td>0.508</td>
</tr>
<tr>
<td>bulk transport</td>
<td>10.4</td>
<td>1.724</td>
<td>1.351</td>
<td>0.005</td>
</tr>
<tr>
<td>other types of transport</td>
<td>13.8</td>
<td>1.501</td>
<td>0.904</td>
<td>ref. group</td>
</tr>
</tbody>
</table>
The frequency of claims increases with the size of the fleet. This result could be explained by a greater exposure to risk (as measured by annual mileage) for the vehicles belonging to large fleets. The same reason probably also explains why gasoline-powered vehicles are much less risky than fuel-powered ones.

If annual mileage was not observed for all vehicles, it was estimated for those which had a recent mechanical check-up (54,699 vehicles). The estimation of the rating model with this supplementary variable leads to the following results, with a level of significance equal to 10%.

² The fuel effect disappears.
² The size effect decreases, but remains significant.
² The ..rm activity sectors are not significant.
² The number of cylinders effect disappears.

Detailed results can be obtained in Desjardins, Dionne, Pinquet (1999).

On the sample, the estimators given in the preceding section are equal to

\[ V_{RR} = \frac{\sum_{i,j=1}^{n} \left( n_{fi,ij} - \bar{c}_{fi} \right) \left( n_{0i,ij} - \bar{d}_{f0,i} \right)}{\sum_{i,j=1}^{n} \bar{c}_{fi} \bar{d}_{f0,i}} = 0.153; \]

\[ V_{UU} = \frac{\sum_{i,j=1}^{n} \left( n_{fi,ij} - \bar{c}_{fi} \right)^2 \left( n_{0i,ij} - \bar{d}_{f0,i} \right) \bar{c}_{fi}^{2}}{\sum_{i,j=1}^{n} \bar{c}_{fi} \bar{d}_{f0,i}} = 1.121; \]

(7)
The variance of the vehicle-specific random effect is important. This suggests that most of the vehicles are used by few drivers. The history of a vehicle will have much more ability to predict the risk level of this vehicle than that of the other vehicles in the fleet. Bonus-malus coefficients are computed at the fleet level in Table 2, for the two limit values of the turnover. Credibilities of the histories and standard deviations of the bonus-malus coefficients are given for each size level retained in Table 1.

**Table 2**

**AVERAGE CREDIBILITIES FOR FLEETS AND VEHICLES**

**STANDARD DEVIATIONS OF BONUS-MALUS COEFFICIENTS AT THE FLEET LEVEL**

<table>
<thead>
<tr>
<th>Fleet size</th>
<th>0 or 1 vehicle-year</th>
<th>2 vehicle-years</th>
<th>3 vehicle-years</th>
<th>from 4 to 9 vehicle-years</th>
<th>from 10 to 20 vehicle-years</th>
<th>more than 20 vehicle-years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.003</td>
<td>0.006</td>
<td>0.009</td>
<td>0.019</td>
<td>0.048</td>
<td>0.245</td>
</tr>
<tr>
<td></td>
<td>0.019</td>
<td>0.026</td>
<td>0.030</td>
<td>0.041</td>
<td>0.072</td>
<td>0.262</td>
</tr>
<tr>
<td></td>
<td>0.020</td>
<td>0.030</td>
<td>0.037</td>
<td>0.053</td>
<td>0.083</td>
<td>0.189</td>
</tr>
<tr>
<td></td>
<td>0.136</td>
<td>0.126</td>
<td>0.122</td>
<td>0.116</td>
<td>0.129</td>
<td>0.203</td>
</tr>
</tbody>
</table>

The standard deviations of the bonus-malus coefficients are related to a between-fleets dispersion of these coefficients. All the averages computed in Table 2 are weighted by the frequency premiums of the fleets. Due to the important value of the variance of the vehicle-specific random effect, the credibility strongly depends on the turnover for fleets with little or medium size. The same result holds for the dispersion of the bonus-malus coefficients. As expected from the conclusion of Section 3.1, the standard deviation of the bonus-malus coefficients is not a monotonic function of the size of the fleet when the turnover is equal to zero.

**3.5 Experience rating with gamma distributions for the random effects (expected value principle)**

Bonus-malus coefficients obtained at the vehicle level from the approach retained in the preceding section have a very low within-fleets dispersion. This is due to the fact that the vehicle only influences these coefficients through the credibility given to the history of the fleet. The within-fleets dispersion of the bonus-malus coefficients, as measured by the standard deviation, is at most equal to three per cent of the total dispersion for the different size levels.

A prediction approach derived from an expected value principle (Lemaire (1985), Dionne and Vanasse (1989), Pinquet (1997)) does not constrain ex ante the shape of the bonus-malus coefficients. A greater within-fleets dispersion of these coefficients can be expected, as confirmed in Table 3 which contains between-fleets and total dispersions of bonus-malus coefficients computed at the vehicle level.
### TABLE 3
**TOTAL AND BETWEEN FleETs STANDARD DEVIATIONS OF BONUS-MALUS COEFFICIENTS**

<table>
<thead>
<tr>
<th>Fleet size</th>
<th>$\sigma_{\text{between}}$</th>
<th>$\sigma_{\text{total}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 or 1 vehicle-year</td>
<td>0.134</td>
<td>0.137</td>
</tr>
<tr>
<td>2 vehicle-years</td>
<td>0.122</td>
<td>0.150</td>
</tr>
<tr>
<td>3 vehicle-years</td>
<td>0.122</td>
<td>0.158</td>
</tr>
<tr>
<td>from 4 to 9 vehicle-years</td>
<td>0.117</td>
<td>0.174</td>
</tr>
<tr>
<td>from 10 to 20 vehicle-years</td>
<td>0.127</td>
<td>0.194</td>
</tr>
<tr>
<td>more than 20 vehicle-years</td>
<td>0.202</td>
<td>0.309</td>
</tr>
</tbody>
</table>

Bonus-malus coefficients were obtained with random effects drawn from gamma distributions, with the estimators obtained in (7). The maximum likelihood estimators of the parameters of the Poisson model are consistent estimators in the model with random effects (Gouriéroux et al. (1984)). The negative binomial model with random effects (Hausman et al. (1984)) provides a specification which is close to the preceding one (Pinquet (1999)). The turnover of the vehicles was supposed equal to zero in the computations. The between fleets dispersions of the bonus-malus coefficients are very close to those obtained in Table 2 for the same value of the turnover. This means that using only the history of the fleet in the prediction did not entail a loss of efficiency for bonus-malus coefficients computed at the fleet level.

### 4 Bonus-malus systems from the number of safety offences

#### 4.1 Safety offences used as regression components

Owing to the no-fault setting, the history of claims is not used by the SAAQ. Safety offences can be used to perform experience rating. In our database, safety offences of different types were recorded at the carrier level and at the driver level. Those which were recorded in 1995 are added here as regression components in the Poisson model estimated in Table 1. Hence the number of claims reported in 1996 is explained by rating factors and by the safety offences recorded the year before. Each estimated coefficient related to a given type of safety offence leads to a relative malus, if this coefficient is positive. The safety offences which did entail a malus are presented in Table 4.
We retained the vehicles with a positive duration of authorization for the licence plate during 1995 and 1996. Other safety offenses which were not retained by the model are the following: exceeding size limits, not respecting bulk trucking regulation, not respecting mechanical check-up rules, driving with a sanction, not stopping at a red light. Many of them are significant when we consider all types of road accidents (property damages and bodily injuries). See Dionne et al. (1999) for more details, including regression results related to the rating factors. An optimal bonus-malus system is designed in the next section from a model with random effects on two types of events, namely the claims and the safety offenses.

### 4.2 The model with random effects

Let $\text{INF}_{fi}$ be the number of safety offenses recorded on the vehicle $i$ belonging to the fleet $f$. We write

$$ \text{INF}_{fi} \sim P(\xi_{fi} t_{fi}) $$

where $\xi_{fi} = d_{fi} \exp(x_{fi} + z_{fi})$ is the component of $E(\text{INF}_{fi})$ which is explained by the duration of exposure to safety offenses and by both fleet-specific and vehicle-specific regression components, and where $t_{fi}$ is the .xed effect. The hierarchical structure of the portfolio is taken into account by writing $t_{fi} = p_{f} q_{i}$; where $p_{f}$ and $q_{i}$ are the fleet-specific and vehicle-specific .xed effects. All the number variables are supposed independent in the .xed effects model. Let $U; R; T$ and $P$ be random variables with the same joint distribution that any random vector such as $(U_{fi}; R_{f}; T_{fi}; P_{f})$ (we use the notations of Section 3.2). The assumption $E(U) = 1$ made in Section 3.2 is relaxed now, because explicit and joint distributions for the random effects lead to log-normal distributions, and the expectation depends then on the variance. Let $\xi_{fi} = d_{fi} \exp(x_{fi} + z_{fi})$ be the estimation of $E(\text{INF}_{fi})$ computed in the Poisson model without .xed or random effects. If data are generated in the random effects model, we have

$$ E(N_{fi}) = \xi_{fi} E(U); \xi_{fi} E(\text{INF}_{fi}) = \xi_{fi} E(T); \quad (8) $$
The expectation is computed in the random effects model. From (8) and results similar to those given in (2), we obtain the following limits

\[
\mathbb{V}_{UT}^1 = \frac{P_{f;i} \ N_{f;i} \ C_{f;i} \ (\inf_{i} \ g_{i})}{E(U)E(T)}
\]

\[
\mathbb{V}_{RP}^1 = \frac{P_{f;i} \ P_{1;i} \ m_{f;i} \ N_{f;i} \ C_{f;i} \ \inf_{i} \ g_{i}}{E(R)E(P)}
\]

\[
\mathbb{V}_{PP}^1 = \frac{P_{f;i} \ P_{1;i} \ m_{f;i} \ \inf_{i} \ g_{i}}{E(P)^2}
\]

\[
\mathbb{V}_{TT}^1 = \frac{P_{f;i} \ g_{i}^2}{E(T)^2}
\]

(9)

The superscript “1” is used for the preceding estimators because they are obtained at the first step of the Newton-Raphson algorithm of likelihood maximization, where the initial value is the m.l.e. for the a priori rating model. For instance, the estimator \(\mathbb{V}_{RP}^1\) reflects the predictive power that safety offences recorded on a given vehicle have on the risk level of every other vehicle in the same fleet. Not surprisingly, the fleet-specific credibility obtained in the next section will depend on this estimator.

### 4.3 Linear credibility predictors

An optimal BMS using both claims and safety offences would be more efficient than those designed in the preceding sections (see Pinquet (1998) for a comparison of short-term effects). We now consider the case where claims cannot be used and the frequency of claims is predicted from the history of safety violations only.

Let us compute the bonus-malus coefficient for the frequency of claims reported by the vehicle \(i_0\) belonging to the fleet \(f_0\): The fleet index is suppressed in order to simplify the expressions. The bonus-malus coefficient is written as \(a_{i_0} + \xi_{i_0} (\inf_{i_1} \ m_{f;i_1} \ g_{i_1})\); with

\[
(a_{i_0}; \xi_{i_0}) = \arg \min_{a, b} \mathbb{E} \left[ \sum_{i=1}^{m} \frac{U_{i_0} \ a_i \ b_i \ \inf_{i_1} \ m_{f;i_1} \ g_{i_1}}{E(U_{i_0})} \right]
\]

(10)

From computations similar to those performed in Section 3.3, we obtain the following bonus-malus coefficient

\[\text{bonmal}_{i_0} = 1 + \xi_{i_0} \ (\inf_{i_1} \ b_i)\]

\[\xi_{i_0} = \frac{\sum_{i=1}^{m} \ m_{f;i_1} \ (\inf_{i_1} \ m_{f;i_1} \ g_{i_1})}{\sum_{i=1}^{m} \ m_{f;i_1} \ (\inf_{i_1} \ g_{i_1})}\]
\[
\begin{align*}
\mathbf{\tilde{A}}^{\mathbf{U}_{i_0}} & \quad \mathbf{X}^n & \quad \mathbf{\tilde{A}}^{\mathbf{U}_{i_0}} & \quad \mathbf{X}^n \\
\mathbf{C}_{\mathbf{U}_{i_0}} & \quad \frac{\mathbf{E}(\mathbf{U}_{i_0})}{\mathbf{E}} & \quad \mathbf{I N F}_i = \mathbf{\mathbf{\varphi}_R}^1 \mathbf{X}^n & \quad \mathbf{3} \left( \mathbf{\varphi}_T^1 \mathbf{\varphi}_R^1 \right) \mathbf{E}_{i_0} \\
\end{align*}
\]

The last term must be suppressed if the vehicle \(i_0\) is not observed during the first period. Following the computations of Section 3.3, we obtain then

\[
\begin{align*}
\mathbf{\Psi}^{\mathbf{\Bbb{I}} \mathbf{N F}_i} = \mathbf{\Bbb{B}}_{i_0} + \mathbf{\mathbf{\varphi}_P}^1 \mathbf{\Bbb{B}}_{i_0} \mathbf{\Delta} + \left( \mathbf{\mathbf{\varphi}_T}^1 \mathbf{\mathbf{\varphi}_P}^1 \right)^{\mathbf{\Bbb{B}}_{i_0}^2} \\
\end{align*}
\]

\[
\begin{align*}
\text{bonmal}_{i_0} &= (1 \cdot \text{cred}_{i_0}) + \text{cred}_{i_0} \frac{p_m \mathbf{E}_{i_0}}{p_i_{i=1}^m} \\
\text{cred}_{i_0} &= \circ(i_0 \cdot f_1; \ldots; mg) \cdot \text{cred}_{i_0} = \circ + \bar{i}_0 (i_0 \cdot f_1; \ldots; mg); \\
\end{align*}
\]

\[
\begin{align*}
\circ &= \frac{3}{1 + \mathbf{\mathbf{\varphi}_P}^1 \left( p_{i=1}^m \mathbf{B} \right)} \left( \mathbf{\mathbf{\varphi}_R}^1 \frac{p_m \mathbf{E}_{i_0}}{p_{i=1}^m \mathbf{B}} \right)^{\mathbf{\Bbb{B}}_{i_0}^2} \\
\bar{i}_0 &= \frac{3}{1 + \mathbf{\mathbf{\varphi}_P}^1 \left( p_{i=1}^m \mathbf{B} \right)} \left( \mathbf{\mathbf{\varphi}_T}^1 \mathbf{\mathbf{\varphi}_R}^1 \right)^{\mathbf{\Bbb{B}}_{i_0}^2} \\
\end{align*}
\]

The fleet-specific credibility coefficient \(\circ\) increases with \(\mathbf{\mathbf{\varphi}_R}^1\); a term related to the covariance between the two fleet-specific random effects. The coefficient \(\bar{i}_0\) is the vehicle-specific credibility. It makes sense only if \(\mathbf{\mathbf{\varphi}_T}^1 > \mathbf{\mathbf{\varphi}_R}^1\), a condition fulfilled in our data.

### 4.4 Empirical results

The frequency of claims with bodily injury reported in 1996 is predicted from the number of safety offenses recorded in 1995, and we retained the vehicles with a positive duration of authorization for the license plate during 1995 and 1996. The detailed results of the regression explaining the number of safety offenses recorded in 1995 are presented in Table 5. Let us quote two points.

1. The annual frequency of recorded offenses is equal to 22.2%. It is much superior to that of the claims with bodily injury liability. This will explain later the better short-term performance of the prediction designed in this section.

2. The frequency of offenses increases with the size of the fleet, but decreases for fleets with more than 20 vehicle-years.
### TABLE 5
RATING SCORE FOR THE FREQUENCY OF SAFETY OFFENCES

<table>
<thead>
<tr>
<th>VARIABLE: FIRM'S ACTIVITY SECTOR</th>
<th>WEIGHT (%)</th>
<th>REL. F.RE.</th>
<th>ST. COFF.</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>general merchandise transport</td>
<td>13.3</td>
<td>1.269</td>
<td>1.048</td>
<td>0.013</td>
</tr>
<tr>
<td>bulk transport</td>
<td>10.7</td>
<td>2.045</td>
<td>0.997</td>
<td>0.314</td>
</tr>
<tr>
<td>short term rental</td>
<td>2.5</td>
<td>1.297</td>
<td>1.742</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>independent trucker, other sector</td>
<td>73.5</td>
<td>0.789</td>
<td>0.967</td>
<td>ref. group</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE: VEHICLES-YEARS</th>
<th>WEIGHT (%)</th>
<th>REL. F.RE.</th>
<th>ST. COFF.</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 or 1 vehicle-year</td>
<td>32.7</td>
<td>0.953</td>
<td>1.055</td>
<td>ref. group</td>
</tr>
<tr>
<td>2 vehicle-years</td>
<td>11.4</td>
<td>1.022</td>
<td>1.119</td>
<td>0.008</td>
</tr>
<tr>
<td>3 vehicle-years</td>
<td>7.1</td>
<td>1.147</td>
<td>1.174</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>4 to 9 vehicle-years</td>
<td>17.4</td>
<td>1.262</td>
<td>1.210</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>10 to 20 vehicle-years</td>
<td>9.7</td>
<td>1.256</td>
<td>1.056</td>
<td>0.725</td>
</tr>
<tr>
<td>more than 20 vehicle-years</td>
<td>21.7</td>
<td>0.686</td>
<td>0.606</td>
<td>&lt;0.001</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE: TYPE OF FUEL</th>
<th>WEIGHT (%)</th>
<th>REL. F.RE.</th>
<th>ST. COFF.</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>gasoline</td>
<td>22.2</td>
<td>0.392</td>
<td>0.582</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>fuel oil</td>
<td>77.8</td>
<td>1.174</td>
<td>1.119</td>
<td>ref. group</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE: WEIGHT OF THE VEHICLE</th>
<th>WEIGHT (%)</th>
<th>REL. F.RE.</th>
<th>ST. COFF.</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>from 3000 to 3870 kgs</td>
<td>20.0</td>
<td>0.653</td>
<td>0.991</td>
<td>0.371</td>
</tr>
<tr>
<td>from 3871 to 6220 kgs</td>
<td>20.0</td>
<td>0.654</td>
<td>0.939</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>from 6221 to 7620 kgs</td>
<td>20.5</td>
<td>1.112</td>
<td>0.960</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>from 7621 to 8850 kgs</td>
<td>19.2</td>
<td>1.517</td>
<td>1.061</td>
<td>0.473</td>
</tr>
<tr>
<td>more than 8850 kgs</td>
<td>20.3</td>
<td>1.082</td>
<td>1.050</td>
<td>ref. group</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE: TYPE OF USE</th>
<th>WEIGHT (%)</th>
<th>REL. F.RE.</th>
<th>ST. COFF.</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>commercial use</td>
<td>76.0</td>
<td>0.776</td>
<td>0.927</td>
<td>0.644</td>
</tr>
<tr>
<td>bulk transport</td>
<td>10.2</td>
<td>2.356</td>
<td>1.667</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>other types of transport</td>
<td>13.8</td>
<td>1.234</td>
<td>0.911</td>
<td>ref. group</td>
</tr>
</tbody>
</table>
### VARIABLE: NUMBER OF AXLES

<table>
<thead>
<tr>
<th>WEIGHT (%)</th>
<th>REL.FRE.</th>
<th>ST.COFF.</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>unknown</td>
<td>2.3</td>
<td>0.998</td>
<td>1.119</td>
</tr>
<tr>
<td>2 axles, less than 4000 kgs</td>
<td>20.6</td>
<td>0.651</td>
<td>0.984</td>
</tr>
<tr>
<td>2 axles, more than 4000 kgs</td>
<td>27.6</td>
<td>0.600</td>
<td>0.751</td>
</tr>
<tr>
<td>3 axles</td>
<td>18.3</td>
<td>0.802</td>
<td>0.696</td>
</tr>
<tr>
<td>4 axles</td>
<td>5.7</td>
<td>0.893</td>
<td>0.856</td>
</tr>
<tr>
<td>5 axles</td>
<td>8.2</td>
<td>0.987</td>
<td>0.865</td>
</tr>
<tr>
<td>6 axles and more</td>
<td>17.3</td>
<td>2.305</td>
<td>1.836</td>
</tr>
</tbody>
</table>

### VARIABLE: NUMBER OF CYLINDERS

<table>
<thead>
<tr>
<th>WEIGHT (%)</th>
<th>REL.FRE.</th>
<th>ST.COFF.</th>
<th>P-VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 5 cylinders</td>
<td>1.4</td>
<td>0.714</td>
<td>0.834</td>
</tr>
<tr>
<td>6 to 7 cylinders</td>
<td>59.0</td>
<td>1.294</td>
<td>1.130</td>
</tr>
<tr>
<td>8 cylinders and more</td>
<td>39.6</td>
<td>0.572</td>
<td>0.812</td>
</tr>
</tbody>
</table>

Number of vehicles: 100,048

As for the random effects, the numerical values of the estimators are

\[
\nu_{UT} = 0.519; \quad \nu_{PP} = 0.465; \quad \nu_{RP} = 0.141; \quad \nu_{TT} = 1.263;
\]

These moment-based estimators can be connected to explicit distributions. If log-normal distributions are retained for the random effects, we can write

\[
R = \exp(a_1 N_1); \quad U_i = \exp(a_1 N_1 + a_2 N_i^2); \quad S_i = \exp(a_2 N_i^2);
\]

The fleet index is suppressed, and the random variables \(N_1; (N_i^2)_{i=1;\ldots;m}\) follow independent standard normal distributions. In the same way, we can write

\[
P = \exp(a_3 N_1 + a_4 N_3); \quad T_i = \exp(a_3 N_1 + a_4 N_3 + a_5 N_i^2 + a_6 N_i^4); \quad T_i = P Q_i;
\]

\[
Q_i = \exp(a_5 N_i^2 + a_6 N_i^4);
\]

with similar assumptions on the random variables \(N_3; (N_i^4)_{i=1;\ldots;m}\). It is easily seen that

\[
N \sim N(0; I_q) \quad \text{Cov}^{\mathbb{E}^t a N \mathbb{E}^t b N} = \exp^{\mathbb{E}^t a b} = 1.8a; b 2. R^q;
\]

The moment-based estimators are then connected with the following values

\[
a_1 = 0.381; \quad a_2 = 0.828; \quad a_3 = 0.346; \quad a_4 = 0.512; \quad a_5 = 0.346; \quad a_6 = 0.562;
\]

The predictor computed in this section cannot be consistent with respect to the fleet specific component, since the event for which the frequency is predicted is not retained in the history. When the size of the fleet \(m\) converges towards infinity, we have

\[
\lim_{m \to +1} \frac{\nu_{RP}^1}{\nu_{PP}^1} = 1.303 \quad B_i^0;
\]
The credibility coefficient \( \text{cred}_i \) is defined in (10). As we have the following limit
\[
\lim_{m \to +1} \frac{P}{E(P)} = \frac{1}{\sum_{i=1}^{\infty} \frac{P_i}{E(P)}} = \frac{1}{\sum_{i=1}^{\infty} \frac{1}{E(P)}}
\]
in the random effects model, the limit of the bonus-malus coefficient is
\[
\lim_{m \to +1} \text{bonmal}_i = \mu \left[ 1 + \frac{V_{P - P}^i}{V_{P - P}^i} \right] \left[ 1 + \frac{V_{R - P}^i}{V_{R - P}^i} \right]
\]
Hence, the bonus-malus system is not consistent (the limit should be \( R \equiv E(R) \) for a consistent predictor). The limit is the estimated affine regression of \( R \equiv E(R) \) with respect to \( P \equiv E(P) \).
Although this bonus-malus system is less efficient in the long run than the one based on the number of claims, it is more efficient after one year, as shown in Table 6.

**Table 6**

**AVERAGE CREDIBILITIES FOR FLEETS AND VEHICLES**

<table>
<thead>
<tr>
<th>Fleet size</th>
<th>( \text{cred}_{\mu} )</th>
<th>( \text{cred}_{\mu +} )</th>
<th>( \frac{1}{\text{bonmal}_{\mu}} )</th>
<th>( \frac{1}{\text{bonmal}_{\mu +}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 or 1 vehicle-year</td>
<td>0.027</td>
<td>0.094</td>
<td>0.064</td>
<td>0.216</td>
</tr>
<tr>
<td>2 vehicle-years</td>
<td>0.049</td>
<td>0.113</td>
<td>0.087</td>
<td>0.198</td>
</tr>
<tr>
<td>3 vehicle-years</td>
<td>0.070</td>
<td>0.132</td>
<td>0.107</td>
<td>0.196</td>
</tr>
<tr>
<td>from 4 to 9 vehicle-years</td>
<td>0.114</td>
<td>0.168</td>
<td>0.136</td>
<td>0.197</td>
</tr>
<tr>
<td>from 10 to 20 vehicle-years</td>
<td>0.175</td>
<td>0.209</td>
<td>0.186</td>
<td>0.222</td>
</tr>
<tr>
<td>more than 20 vehicle-years</td>
<td>0.242</td>
<td>0.252</td>
<td>0.220</td>
<td>0.235</td>
</tr>
</tbody>
</table>

Table 6 is obtained in the same way as Table 2. Standard deviations of bonus-malus coefficients are more important in this table for fleets with little or medium size. This BMS is less efficient in the long run than the one presented in Section 3, but it is closer to its limit, due to the higher frequency of safety offenses.

**5 Conclusion**

The objective of this paper was to propose bonus-malus systems for fleets of vehicles. The models were applied to fleets of trucks, but they could be used for other stratified portfolios if individual information on the insurance contracts was available.
Two systems were presented: one based on past accidents and the other based on past safety offenses. It was shown that the former system is more efficient in the long run, while the second is closer to its limit in the short run, a result explained by the higher frequency of safety offenses.
Many extensions of this article can be done. We plan to use information on many periods in order to build up a panel. This panel will be very useful to analyze the stability of the bonus-malus systems over time. It will also permit to verify for how long period the system based on safety offenses will dominate the one based on accidents. However such extensions will not be straightforward since we will have to introduce dynamic random effects in order to take into account the serial correlations.
References


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Figure 1: Links between the SAAQ files