Dynamics of Realized Volatility and Correlations: An Empirical Study Using Interest Rate Spread Options

René Ferland[1] and Simon Lalancette[2][3]

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[1]Associate Professor, Department of Mathematics, University of Quebec in Montreal. PO Box 8888, Downtown Station, Montreal, H3C 3P8. Research supported in part by NSERC grant 105740-98.

[2]Hydro-Québec Professor of Finance and Financial Engineering, Finance Department, HEC-School of Business in Montreal. 3000 Chemin de la Cote-Ste-Catherine, Montreal, H3T 2A7. Financial support for this research is provided by Hydro-Québec.

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Abstract

This study empirically examines the competitiveness of different forecasting sets of realized volatilities and correlations using linear and nonlinear specifications of time series based on high frequency data. The linear specification uses lagged explanatory variables to explain fractionally integrated series of realized volatilities and correlations. The nonlinear specification consists of a two-step approach. In the first step, joint time series of realized volatilities and correlations are filtered using a multivariate singular system analysis approach. Based on the cleaned series, vectors of nearest neighbors are identified in space and casted into a local linear regression to generate forecasts in the second step. The empirical performance of those specifications is compared to a GARCH diagonal-BEKK model in the context of a trader who would simultaneously quote a call spread option price based on the forecasted parameters and delta-hedge her position with a replicating portfolio. More traditional loss functions based on the absolute forecasting error are also used. The forecasting methodologies based on time series of realized volatilities and correlations generally (but not unanimously) dominate the GARCH approach. Evidence of nonlinearity seems apparent for time series of volatilities irrespective of the return sampling frequency. General performance ranking for the approaches based on realized volatilities and correlations is not robust to the chosen loss function and the return sampling frequency.

Keywords: Futures interest rate, high-frequency data, realized volatility, realized correlation, fractionally integrated linear model, multivariate singular spectrum analysis, local linear regression, GARCH diagonal-BEKK, nonlinear forecasting evaluation, vega-weighted forecasting error, replication error, pricing error.

1 Introduction

Volatility and correlation forecasts are fundamental statistical parameters for many finance practitioners. For instance, extensions from the seminal contribution of Markowitz (1959) suggest that the conditional variance-covariance matrix of asset returns plays a determinant role in wealth allocation and portfolio management issues.[1] In a similar fashion, these forecasts also play a central role in risk management insofar as they capture the shifts in financial variables required for setting risk limits.[2] Volatility and correlation forecasts are also critical to devise profitable option trade rules since they are the only unknown parameters amongst traders. Accordingly, profitable trades can emerge as the difference between the option sell (buy) quoted price which depends on the anticipated volatility over the option's life, and the costs (gains) to linearly replicate the position. This category of trades, often implemented by traders, generates profits proportional to the difference between the forecasted volatility and its realized value weighted by the gamma of the position [Baz *et al.* (2000) and Fitzgerald (1998)]. Traders responsible for exotic option books face an even more difficult challenge since the realization of profitable trades depends on the entire conditional variance-covariance matrix.

Most of the academic work produced in the area of volatility and correlation parametrization falls either within the categories of stochastic volatility models[3] or GARCH models [4]. However, the non-observability of the underlying true processes is such that these competing latent models are subject to misspecification. Andersen, Bollerslev, Diebold and Labys (2001a, henceforth ABDL) make a very important contribution on that issue. They resort to semi-martingale processes and demonstrate that ex-post estimators of volatility and covariance parameters can be obtained simply by considering the sum squares and cross-products of continuously recorded returns. These estimators display important attributes: when the sampling frequency tends to infinity, the volatility and covariance estimators become free of measurement error and are not influenced by any econometric or mathematical specification. This approach permits volatility and correlation forecasts to be generated from the empirical detection of the dynamics at work in the time series of realized volatilities and correlations. Based on high frequency currency returns, ADBL (2001a,b,c) and Andersen, Bollerslev, Diebold and Ebens (2000, henceforth ADBE) modelize times series of realized volatilities and correlations of stock and currency returns as fractionally integrated processes to capture the persistence in the time series. Forecasts follow from the estimation of a linear regression model with explanatory variables or from an AR specification.

The first goal of this study is to formally extend the work undertaken by ABDL (2001c) into a multivariate context so the issue of volatility *and* correlation forecasts is formally addressed. However, many market practitioners are as much concerned with forecast assessments as they are with the technology used to produce the forecasts. They must ensure that the characteristics of the approach employed to appreciate forecasts matches their preoccupations[5]. Further complexity is added when a multivariate framework is considered. The central loss function implemented in this study is inspired by option trading practices. In

[1] See, for instance, Huang and Litzenberger (1998) and Campbell, Lo and MacKinlay (1997) for formal treatment.

[2] For a broader perspective on risk management and related issues, see Alexander (1996) and Riskmetrics (1996).

[3] See Ghysels *et al.* (1996) for a review.

[4] See Bera and Higgins (1995) for a review.

[5] Diebold and Lopez (1996) provide an extensive and interesting survey on various forecasting evaluation approaches.

the same spirit, Noh *et al.* (1994) generate GARCH volatility forecasts and implied volatility forecasts to motivate daily buy or sell positions on option straddles. Gibson and Boyer (1998) address the multivariate case via the trading of one-day rainbow options. In both cases, better forecasts imply more profitable trading strategies. Our approach differs from Gibson and Boyer (1998) and adopts the point of view of a trader acting as a market maker of spread options. It is assumed that when the trader engages in a buy or sell trade of the call spread option, she simultaneously sets a replication portfolio to cover the risk. This is performed through the linear replication ('delta-hedging') of the option to delta-neutralize her position until the option reaches expiration. If the trader is capable of providing an option price quote based on volatility and correlation forecasts that exactly match their realized counterparts over the option life, the position should not generate any profit or loss if delta-hedging is performed at a reasonably high frequency. Any discrepancy between the two sets of parameters leads to nonzero profit whose magnitude is governed by the difference between the forecasted and realized parameters and by the price path followed by the underlying assets over the life of the option. The numerical approach introduced by Pearson (1995) is used for that purpose. Overall, three reasons motivate this forecast evaluation approach. First, it places forecast evaluation into the context of a practical financial situation. Second, spread option premia and delta-hedging implementation depend on the entire variance-covariance matrix and constitute a natural multivariate criterion. Finally, this criterion is a nonlinear function of the second moment. Thus, performance ranking might not be insensitive to the transition from a linear to a nonlinear loss function.[6]

To enrich the analysis, we also propose a vega-weighted absolute forecasting error measure. This criterion contrasts with the previous one because it is linear in the parameter distance, while each error component depends proportionally on the option premium sensitivity to shifts in volatilities or correlations.

The second goal of this study is to inquire whether superior volatility and correlation forecasts can be generated by a nonlinear specification of realized volatilities and correlations. While many stylized facts have been observed with respect to volatility such as volatility clustering, leverage effect and volatility persistence, realized volatilities are apparently well modelled by fractionally integrated processes. However, a broader view on this issue suggests that unknown nonlinear but potentially noisy dynamic processes drive the conditional variance-covariance matrix. To empirically examine this possibility, a two-stage methodology is proposed. In the first stage, the filtering methodology advocated by Lisi *et al.* (1994) and Lisi and Medio (1997) is implemented. Their method is structured around the fact that joint time series of two different variables may share the same nonlinear dynamics. The method is flexible and can accommodate different market phenomenon such as nonlinear volatility spillover effects. To eliminate the noise and to reconstruct clean series, they resort to orthogonal functions truncated at the point where the signal-to-noise ratio maintains a satisfactory level. In the second step, forecasting is performed by means of local linear regressions based on the reconstructed series of volatilities and correlations. Competing models to the nonlinear approach include a GARCH-diagonal BEKK model (Engle (2000)) and a naive estimator based on previous week realized estimates.

The remainder of the paper is organized as follows. Section 2 describes the data. In section 3, the different forecasting models and loss functions are presented; section 4 presents the empirical findings while section 5 offers a conclusion.

[6] See Christoffersen et Jacobs (2001) for related issues.

2 Data

The data set used in this study consists of two similar series of futures contracts on interest rates: the Eurodollar futures contract on the LIBOR 3-month interest rate (henceforth *Euro*) listed on the Chicago Mercantile Exchange and its Canadian counterpart, the BAX futures contracts on the 3-month banking acceptance interest rate listed on the Montreal Exchange. The underlying 'asset' of the *Euro* contract is an index whose value corresponds to 100 minus the LIBOR 3-month futures interest rate implied by the market. The contract is settled in cash and is linked to a notional value of $1,000,000. Thus, a fluctuation of one basis point in the rate results in a $25 variation in the contract. The contract is issued according to a quarterly expiry cycle based on the months of March, June, September and December up to ten years in advance. Although contracts with shorter maturities are available, market activity is mainly concentrated in the first two contracts of a quarterly cycle. The BAX contract possesses similar characteristics but the underlying interest rate is the 3-month Canadian bankers' acceptance rate. The BAX strip covers maturities up to three years. One of the original motivations of the BAX design was to exploit possible arbitrage opportunities with the *Euro*. Arbitrage attempts would, for instance, replicate the BAX position with appropriate combinations of *Euro* and foreign exchange forwards. To facilitate such trade, both series of contracts trade at similar times during the day[7].

Tick-by-tick data for the entire strip of *Euro* (BAX) are obtained from the Futures Industry Institute (Montreal Exchange), for the period January, 1992 to December, 1999. Futures returns are calculated as $r_{c,t} = \ln(i_{c,t}/i_{c,t-1})$ where c is either *Euro* or BAX while $i_{c,t}$ is the quoted futures interest rate at time t for contract c observed at the 30- and 90-minute sampling frequencies. To follow market activity, nearest maturity contracts series are created by calculating futures returns on the front month contract until the last trading day of the month before the expiration month of the contract is reached. At that point, the second front month contract becomes the lead contract. The procedure is repeated until December, 1999. The resulting *Euro* and BAX series of nearest maturity contracts are further trimmed to ensure that both sets of returns are available at each period t. Contiguous time series of weekly realized volatilities and correlations are then constructed according to equations (1) and (2) of the next section. The 30-minute return trading frequency and the weekly horizon emerge as a compromise to remove as much as possible microstructure effects and, on the other hand, to preserve an adequate level of precision with which the estimators can represent reasonable theoritical approximations. This choice is mainly motivated by ABDE (2000) and ABDL (2001b) where they respectively identify 20- and 30-minute as the optimal trading frequencies. Therefore, for each trading day, fourteen 30-minute returns are collected, and weekly estimates of realized volatilities and correlations comprise seventy 30-minute returns.[8] Weekly realized volatilities and correlations based on 90-minute returns are also computed to further assess the impact of measurement errors on volatility and correlation forecasting.[9] Along with the one-week-ahead forecast horizon, the out-of-sample period begins on January

[7]On a typical trading day during the time period considered herein, the average volume peak reaches 150,000 to 200,000 (10,000 to 15,000) contracts on the *Euro* (BAX) market. Trading volume may go beyond 1,000,000 (50,000) contracts.

[8]Some of the weeks included in the sample comprise only four days of trading. In this case, weekly realized volatilities and correlations are based on fifty-six 30-minute returns which is comparable to ABDL (2001b) who based their daily estimates on forty-eight 5-minute returns. Therefore, these cases are included in the time-series.

[9]An experiment drawn from ABDE (2000) was conducted. BAX and *Euro* monthly realized volatilities were calculated for three

1998 and ends on December 1999 for a total of 98 weeks.[10] The remaining in-sample period begins on January 1992 and ends on December 1997 for a total of 298 observations. The different sets of forecasts produced in section 4 are compared to realized volatilities and correlation calculated from the 30-minute sampling frequency since the theory suggests that those estimates are the closest to the true *Euro* and BAX volatilities and correlation. We use a rolling sample procedure to produce sets of one-week ahead forecasts.

3 Forecasting models and forecast evaluation

The main inputs are the time series of realized volatilities and correlations based on the BAX and *Euro*. The work of ABDL (2001a,b,c) and ABDE (2000), amongst others, indicates that the realized *Euro* and BAX volatilities at week t are given by:

$$\sigma_{c,t} = \left[\sum_{j=1}^{n_f} r_{c,t_j}^2 \right]^{1/2} \tag{1}$$

where $c = \text{BAX}, \textit{Euro}$; $f = 30, 90$; $n_f = 70, 14$ and the r_{c,t_j}'s are the intra-week returns. In a manner consistent with equation (1), the weekly realized correlation ρ_t is inferred from:

$$\rho_t = \frac{\left[\sum_{j=1}^{n_f} r_{\text{BAX},t_j} \cdot r_{\textit{Euro},t_j} \right]}{\left[\sum_{j=1}^{n_f} r_{\text{BAX},t_j}^2 \right]^{1/2} \left[\sum_{j=1}^{n_f} r_{\textit{Euro},t_j}^2 \right]^{1/2}} \,. \tag{2}$$

The central issues addressed in this paper consist of forecasting those quantities by different methods and assessing their performance mainly on financial grounds.

3.1 The linear model

The findings reported by ABDL (2001c), the volatility clustering effect retrieved by GARCH models and the analysis of French *et al.* (1987) and Hull and White (1987), amongst others, suggest that *Euro* and BAX lagged weekly realized volatilities are promising candidates for forecasting future realized volatilities. Furthermore, the presence of volatility spillovers must be considered given the findings of Fong *et al.* (1997) and Laopodis (2000) who observe volatility linkages in interest rate markets and the general perception entertained in the Canadian trading community that the *Euro* volatility significantly influences the BAX volatility. Thus, lagged *Euro* and BAX realized volatilities are introduced in both volatility equations. Research efforts by Chan (1992) and Koutmos (2000) demonstrate that interest rate volatility dynamics can also be driven by the level of interest rate. This is consistent with the theoritical contribution of Cox *et al.* (1985) where the volatility of the instantaneous interest rate differential depends on the level of the instantaneous interest

different months. For each month, volatility signature plots were constructed through a sequential calculation of the volatilities using sampling interval multiples of the 5-minute interval up to a daily interval. Generally, the volatilities begin to stabilize at the 30-minute sampling frequency while full stability is reached before the 90-minute sampling frequency. The phenomenon is particularly noteworthy for the BAX volatilities.

[10] Four weeks where only three trading days or less occurred were deleted from the out-of-sample period.

rate. Inspired by this conjecture, the volatility equations also include the corresponding futures interest rate observed at the beginning of each week.

Intuitively, the autoregressive nature of volatility dynamics should translate to covariance and correlation dynamics. Therefore, lagged realized correlations are included as explanatory variables in the correlation regression equation. Based on the findings of Solnik *et al.* (1996) whereby international correlations between stock markets tend to follow volatility cycles, lagged realized *Euro* and BAX volatilities are also included in the regression model. An alternative approach taken by Erb *et al.* (1994) relates correlations to business cycles. They observe that correlations typically decrease when economies are expanding. Perhaps, fluctuations in the weekly realized correlation between *Euro* and BAX returns reflect timing differences in the US and Canadian business cycles. To account for this possibility in a weekly setting (where most macro indicators are unavailable), the slope of the US and Canadian yield curves are considered.[11] Accordingly, weekly differences in the slopes of the US and Canadian yield curves are included in the regression model. For each market, the slope is calculated from the difference between the yields on a constant 10-year maturity government bond and the 3-month T-bill available on Bloomberg.[12]

The claims made in the previous paragraph, and the presumption that the time series of realized volatilities and correlations should be fractionally integrated, lead to the following system of linear equations:

$$(1-L)^{1-\hat{d}}\sigma_{\text{BAX},t} = \alpha_{11} + \sum_{i=1}^{m_1}\alpha_{1i}\sigma_{\text{BAX},t-i} + \sum_{j=1}^{m_1}\alpha_{1j}\sigma_{\textit{Euro},t-j} + \theta_1 i_{\text{BAX},t-1} + \varepsilon_{1t} \quad (3)$$

$$(1-L)^{\hat{d}}\sigma_{\textit{Euro},t} = \alpha_{21} + \sum_{i=1}^{m_1}\alpha_{2i}\sigma_{\text{BAX},t-i} + \sum_{j=1}^{m_1}\alpha_{2j}\sigma_{\textit{Euro},t-j} + \theta_2 i_{\textit{Euro},t-1} + \varepsilon_{2t} \quad (4)$$

$$(1-L)^{\hat{d}}\rho_t = \alpha_{31} + \sum_{i=1}^{m_2}\alpha_{3i}\rho_{t-i} + \sum_{i=1}^{m_3}\alpha_{2i}\sigma_{\text{BAX},t-i} + \sum_{j=1}^{m_3}\alpha_{3j}\sigma_{\textit{Euro},t-j} + \theta_3\Delta y_{t-1} + \varepsilon_{3t} \quad (5)$$

where $i_{\text{BAX},t-1}$ ($i_{\textit{Euro},t-1}$) and Δy_{t-1} respectively designate the BAX (*Euro*) implied interest rate and the difference between the US and Canadian yield curve slopes observed at the beginning of week t. The parameter d denotes the level of fractional integration. This parameter is obtained by running the regression test procedure of Geweke and Porter-Hudack (1983)[13]. This system of equations (3) to (5) contrasts with ABDL (2001c) who resort to a standard vector autoregressive approach. However, in an effort to make the system of equations as parsimonious as possible, equations (3) to (5) include only explanatory variables that can conceptually contribute to volatility and correlation forecasts. One and two period lagged realized volatilities and the lagged level of interest rate, and the one period lagged realized volatilities and correlations as well as the lagged US-Canada interest yield curve slope differential are respectively introduced in the volatility and correlation equations. Since the explanatory variables differ across equations and that ε_{1t}, ε_{2t}

[11]Many market participants believe that an inversion of the interest rate yield curve is a signal for an impending recession. Stock and Watson (1989, 1990a,b and 1993) and Lahiri and Jiazhuo (1996), amongst others, confirm the predictive power of the yield curve slope. The conditional estimation of asset pricing model often uses the slope of the yield curve as an instrumental variable to predict the expected return of assets. See, for instance, Kryzanowski *et al.* (1997) and Ferson and Schadt (1996).

[12]These series were kindly provided by the HEC-School of Business trading room.

[13]See also Robinson (1995, 1994).

and ε_{3t} might be correlated, the system is estimated via a seemingly unrelated regression procedure. The residual variances are adjusted following the procedure of Newey and West (1987).

3.2 The nonlinear model

In this approach the realized volatilities and correlations are considered to be generated by an unknown nonlinear dynamical system. Formally, it is assumed that there exists an unknown function f such that

$$x_t = f(x_{t-1}, \ldots, x_{t-M}, y_{t-1}, \ldots, y_{t-M'}) \tag{6}$$

where x_t is either $\sigma_{\text{BAX},t}$, $\sigma_{\textit{Euro},t}$ or ρ_t as calculated by equations (1) and (2) and where $y_{t-1}, \ldots, y_{t-M'}$ denotes the lagged values of an auxiliary variable generated from a similar system. It is possible, however, that the conditions under which equation (6) holds might not be observed in the presence of noisy time series. To circumvent this obstacle, Lisi *et al.* (1994) and Lisi and Medio (1997) propose to jointly filter x_t and y_t to eliminate the noise in the series.[14] In this setting, it is hoped that the forecasting model should yield better results.

The first step is to represent x_t and y_t into a double Hankel trajectory matrix[15] such that:

$$Z = \left[\begin{array}{cccc|cccc} x_1 & x_2 & \cdots & x_M & y_1 & y_2 & \cdots & y_M \\ x_2 & x_3 & \cdots & x_{M+1} & y_2 & y_3 & \cdots & y_{M+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{T-M+1} & x_{T-M} & \cdots & x_T & y_{T-M+1} & y_{T-M} & \cdots & y_T \end{array}\right]$$

Now let $N = T - M + 1$ and $m = 2M$. Using the singular value decomposition, the rectangular $N \times m$ matrix Z can be decomposed as

$$Z = S\Sigma C^T$$

where S is a $N \times m$ matrix whose columns s_j ($j = 1, \ldots, m$) are the eigenvectors of the symmetric matrix ZZ^T; C is a $m \times m$ matrix whose columns c_j ($j = 1, \ldots, m$) are the eigenvectors of the symmetric $m \times m$ matrix $Z^T Z$; Σ is an $m \times m$ diagonal matrix whose elements are the positive square roots of the eigenvalues of $Z^T Z$. Therefore, each row z_i of Z can be expressed as:

$$z_i = \sum_{r=1}^{m} \sigma_r s_{ir} c_r \tag{7}$$

for $i = 1, \ldots, T$. Insofar as the series are contaminated by noise, it is assumed that after a threshold d, the principal components are dominated by noise. Thus, the reconstructed noise-free series x_t^* and y_t^* are based on equation (7) where the expansion is truncated at term d with $d < m$. But, as one should expect, the reconstructed trajectory matrix W based on the d-truncated expansion may no longer be Hankel. Therefore,

[14] The filtering approach of Lisi and Medio (1987, 1984) is based on singular spectrum analysis (SSA) initially introduced by Broomhead and King (1986a,b) and Vautard and Ghil (1989). Lisi and Medio (1987, 1984) focus on the multivariate case.

[15] To simplify the notation in what follows, M' is taken equal to M although the method does not depend on this assumption.

one looks for the closest double Hankel matrix with respect to the matrix norm. In other words, the trajectory matrix corresponding to the noise-free series is the unique double Hankel matrix Z^* which minimizes

$$\sum_{i=1}^{T-M+1} \sum_{j=1}^{m} \left[Z^*_{ij} - W_{ij} \right]^2 \tag{8}$$

and is obtained by the so-called *diagonal averaging* of W. The reconstructed noise-free series x^*_t and y^*_t are then given by the formulas:

$$x^*_t = \frac{1}{t} \sum_{j=1}^{t} W_{t-j+1,j} \quad \text{and} \quad y^*_t = \frac{1}{t} \sum_{j=1}^{t} W_{t-j+1,j+M}$$

for $l \leq t \leq M-1$,

$$x^*_t = \frac{1}{M} \sum_{j=1}^{M} W_{t-j+1,j} \quad \text{and} \quad y^*_t = \frac{1}{M} \sum_{j=1}^{M} W_{t-j+1,j+M}$$

for $M \leq t \leq T-M+1$, and

$$x^*_t = \frac{1}{T-t+1} \sum_{j=t-T+M}^{M} W_{t-j+1,j} \quad \text{and} \quad y^*_t = \frac{1}{T-t+1} \sum_{j=t-T+M}^{M} W_{t-j+1,j+M}$$

for $T-M+2 \leq t \leq T$.

Once the reconstruction has been achieved, the next step is to forecast x_{T+1}. To reach that goal, nearest neighbors are identified and forecasting is performed by means of local linear regression. The idea of the nearest neighbors method is to identify, among the vectors $(x^*_j, x^*_{j+1}, \ldots, x^*_{j+M-1})$, the closest neighbors to $(x^*_{T-M+1}, x^*_{T-M+2}, \ldots, x^*_T)$ as they are anticipated to have successors x^*_{j+M} similar to x^*_{T+1}.

Once the local linear regression econometric specification that relates x^*_{j+M} to its predecessors is estimated, prediction of x^*_{T+1} results from a linear combination of x^*_{T-M+1}, x^*_{T-M+2},...,x^*_T. The forecast is further enriched by the presence of auxiliary vectors $(y^*_j, y^*_{j+1}, \ldots, y^*_{j+M-1})$ that contribute to the prediction of x^*_{T+1}. Overall, based on the k nearest $(x^*_j, x^*_{j+1}, \ldots, x^*_{j+M-1})$ vectors, the following local linear regression is performed:

$$\begin{aligned} x^*_{t(1)} &= \beta_o + \sum_{i=1}^{M} \beta_i x^*_{t(1)-i} + \sum_{i=1}^{M} \alpha_i y^*_{t(1)-i} + e_{t(1)} \\ &\vdots \\ x^*_{t(k)} &= \beta_o + \sum_{i=1}^{M} \beta_i x^*_{t(k)-i} + \sum_{i=1}^{M} \alpha_i y^*_{t(k)-i} + e_{t(k)} \end{aligned} \tag{9}$$

Evidently, because the nearest neighbors are identified in space, not in time, the system of equations (9) captures nonlinear dynamics.

One question remains: what are the best candidates for y^*_t? Market dynamics impose a natural choice when $x_t = \sigma_{\text{BAX},t}$ or ρ_t. In this case, $y^*_t = \sigma_{\mathit{Euro},t}$ naturally stands out. The significant impact of US markets over Canadian markets translates into a possible influence of the nonlinear dynamics exhibited by $\sigma_{\textit{Euro},t}$ on that of $\sigma_{\text{BAX},t}$ and ρ_t. When $x_t = \sigma_{\textit{Euro},t}$, the situation seems more complex. However, Rebonato (1998), amongst others, contends that interest rate yield curves embody a rich set of information that includes future market interest rate and volatility assessments. Since the interest level explains the largest proportion of yield curve movements (see Rebonato (1998) again), it seems a warranted choice for y_t^* when $x_t = \sigma_{\textit{Euro},t}$.

Since no guideline seems to exist for the optimal choice of d, M, M' and k, the nonlinear approach described by equations (7) to (9) displays an intensive data driven nature. The optimal set of d, M, M' and k minimizes the following mean absolute forecasting error measure (MAE):

$$\text{MAE} = \frac{1}{98}\sum_{t=1}^{98} |\hat{x}_t - x_t| \tag{10}$$

where x stands for σ_{BAX}, $\sigma_{\textit{Euro}}$ or ρ and the $\hat{x}_t$ denotes the forecast produced at time t. To ensure total independence between the estimation and forecasting steps, the in-sample period is split into rolling samples of 200 observations dedicated to the estimations of equations (7) to (9) for the production of 98 'pseudo' out-of-sample sets of volatilities and correlation forecasts over which the criterion (10) is applied. Based on the criterion (10), the optimal sets of d, M, M' and k parameters for the BAX and *Euro* realized volatilities and correlations at the 30-minute and 90-minute return sampling frequencies are: (7,3,5,170), (7,5,4,140), (5,3,4,40), (5,3,4,100), (6,4,3,170), and (7,5,3,90).

3.3 The multivariate GARCH model

Recognizing, as many market operators believe, that *Euro* and BAX contracts move together across time, the estimation of the entire conditional covariance matrix into one GARCH-type system of equations might be well advised. However, since full multivariate GARCH estimations raise many numerical difficulties, a diagonal BEKK representation was chosen (see, for instance, Engle (2000)). According to this specification, the conditional variance-covariance matrix is parametrized as:

$$H_t = V'V + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'H_{t-1}B \tag{11}$$

where

$$H_t = \begin{bmatrix} h_{\text{BAX},\text{BAX},t} & h_{\text{BAX},\textit{Euro},t} \\ h_{\textit{Euro},\text{BAX},t} & h_{\textit{Euro},\textit{Euro},t} \end{bmatrix}$$

is a 2×2 conditional covariance matrix, A and B are 2×2 diagonal parameter matrices, ε_t is the innovation at time t and the matrix $V'V$ is determined via a variance targeting procedure. The s-step-ahead forecast at time t of the conditional matrix, $E_t\left[\text{vech}(H_{t+s})\right]$, is given by:

$$V'V + A\text{vech}(\varepsilon_t\varepsilon_t') + B\text{vech}(H_t) \tag{12}$$

when $s = 1$, and by:

$$V'V\left[\sum_{i=0}^{s-1}(A+B)^i\right] + (A+B)^{s-1}E_t\left[\text{vech}(H_{t+1})\right] \tag{13}$$

when $s > 1$.

The bivariate system depicted by equation (11) is estimated using daily returns. Other frequencies are disregarded since Anderson *et al.* (2001) demonstrate that GARCH specifications are incapable of adequately capturing intra-day fluctuations.

The time index t in equation (11) denotes a daily frequency. Weekly estimates are obtained by adding daily estimates over the week. The weekly forecast at time t is given by:

$$\sum_{s=1}^{5} E_t \left[H_{t+s} \right] \tag{14}$$

from which *Euro* and BAX volatilities are directly extracted. The corresponding correlation forecast is:

$$\frac{\sum_{s=1}^{5} E_t \left[h_{\mathrm{BAX},Euro,t+s} \right]}{\sqrt{\sum_{s=1}^{5} E_t \left[h_{\mathrm{BAX},\mathrm{BAX},t+s} \right]} \sqrt{\sum_{s=1}^{5} E_t \left[h_{Euro,Euro,t+s} \right]}} . \tag{15}$$

3.4 Forecast evaluation

There exists a variety of loss functions available to compare competitive sets of forecasts. This lack of a unique reference point suggests that forecasting evaluation may not be robust to the chosen criterion. At the same time, it creates opportunities to use different evaluation methodologies based on financial foundations like derivative trading as in Noh *et al.* (1994) or Gibson and Boyer (1998). There are essentially two categories of derivative trading based on volatility. The first assumes that the trader has a view on the implied volatility and the expected realized volatility levels. The second focuses on the dynamic behavior of the implied volatility. The work of Noh *et al.* (1994) falls within the second category. In their contribution, future volatilities are provided by a GARCH and compared to the future implied volatility estimated from a linear regression model. Based on the forecasted direction of future volatility, buy or sell trades in one-day straddle are engaged using option market data. This approach cannot really be extended to exotic options based on the entire variance-covariance matrix insofar as market data are not available. The work of Gibson and Boyer (1998) belongs to the first category. They examine forecasting issues with respect to the entire variance-covariance matrix. In their artificial market, there exists a community of traders who take buy and sell positions amongst themselves on one-day rainbow options based on their respective variance-covariance matrix forecasts. The best forecasts should generate the largest profits.

Our approach belongs also to the first category and it is an attempt to add further realism to volatility-based option trading in the presence of volatility and correlation forecasts. The criterion adopts the view of an exotic option trader who quotes prices on a weekly at-the-money interest rate spread option using forecasted volatilities and correlations and who simultaneously implements and manages a replication portfolio using delta-hedging. Delta-hedging is performed in order to insulate the positions from underlying BAX and *Euro* movements. Basically, perfect accuracy in volatility and correlation forecasting should result in no trading profit. Since the motivation is the assessment of forecasting abilities, no suitable margin for the cost of hedging and for extra profits are added to the estimates.

The terminal payoff on a European call spread option with a one week maturity is based on the differential BAX-*Euro* against the strike differential:

$$\text{Payoff} = \max\Big\{(i_{\text{BAX}} - i_{\textit{Euro}}) - K, 0\Big\} \tag{16}$$

where i_{BAX} and $i_{\textit{Euro}}$ designate the BAX and *Euro* level at maturity while K is the at-the-money strike level[16]. While there are several types of exotic options sensitive to the entire variance-covariance matrix of underlying assets, our choice to focus on an interest-rate call spread option is mainly motivated by the depth of over-the-counter market for that type of option. Many dealers active in the US and Canadian interest rate markets can readily provide quotes on this instrument. The simultaneous quoting of an option price and implementation of the replication portfolio over the life of the option gives an absolute replication error for week t:

$$|e_t| = \Big|C_t(\hat{\sigma}_{\text{BAX},t}, \hat{\sigma}_{\textit{Euro},t}, \hat{\rho}_t) - \text{Payoff} + \text{Delta-hedging costs}\Big| \tag{17}$$

where C_t is the theoretical premium on the call spread option quoted at the beginning of the week t. This quote depends on the set of forecasts $\{\hat{\sigma}_{\text{BAX},t}, \hat{\sigma}_{\textit{Euro},t}, \hat{\rho}_t\}$ for that week. The delta-hedging cost is given by the gains and losses that result from the systematic rebalancing of the replication portfolio in order to match BAX and *Euro* positions with the option's deltas[17]. Now observe that the replication error can be disentangled as:

$$\begin{aligned} e_t = &\Big(C_t(\hat{\sigma}_{\text{BAX},t}, \hat{\sigma}_{\textit{Euro},t}, \hat{\rho}_t) - C_t(\sigma_{\text{BAX},t}, \sigma_{\textit{Euro},t}, \rho_t)\Big) \\ &+ \Big(C_t(\sigma_{\text{BAX},t}, \sigma_{\textit{Euro},t}, \rho_t) - \text{Payoff} + \text{Delta-hedging cost}\Big) \end{aligned}$$

that is[18]

$$\text{Replication error} = \text{Pricing error} + \text{Delta-hedging error} \tag{18}$$

While the pricing error is essentially a distance between two nonlinear functions, the delta-hedging error reflects the imperfections involved in the replication of the call spread payoff. Equation (17) shows that discrete rebalancing brings the identification of optimal forecasts beyond a simple pricing differential criterion and reveals that the trader's preferences for the optimal sets of forecasted of volatilities and correlation can be influenced by the delta-hedging errors.

To enrich the analysis, an absolute vega-weighted forecast error for each week of the out-of-sample period is given by[19]:

$$e_t = \mathcal{V}_{2t}|\hat{\sigma}_{\text{BAX},t} - \sigma_{\text{BAX},t}| + \mathcal{V}_{2t}|\hat{\sigma}_{\textit{Euro},t} - \sigma_{\textit{Euro},t}| + \mathcal{V}_{3t}|\hat{\rho}_t - \hat{\rho}_t| \tag{19}$$

---

[16] The call spread payoff is invariant to the side of the spread. A payoff given by $\max\{(i_{\textit{Euro}} - i_{\text{BAX}}) - K, 0\}$ where K is set at-the-money possesses the same properties.

[17] Details on the computation of premiums, deltas and vegas can be found in the Appendix.

[18] To conform with market practice with respect to the quotations of options on BAX and *Euro*, all components in equation (18) are sequentially multiplied by \$ 25 and 100.

[19] See Diebold and Lopez (1996) for a discussion on the traditional statistical procedures available for assessing the quality of forecasts.

where $\mathcal{V}_{it} = \partial C/\partial x_{it}$, $x_{1t} = \sigma_{\text{BAX},t}$, $x_{2t} = \sigma_{\textit{Euro},t}$, and $x_{3t} = \rho_t$. The vegas are standardized to yield $\sum_{i=1}^{3} \mathcal{V}_{it} = 1$. As opposed to the nonlinear criterion proposed in equation (16), this loss function is linear in the parameter distances. The goal pursued with this criterion is the natural behavior of a trader who would put more energy into forecasting the parameters that most affect the option premium. A more traditional evaluation approach is performed by applying descriptive statistics to the individual forecasting error terms of equation (19).

4 Empirical Findings

Descriptive statistics and a statistical evaluation of the time series properties of the realized volatilities and correlations calculated for the 30-minute and 90-minute sample frequencies using equations (1) and (2) are presented in Table 1. Two sampling frequencies are used to assess the impact of modifying the trade-off between the number of observations available for moment estimation and potential market microstructure effects. The summary statistics for the two sets of realized volatilities seem relatively robust to the sampling frequency. Such is not the case however for the correlation series. These claims are confirmed in Figure 1. Each graph respectively reports the 30-minute and 90-minute-based realized weekly volatilities and correlations. While the BAX and euro 30-minute and 90-minute estimates are virtually indistinguishable, the correlation estimates differ substantially. As expected, higher estimates of the mean and standard deviation are obtained for the BAX volatility, reflecting the general market consensus that the BAX market is more volatile than its *Euro* counterpart. In fact, the sample contains weeks where the BAX volatility exhibits values beyond 100%. The mean of the weekly correlations is positive, which is consistent with the closeness of these two markets. The skewness and excess kurtosis coefficients suggest that the densities of the estimates are not Gaussian with predominant fat tails in the case of the BAX volatility density. The next block of statistics shows the correlation matrices. As expected, the BAX and *Euro* volatilities tend to fluctuate in a correlated manner, but the correlation series fluctuate relatively independently[20].

One of the most distinctive features of ABDL (2001a,b,c) and ABDE (2001) is the parametrization of return volatility and correlation dynamics as fractionally integrated processes. The first eight autocorrelation coefficients and (to save space) the average autocorrelation coefficients for lags 10 to 24 shown in Table 1 raise the possibility that a similar characterization is taking place with this data set. The autocorrelations start around 0.5 and decrease progressively while the Ljung-Box statistics clearly reject the null hypothesis of white noise at the 1% level for 24 lags[21]. To formalize this intuition, the log-periodogram estimator d proposed by Geweke and Porter-Hudack (1983) is estimated by running this regression on each of the six times series:

$$\log(I(\omega_j)) = \alpha_0 + \alpha_1 \log(\omega_j) + \eta_j \tag{20}$$

where $I(\omega_j)$ denotes the sample periodogram at the j^{th} frequency, $\omega_j = 2\pi j/T$ with $j = 1, \ldots, m$, and where the GPH estimator corresponds to $\widehat{d} = -0.5\widehat{\alpha}_1$ with its standard error defined as $\pi(24m)^{-0.5}$ where

[20] This finding contrasts with some empirical evidence pointing out that high volatility regimes are associated with correlation coefficients closer to unity. See Karoyli and Stulz (1996), Solnik *et al.* (1996), and Longin and Solnik (2001) for a discussion.

[21] Similar significant results are obtained using 48 lags.

$m = T^{4/5}$ and T is the total number of observations. The $\hat{d}$ estimates reported in the second to last block in Table 1 are all significant and around 0.4 for the *Euro* volatility and correlation series. In contrast, $\hat{d}$ estimates for BAX are around 0.6. Again, the sampling frequency does not seem to substantially influence the estimates. To investigate whether a fractional difference filter can eliminate the bulk of autocorrelation in the series, the Ljung-Box statistics are calculated again for 24 lags by applying the filters $(1-L)^{\hat{d}}$ to the *Euro* volatility and correlation series and $(1-L)^{-\hat{d}}(1-L)$ to the BAX volatility series. These findings are reported at the bottom of Table 1. In spite of the rejection of the null hypothesis of white noise series in all cases, the magnitude of the Ljung-Box statistics indicates that the transformed series are left with little autocorrelation. ABDL (2001c) report similar results for currency data.

The in-sample results from the estimation of the GARCH diagonal-BEKK (equation (11)) based on daily returns and the system of linear equations (equations (3) to (5)) are reported in Table 2. Given the non-parametric nature of the nonlinear approach, in-sample intermediary analysis does not appear relevant. Furthermore, it is impossible for that model to provide similar comparative statistics as those presented in Table 2. The analysis focuses more on out-of-sample forecasting performance. Based on the magnitude of the individual gradients upon convergence, the GARCH diagonal-BEKK explains the data well. Estimated coefficients exhibit the typical behavior found in GARCH estimations whereby the autoregressive parameters are much closer to one than those of the squared noise components. The linear system appears to reasonably explain the data and is robust to the sampling frequency. There are many significant regression coefficients and the Lyung-Box statistics applied to the residuals of the regressions indicate that the null hypothesis of no serial correlation is not rejected for the vast majority of cases. Note that the empirical findings also emphasize a spillover effect from lagged Euro volatility to BAX volatility, which confirms the intuition of many Canadian traders on the influence of the US markets over Canadian markets. Interestingly, the fluctuations in the realized correlations seem mainly driven by the yield curve slope differential. Lagged correlation and volatilities marginally affect the current fractionally differenced correlation.

Summary statistics for the absolute forecast error (AFE) are reported in Table 3. This analysis is based on the individual terms of the loss function corresponding to equation (19) where the 30-minute frequency estimates form the benchmark of the analysis. The ability of the GARCH to produce forecasts differs across the parameters since the mean AFE is 0.5% for the *Euro*, 1.5% for the BAX and 0.188 for the correlation. ABDL (2001c) contend that the incorporation of intra-day information in the realized volatility and correlation time series is such that approaches based on this information should outperform GARCH modelizations. Contrasting findings to those of ABDL (2001c) are found for the linear model, since it outperforms (underperforms) the GARCH for the BAX (*Euro*) volatilities based on the mean AFE, irrespective of the sampling frequency. A two-tailed binomial t-test of the null hypothesis that the 50% probability of having a weekly AFE from the linear model (AFE-linear) different from the weekly AFE-GARCH, is implemented. Given that there are respectively 70 (13) and 67 (13) weeks for which the AFE-linear is lower under the 30-minute and 90-minute sampling frequencies for the BAX (*Euro*), the null hypothesis is rejected. It is interesting to note that although correlation forecasts lead to lower mean AFE-linears, the null hypothesis of 50% probability cannot be rejected at the 5% level since there are only 48 weeks where the weekly AFE-linear dominates the weekly AFE-GARCH. Nevertheless, adopting the point of view of call spread trader by focusing on the mean vega-weighted AFE, the findings indicate that the forecasts produced by the linear model are *collectively* more accurate. This is further supported by the rejection of the associated null hypothesis, since the number of weeks where the vega-weighted-AFE-linear dominates the vega-weighted-AFE-GARCH lies between 60 and 62 weeks.

Similar claims can be made when the performance of the nonlinear model is considered in comparison with the GARCH model. On the other hand, it seems that the nonlinear approach performs better than the linear one. Looking at the mean AFE, the nonlinear model seems to do a better job at forecasting the volatilities and producing correlation forecasts of a comparable quality at the 30-minute frequencies. Overall, a multivariate appreciation of the performance based on the mean vega-weighted AFE favors the nonlinear model for both sampling frequency. This is consistent with the rejection of the null hypothesis of 50% probability since there are respectively 67 weeks where the weekly vega-weighted AFE-nonlinear is inferior to the weekly vega-weighted AFE-linear. It is noteworthy that in the case of the correlation series, the null hypothesis cannot be rejected irrespective of the model considered in the analysis.

To make the empirical analysis more complete, forecasts based on previous-week realized estimates are considered. These forecasts yield interesting findings. The historical estimates based on a 90-minute return sampling frequency deliver the worst individual (collective), forecasts as indicated by the different mean AFE's (the mean vega-weighted AFE). In most cases, the null hypothesis is rejected and the number of weeks where the AFE-previous-week is lower to that of AFE-model, being systematically lower than 50%. In contrast, the previous-week estimates based on the 30-minute return sampling frequency appear relatively accurate. When the mean AFE and the mean vega-weighted AFE are considered, the previous-week estimates outperform the GARCH forecasts everywhere. In addition, more than 50% of the time, the previous-week estimates produce lower absolute weekly errors, leading to a rejection (non-rejection) of the null hypothesis in the case of the BAX volatility and vega-weighted (Euro and correlation) absolute errors[22]. This suggests that the nature of the information available is a critical aspect when attempts are made to capture the persistence nature of volatility.

The ability of the previous-week estimates to compete further extends to volatility forecasts for the 30-minute return sampling frequency. In short, these forecasts deliver a mean AFE for BAX and Euro volatilities and vega-weighted errors of order similar to that of the nonlinear model, although such is not the case for the correlation forecasts. When the binomial t-test is computed against the nonlinear model, the null hypothesis of 50% probability cannot be rejected in all cases.

Summary statistics for the absolute replication errors inherent in the different forecasting candidates sequentially utilized for dynamically replicating the call option spread are reported in Table 4. The errors are grouped under three different categories according to equation (18). Since the pricing and replication of the call option depend on BAX and *Euro* volatilities and correlations, this financial approach provides a natural multivariate criterion to globally evaluate forecasting abilities. The analysis begins with an examination of the absolute pricing error (APE) which captures the discrepancies observed when the option premium is calculated from the forecasted as opposed to the realized volatilities and correlations. The GARCH produces a mean APE of \$66.92 with a standard deviation of \$60.63 across the 98 weeks of the out-of-sample period. The linear-model-based forecasts of volatilities and correlations using either the 30-minute or 90-minute

[22]In a similar spirit, Poteshman (2000) observes that when the SPX realized volatility is regressed on a volatility forecast, the historical estimator based on 5-minute time intervals outperforms the GARCH(1,1) volatility.

sampling frequencies produce substantially lower mean APE's of $37.07 and $35.79, respectively. This is consistent with a rejection of the null hypothesis of the binomial t-test as there are respectively 78 and 76 weeks out of 98 where the APE-linear is inferior to the APE-GARCH. The nonlinear model performance does not appear robust to the choice of the loss function. While it outperforms the GARCH modelization, it produces mean APE's of $45.65 and $42.02, respectively. However, the null hypothesis of a 50% probability of observing the weekly APE-nonlinear being lower than the APE-linear cannot be rejected.

The historical one-week forecasts do quite well with mean APE's between $36 and $39. Somehow, the forecasting errors of this naive estimator behave in such a way that their collective impact seems diminished when filtered by a nonlinear loss function such as an option.

The statistics that describe the delta-hedging errors are presented in the last column of Table 4. The analysis focuses on the algebraic errors in order to address the ability of model (21)(presented in the Appendix) to replicate the call spread option and, accordingly, to acquire an understanding of their impact on the replication errors. As the theory indicates, the continuous rebalancing of the replication portfolio should eliminate any delta-hedging error if the option pricing model is satisfactory. Thus, these summary statistics strictly reflect the imperfections from discrete delta-hedging assumed to occur four times a day[23]. Since these errors are invariant with respect to the forecasting technology, the set of descriptive statistics is repeated several times in Table 4 only to facilitate the reading. The evidence reveals that the mean delta-hedging error deviates slightly from zero at a value of -$2.08. This mean is accompanied by a much larger standard error of $69.39. In addition, the delta-hedging error appears asymmetrical since the maximum gain and loss are respectively $81.30 and -$444.09. This skewness is probably affected by the asymmetrical nature of the terminal payoff value[24] and may impact on the weekly absolute replication error. Overall, the insights provided by the descriptive statistics and the empirical evidence displayed by the histogram in Figure 2, support the contention that model (21) provides an adequate description of BAX and *Euro* futures interest rates and a satisfactory replication of the option payoffs.

Finally, the second and third column of Table 4 show the descriptive statistics for the weekly absolute replication error (ARE). Recall that the ARE corresponds to the weekly gains or losses a trader would experience from simultaneously quoting a call spread option premium based on a set of volatility and correlation forecasts and creating a replication portfolio. Most importantly, equation (18) emphasizes the fact that performance ranking based on the APE and ARE may differ because of the influence of the delta-hedging error. To some extent, this is confirmed by the findings shown in Table 4. Irrespective of the forecasting methodology, all mean ARE's are higher than their APE counterparts. Based on this statistic and on the binomial t-test, the different modelization approaches based on time series of realized volatilities and correlations dominate the GARCH approach, although the performance ranking within the group is relatively robust to the inclusion of the absolute delta-hedging errors in the analysis. The mean ARE tends to favor the linear model as the preferred forecasting methodology. As far as the ARE is concerned, it must be underscored that the standard deviation parameter is as important as the mean. Consider a trader whose trading activities are regularly bounded by risk limits. A lower standard deviation of the errors suggests that the probability

[23] In practice, traders usually delta-hedge on an infrequent basis depending on market conditions.

[24] The estimated skewness coefficient of the delta-hedging errors with (without) the option terminal payoff value is -4.916 (-1.086).

of reaching those bounds through the realization of huges losses is less. This conjecture implies that some traders may be indifferent to the choice between the linear and the nonlinear models since they produce similar ARE standard deviations. Given this broader perspective, the previous-week estimates are unanimously dominated by that of the linear model. However, it should be emphasized that the linear and the nonlinear models generate virtually identical ARE standard deviations. Finally, the null hypothesis of 50% probability measured against the linear and the nonlinear models cannot be rejected in essentially all cases.

5 Concluding Remarks

The ability of different methodologies to exploit patterns in time series of weekly realized volatilities and correlations based on high-frequency returns for the BAX and *Euro* futures contracts was assessed using two performance criteria. The first criterion simply consists of an absolute forecasting error which is linear over the distance that separates the estimated and realized estimates. A multivariate version of this criterion weights each forecasting error by its corresponding call spread option vega. The second criterion adopts the point of view of an option trader who simultaneously quotes prices on a call spread option based on the forecasted set of volatilities, and correlations and dynamically replicates the option payoffs. As opposed to the first criterion, this second one is strongly nonlinear in the forecasts and provides a natural multivariate evaluation approach. Three approaches are used to model the time series of realized volatilities and correlations: a linear model where explanatory variables forecast the fractionally integrated series of volatilities and correlations, a nonlinear approach where jointly filtered series based on a multivariate singular system analysis of realized volatilities and correlations are casted into autoregressive linear local regressions, and a set of naive forecasts that consist of previous-week realized volatilities and correlations.

In a manner consistent with ABDL (2001c), the forecasts produced by the methodologies that resort to time series of weekly realized volatilities and correlations generally dominate those produced by a GARCH diagonal-BEKK. These findings are robust to the choice of a loss function and return sampling frequency. The findings do not necessarily discredit GARCH modeling but confirm the importance of accounting for the extra information contained in intra-day returns. GARCH modeling was limited to daily information whereby the noise contained in daily squared returns mitigate the performance of the model.

Performance ranking based on the absolute forecasting error demonstrates that no approach based on the realized volatilities and correlations unanimously dominates the other ones and sensitivity to the return sampling frequency is somewhat observed. Evidence of nonlinear dynamics in the volatility time series is observed and the nonlinear approach is favored with this performance criterion. Performance ranking based on the absolute replication errors are different. Forecast evaluation based on the distance between option premia using the forecasted parameters and those using the realized parameters appears robust to the inclusion of delta-hedging errors. With this criterion, the linear model delivers the lowest mean and standard deviation forecasting error. At the 30-minute sampling frequency, the forecasting performance of the naive forecasts is impressive irrespective of the evaluation criterion although the related ARE standard deviations are the highest amongst all approaches considered.

Finally, we are left with the impression that volatility and correlation forecasting are two different issues.

6 Appendix

Consider a call option whose payoff is dependent on the spread between the Eurodollar three-month futures rate and the BAX futures rate. At the expiration date T, the payoff of the spread call option is

$$C_T = \max\{\, i_{\textit{Euro},T} - i_{\mathrm{BAX},T} - K, 0\,\}$$

where K is the strike level.

In this study, the two rates are assumed to be log-normal and correlated. Under the risk-neutral measure the rates follow a bivariate geometric Brownian motion process:

$$\begin{bmatrix} di_{\mathrm{BAX},t} \\ di_{\textit{Euro},t} \end{bmatrix} = \begin{bmatrix} \sigma_{\mathrm{B}}\ i_{\mathrm{BAX},t} & 0 \\ \rho\sigma_{\mathrm{E}}\ i_{\textit{Euro},t} & \sqrt{1-\rho^2}\sigma_{\mathrm{E}}\ i_{\textit{Euro},t} \end{bmatrix} \begin{bmatrix} dW_1(t) \\ dW_2(t) \end{bmatrix} \tag{21}$$

and the value of the call spread option at the current date t is the double integral of the payoff over the risk-neutral joint distribution h of $i_{\mathrm{BAX},T}$ and $i_{\textit{Euro},T}$:

$$C_t = \int_0^\infty \int_0^\infty \max\{y - x - K, 0\} h(x,y) dy dx$$

where

$$h(x,y) = \frac{1}{2\pi|\Sigma|^{1/2}xy} \exp\left\{ -\frac{1}{2} \begin{bmatrix} \ln(x) - m_{\mathrm{BAX}} \\ \ln(y) - m_{\textit{Euro}} \end{bmatrix}' \Sigma^{-1} \begin{bmatrix} \ln(x) - m_{\mathrm{BAX}} \\ \ln(y) - m_{\textit{Euro}} \end{bmatrix} \right\}$$

and

$$\begin{aligned} m_{\mathrm{BAX}} &\equiv \ln(i_{\mathrm{BAX},t}) - \frac{\sigma_{\mathrm{B}}^2}{2}(T-t) \\ m_{\textit{Euro}} &\equiv \ln(i_{\textit{Euro},t}) - \frac{\sigma_{\mathrm{E}}^2}{2}(T-t) \\ \Sigma &\equiv \begin{bmatrix} \sigma_{\mathrm{B}}^2 & \rho\sigma_{\mathrm{B}}\sigma_{\mathrm{E}} \\ \rho\sigma_{\mathrm{B}}\sigma_{\mathrm{E}} & \sigma_{\mathrm{E}}^2 \end{bmatrix} (T-t)\,. \end{aligned}$$

To compute the integral, the approach of Pearson (1995) is followed. The first step is to factorize h as a product of the marginal density $f(x)$ of $i_{\mathrm{BAX},T}$ and the conditional density of $g(y \mid x)$ of $i_{\textit{Euro},T}$ given $i_{\mathrm{BAX},T}$:

$$C_t = \int_0^\infty \left[\int_0^\infty \max\{y - x - K, 0\}\, g(y \mid x)\, dy \right] f(x)\, dx$$

The inner integral may be computed and explicitly leads to the formula:

$$C_t = \int_0^\infty F(x) f(x)\, dx \tag{22}$$

where

$$F(x) \quad = \quad e^A i_{\textit{Euro},t} \left(\frac{x}{i_{\mathrm{BAX},t}} \right)^{\frac{\rho\sigma_{\mathrm{E}}}{\sigma_{\mathrm{B}}}} N(x_1) - (x+K)N(x_2)$$

$$
\begin{aligned}
A &= \rho\left(\frac{\sigma_{\mathrm{B}}}{2}\right)(\sigma_{\mathrm{E}} - \rho\sigma_{\mathrm{E}})(T-t) \\
x_1 &= \frac{M_2(x) + \sigma^2 - \ln(x+K)}{\sigma} \\
x_2 &= \frac{M_2(x) - \ln(x+K)}{\sigma} \\
M_2(x) &= m_{\mathit{Euro}} + \frac{\rho\sigma_{\mathrm{E}}}{\sigma_{\mathrm{B}}}(\ln(x) - m_{\mathrm{BAX}})
\end{aligned}
$$

and where $N(\cdot)$ is the cumulative standard normal distribution function.

The next step is to approximate F by a piecewise linear function. Since this function may be interpreted as the payoff of a portfolio of bonds, puts and calls, C_t can be arbitrarily closely approximated by the value of such a portfolio.

The approximation also applies to the computation of Greek letters. For example, to calculate the delta $\partial C_t / \partial i_{\mathit{Euro},t}$ one can pass differentiation through the integral in (22) and write

$$
\frac{\partial C_t}{\partial i_{\mathit{Euro},t}} = \int_0^\infty \frac{\partial [F(x) f(x)]}{\partial i_{\mathit{Euro},t}}\, dx = \int_0^\infty G_{\mathit{Euro}}(x) f(x)\, dx
$$

with

$$
G_{\mathit{Euro}}(x) = \frac{\partial [F(x) f(x)]}{\partial i_{\mathit{Euro},t}} \frac{1}{f(x)}\,.
$$

This expression has the same form as (22), with G_{Euro} replacing F. Therefore, to compute the delta, it is sufficient to apply the piecewise linear approximation to G_{Euro} instead of F. The same approach applies to the option vegas.

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TABLE 1
Descriptive Statistics and GPH Estimates for Time Series of Weekly Realized Volatilities and Correlations Based on 30-minute and 90-minute Sampling Frequencies.

	30-minute sampling frequency			90-minute sampling frequency		
	Vol *Euro*	Vol BAX	Corr	Vol *Euro*	Vol BAX	Corr
Mean	13.812	26.044	0.353	12.438	25.376	0.424
Std	8.175	16.974	0.247	8.001	17.770	0.286
Skew	1.791	3.296	-0.229	1.671	3.246	-0.557
Ex-kurtosis	2.254	12.908	-3.541	0.966	14.030	-3.247
Max	60.112	161.127	0.945	51.614	165.946	0.968
Min	3.228	4.575	-0.295	2.374	4.760	-0.509
	Correlation matrices					
	1.000			1.000		
	0.444	1.000		0.381	1.000	
	0.080	0.014	1.000	0.140	-0.082	1.000
Lag	Autocorrelation coefficients					
1	0.577	0.479	0.509	0.520	0.455	0.438
2	0.512	0.337	0.366	0.413	0.284	0.361
3	0.437	0.247	0.365	0.362	0.234	0.353
4	0.417	0.237	0.286	0.334	0.235	0.331
5	0.376	0.214	0.279	0.296	0.206	0.264
6	0.347	0.154	0.234	0.264	0.137	0.303
7	0.337	0.179	0.216	0.248	0.142	0.199
8	0.371	0.129	0.259	0.282	0.098	0.237
10-24	0.3574	0.0913	0.147	0.268	0.071	0.128
LB-24 lags	1613.100	524.168	523.381	1124.700	477.052	472.045
p-value	0.000	0.000	0.000	0.000	0.000	0.000
	Fractional integration					
d estimates	0.424	0.592	0.403	0.414	0.554	0.360
t-value	6.470	9.038	6.149	6.318	8.447	0.549
	Fractionally differenced series					
LB-24 lags	116.552	44.645	46.498	75.331	40.761	56.056
p-value	0.000	0.000	0.000	0.000	0.017	0.001

LB-24 lags and d estimates respectively refer to the Ljung-Box statistic for 24 lags and the GPH fractional integration parameter estimate.

TABLE 2
In-Sample Estimation Results for the GARCH diagonal-BEKK and the Linear Models.

	DIAGONAL-BEKK GARCH					
	a_1	a_2	b_1	b_2	$mean_1$	$mean_2$
Mean grad. ($\times 10^3$)	-0.245	-0.113	-0.320	-0.400	0.265	-0.021
Mean coeff.	0.441	0.329	0.812	0.919	-0.002	-0.000
Std.	0.077	0.071	0.093	0.036	0.000	0.000
# sign.(5%)	98	98	98	98	0	0

	LINEAR MODEL											
	BAX equation											
	30-minute sampling frequency						90-minute sampling frequency					
	Intercept	Vol BAX (1)	Vol *Euro* (1)	Vol BAX (2)	Vol *Euro* (2)	Level	Intercept	Vol BAX (1)	Vol *Euro* (1)	Vol BAX (2)	Vol *Euro* (2)	Level
Mean coeff.	9.458	-0.152	-0.115	-0.164	0.321	-0.776	9.166	-0.106	-0.185	-0.164	0.261	-0.642
Std.	0.781	0.027	0.181	0.034	0.054	0.493	0.924	0.025	0.172	0.044	0.060	0.479
# sign.(5%)	98	84	27	96	91	28	98	55	65	96	65	27
Residuals	Mean LB(12)		16.126	# of rej.(5%)		22	Mean LB(12)		16.671	# of rej.(5%)		26
	***Euro* equation**											
Mean coeff.	12.730	-0.019	-0.069	0.023	0.058	-1.424	12.834	-0.016	-0.038	0.014	0.023	-1.409
Std.	2.747	0.026	0.081	0.015	0.049	0.415	2.420	0.025	0.070	0.014	0.048	0.384
# sign.(5%)	97	0	33	0	32	92	97	0	27	0	9	90
Residuals	Mean LB(12)		5.986	# of rej.(5%)		0	Mean LB(12)		6.788	# of rej.(5%)		0
	Correlation equation											
	Intercept	Corr (1)	Vol BAX (1)	Vol *Euro* (1)	Diff. shape		Intercept	Corr (1)	Vol BAX(1)	Vol *Euro* (1)	Diff. shape	
Mean coeff.	0.181	-0.057	-0.000	-0.000	-0.030		0.165	-0.013	-0.000	-0.000	-0.287	
Std.	0.013	0.034	0.001	0.001	0.014		0.013	0.025	0.001	0.001	0.014	
# sign.(5%)	98	10	0	0	59		98	0	0	0	59	
Residuals	Mean LB(12)		11.090	# of rej.(5%)		0	Mean LB(12)		12.002	# of rej.(5%)		0

The mean coefficient estimates (Mean coeff.) across the 98 estimation periods, their standard deviations (Std.) and the number of significant coefficients at the 5% level (# sign. (5%)) for the GARCH diagonal-Bekk (equation (11)) based on daily returns and the system of linear equations (equations (3) to (5)) based on time series of realized volatilities and correlations using 30-minute and 90-minute returns are reported in this Table. Mean grad., Mean LB(12) and # of rej. (5%) respectively refer to the average gradients upon convergence of the likelihood function, the mean Ljung-Box statistics based on 12 lags and applied on the residuals of the linear regression equation and to the number of cases for which the null hypothesis of white noise is rejected at the 5% level. Vol BAX (x), Vol *Euro* (x), Corr (x), Level and Diff. shape respectively refer to the realized BAX and *Euro* volatilities and correlation lagged by x periods, to the interest rate level at the beginning of week t and to the US-Canada interest yield curve slope differential.

TABLE 3
Summary Statistics for the Absolute Forecasting Errors for the GARCH Diagonal-BEKK,
the Linear and Nonlinear Models, and Previous-Week Estimates.

	GARCH DIAGONAL-BEKK			
	Mean	Std.	Min.	Max.
Vol Bax	0.015	0.011	0.001	0.066
Vol *Euro*	0.005	0.005	0.001	0.028
Corr	0.188	0.174	0.859	0.859
Vega-weighted	0.016	0.013	0.002	0.093

	30-minute sampling frequency							90-minute sampling frequency						
	Mean	Std.	Min.	Max.	# of cases			Mean	Std.	Min.	Max.	# of cases		
LINEAR MODEL														
					< garch							<garch		
Vol BAX	0.011	0.011	0.001	0.082	70*			0.016	0.011	0.000	0.082	67*		
Vol *Euro*	0.010	0.006	0.000	0.028	13*			0.009	0.006	0.000	0.033	13*		
Corr	0.159	0.112	0.001	0.675	48			0.160	0.106	0.010	0.599	48		
Vega-weighted	0.014	0.015	0.003	0.110	60*			0.014	0.0141	0.003	0.109	62*		
NONLINEAR MODEL														
					< garch	< linear						< garch	< linear	
Vol BAX	0.009	0.009	0.000	0.071	80*	56		0.009	0.010	0.000	0.072	78*	60*	
Vol *Euro*	0.005	0.004	0.000	0.031	55	81*		0.004	0.004	0.000	0.031	64*	81*	
Corr	0.158	0.122	0.006	0.663	52	52		0.189	0.133	0.010	0.729	43	42	
Vega-weighted	0.011	0.012	0.007	0.100	77*	67*		0.011	0.013	0.000	0.101	76*	67*	
PREVIOUS-WEEK														
					< garch	< linear	< nonlinear					< garch	< linear	< nonlinear
Vol BAX	0.009	0.011	0.000	0.077	70*	62*	55	0.014	0.012	0.000	0.089	50	25*	27*
Vol *Euro*	0.005	0.005	0.001	0.039	50	79*	50	0.013	0.011	0.000	0.079	20	33*	25*
Corr	0.179	0.002	0.002	0.666	50	42	46	0.220	0.157	0.009	0.728	40	40	37*
Vega-weighted	0.011	0.013	0.001	0.088	75*	63*	51	0.017	0.013	0.004	0.011	41	22*	19*

The mean absolute forecasting error estimates (Mean), their standard deviations (Std.) and the minimum (Min.) and maximum (Max.) absolute forecasting errors across the 98 weeks of the out-of-sample period under the GARCH diagonal-BEKK, the linear and nonlinear models and the previous-week estimates are reported in this Table. < 'model' and vega-weighted respectively refer to the number of weeks out of 98 where the studied forecast generates an absolute forecasting error lower than that of 'model' and the vega-weighted absolute forecasting error calculated as in equation (19).

TABLE 4

Summary Statistics for the Absolute Replication, the Absolute Pricing and the Delta-Hedging Errors for the GARCH Diagonal-BEKK, the Linear and Nonlinear Models and Previous-Week Estimates.

	Absolute Replication Error		Absolute Pricing Error		Algebraic/Absolute Delta-Hedging Error
GARCH DIAGONAL-BEKK					
Mean	82.818		66.916		-2.076/28.726
Std.	76.34		60.627		69.392
Min.	0.66		0.5		-444.092
Max.	433.61		448		81.301
	Sampling frequency		Sampling frequency		
	30-minute	90-minute	30-minute	90-minute	
LINEAR MODEL					
Mean	56.732	55.468	37.068	35.786	-2.076/28.726
Std.	67.43	68.42	44.707	45.973	69.392
Min.	3.87	0.78	0.25	0.5	-444.092
Max.	425.09	423.34	380	397.5	81.301
# of cases $<$ garch	71*	68*	78*	76*	
NONLINEAR MODEL					
Mean	63.235	58.419	45.647	42.023	-2.076/28.726
Std.	68.419	64.025	45.050	46.930	60.392
Min.	1.712	1.454	1.25	0.500	-444.092
Max.	381.6	389.6	352.5	377.3	81.301
# of cases $<$ garch	68*	66*	73*	71*	
# of cases $<$ linear	41	52	37	42	
PREVIOUS-WEEK					
Mean	58.674	62.552	39.390	36.242	-2.076/28.726
Std.	80.01	83.25	56.461	62.236	60.292
Min.	0.44	0.16	1	0.25	-444.092
Max.	429.34	424.59	397.25	421.25	81.301
# of cases $<$ garch	69*	62*	78*	68*	
# of cases $<$ linear	51	42	47	41	
# of cases $<$ nonlinear	58	49	63*	50	

The mean absolute replication, pricing and delta-hedging error estimates (Mean), their standard deviations (Std.) and the minimum (Min.) and maximum (Max.) values across the 98 weeks of the out-of-sample period for the call spread option pricing and replication based on equation (18) under the GARCH diagonal-BEKK, the linear and nonlinear models and the previous-week estimates are reported in this Table. # of cases $<$ 'model' refers to the number of weeks that the studied approach generates an error lower than that of 'model'.

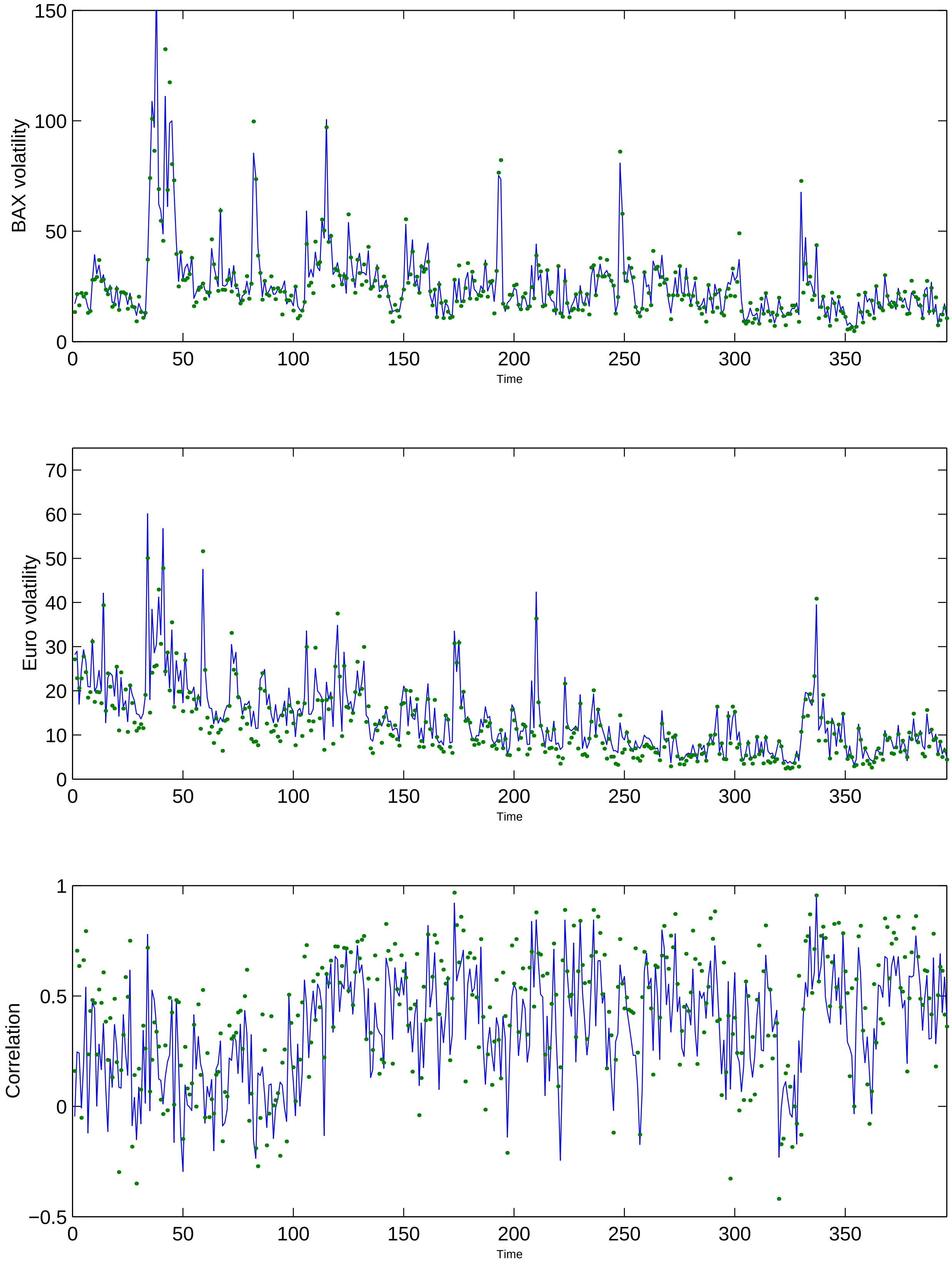

Figure 1: Realized volatilities and correlations based on 30-minute and 90-minute returns

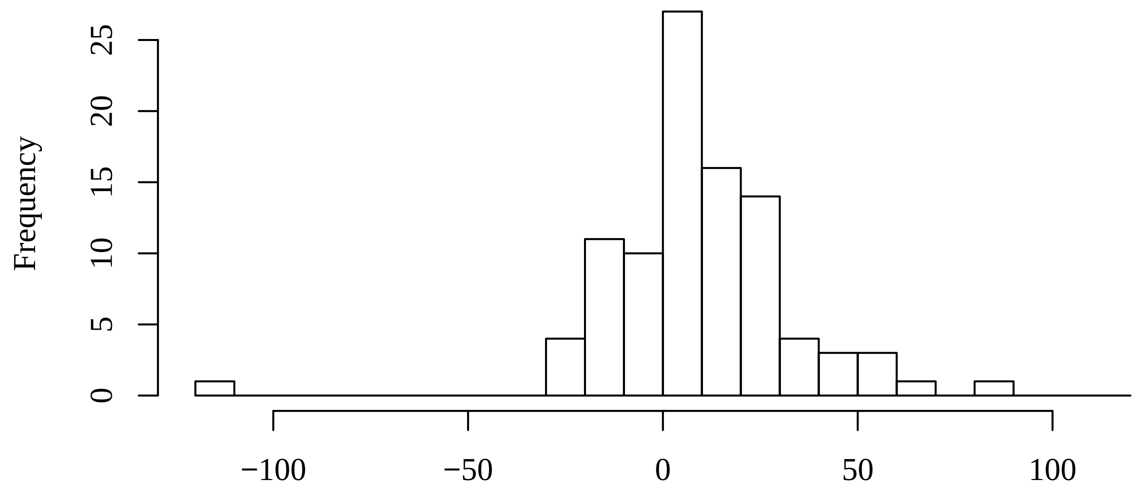

Figure 2: Histogram of Delta-Hedging Errors