Working paper 05-02
April 2005

# Testing Explanations of Preference Reversal: A Model

Yves Alarie* and Georges Dionne*
HEC Montréal

April 5, 2005

**Abstract**

When Cubitt, Munro and Starmer (2004) presented their new experimental investigation of preference reversal, they pointed out that their test results cannot be explained by any of the best-known explanations proposed by economists and psychologists. In this paper we propose a model based on lotteries qualities to explain these new test results.

*JEL classification*: D80.

*Keywords*: Standard preference reversal, counter preference reversal, choice task, money valuation task, probability valuation task, lottery, test.

* Canada Research Chair in Risk Management, HEC Montréal.
georges.dionne@hec.ca. Fax: (514) 340-5019.

## Introduction

When Cubitt, Munro and Starmer (CMS) (2004) presented their new experimental investigation of preference reversal, they pointed out that their test results cannot be explained by any of the best-known explanations proposed by economists and psychologists. In this paper we propose a model based on lotteries qualities to explain these new test results. CMS tests use pairs of simple monetary gambles. In each pair one bet (the P-bet) offers a relatively large chance of a modest prize, while the other (the $-bet) offers a smaller chance of a larger prize. The subjects make a straight choice between the P-bet and the $-bet. They also state a monetary valuation for each bet. A standard preference reversal (SPR) occurs when subjects choose the P-bet over the $-bet in the choice task but place a higher monetary value on the $-bet. A counter preference reversal (CPR) occurs when subjects choose the $-bet over the P-bet in the choice task but place a higher monetary value on the P-bet. The subjects are faced with three different tasks. Table 1 presents an example.

Table 1
Choice and Valuation Tasks

| | |
|---|---|
| *Choice task* | |
| Choose A or B: | |
| Option A: | You get L32.00 for numbers 1 to 31 |
| Option B: | You get L8.00 for numbers 1 to 97 |
| *Money Valuation Task (MV)* | |
| Set the missing amount so that the two options are equally attractive: | |
| Option A: | You get L??? for numbers 1 to 100 |
| Option B: | You get L18.00 for numbers 1 to 19 |
| *Probability Valuation Task (PV)* | |
| Set the missing probability so that the two options are equally attractive: | |
| Option A: | You get L10.00 for numbers 1 to ??? |
| Option B: | You get L18.00 for numbers 1 to 19 |

Both MV and PV groups perform the Choice task in a first step. The MV group then responds to the Money Valuation Task and the PV group, to the Probability Valuation Task. The most frequent preference reversals are the SPR (33.6%) for the MV group and the CPR (21.7%) for the PV group. For the MV group the proportion (3.3%) of CPR is negligible but for the PV group the proportion (19.5%) of SPR is close to that of CPR.

The first important result is the existence of standard preference reversals for the subjects who answer the MV task. This result rules out a large class of models including the expected utility model and the prospect theory model. There are, however, three psychological hypotheses capable of explaining this result: the prominence hypothesis (Tversky *et al*., 1988), the task goal hypothesis (Fisher *et al*., 1999), and the scale compatibility hypothesis (Tversky *et al*., 1988).

As to those performing the PV task, a surprising result is the observation of counter preference reversals. This result rules out the prominence hypothesis and the task goal hypothesis, leaving only the scale compatibility hypothesis as a possible explanation. However, the standard preference reversal is also observed among the PV group, and, as pointed out by CMS, this last result cannot be explained by the scale compatibility hypothesis. So there is no model in the literature that can account for all three test results. We now propose a single model that can capture all these results.

**Qualitative judgment of lotteries**

The model extends Alarie and Dionne (2004) and takes into account explicitly the qualities associated with the set of probabilities $p_i \in P$ and the set of monetary amounts $x_i \in X$ in order to obtain optimal processes and lottery evaluation. More specifically, as in Kahneman and Tversky (1979), we assume that probabilities have the surety (S) and risk (R) qualities. Prelec (1998) has proposed a $w(p_i)$ function that takes into account the qualitative difference between impossibility (I) and risk (R). So we define a partition of the set of probabilities P as $\{S,R,I\}$: $S = \{1\}$, $R = \{p_i / p_i \in ]0,1[\}$ and $I = \{0\}$. Alarie and Dionne (2004) add to the literature two new qualities (H and L) for probabilities that indicate whether a lottery has high chances of winning or not. In the partition $\{H,L\}$ of P, the elements of H have the high quality and those of L have the low quality: $L = \{ p_i / p_i \in [0, p^*[\}$ and $H =\{p_i / p_i \in [p^*,1]\}$. A value of $p^*$ — the fixed point of the inverse S-shape probability weighting function — which is larger than .3 but smaller than .5 is observed in many tests (see Prelec, 1998, for a discussion).

In the evaluation function, the model takes into account the qualities by using a parameter $\alpha \in ]0,\propto[$. The parameter $\alpha$ can multiply either lottery A or lottery B but one lottery must be selected. When $\alpha$ is equal to one the model is equivalent to the expected utility one with risk neutrality. The judgment functions $J(p_A,x_A,p_B,x_B)$ for the comparison of two lotteries where $p_A$ and $p_B$ belong to ]0,1[ are either:

$$\alpha_{HL}\, p_A x_A - p_B x_B \quad \text{if } p_A \in H \text{ and } p_B \in L \tag{1}$$

or

$$\alpha_{HH}\, p_A x_A - p_B x_B \quad \text{if } p_A,p_B \in H,\ p_A > p_B. \tag{2}$$

We observe in (1) that the higher probability $p_A$ is multiplied by the parameter $\alpha_{HL}$ since $p_A \in H$ and $p_B \in L$ while in (2) $\alpha = \alpha_{HH}$ since $p_A \in H$ and $p_B \in H$ and $p_A > p_B$. It should be noted that $\alpha_{HL} > 1$and $\alpha_{HH} > 1$ come from tests results in Tversky et al.

1990) and Kahneman and Tversky (1979). Indeed the lottery with the highest probability is selected.

(1) and (2) are used for the choice tasks and the PV task. For the MV task where $p_B = 1$ (pricing), there exist two judgments: one when $p_A \in L$ and the other when $p_A \in H$:

$$\alpha_{SR}\, p_A x_A - p_B x_B \quad \text{if } p_A \in H; \tag{3}$$

$$\alpha_{IR}\alpha_{SR}\, p_A x_A - p_B x_B \quad \text{if } p_A \in L. \tag{4}$$

It should be noted that $\alpha_{SR} < 1$and $\alpha_{IR} > 1$ come from the tests in Tversky and Kahneman (1992).

## Results

The probabilities and the monetary amounts of the P-bet are noted $p_P$ and $x_P$ and those of the \$-bet are $p_\$$ and $x_\$$.

### *Choice task*

For the choice task, by using (1) when $p_P \in H$ and $p_\$ \in L$, we obtain:

$$\alpha_{HL}\, p_P x_P - p_\$ x_\$ > 0 \tag{5}$$

because $\alpha_{HL} > 1$ for this type of choice which indicates that the quality H is preferred to the quality L. The P-bet is selected when the inequality (5) is respected. In the choice tasks that involve $p_\$ \in H$ and $p_P \in H$ the decision maker uses $\alpha_{HH} > 1$ instead of $\alpha_{HL}$ in (5) and the result remains the same.

***MV task***

For the MV task, the prices $CCE_\$$ and $CCE_p$ are the Choice Certainty Equivalents that correspond respectively to the \$-bet and the P-bet. By using (3) and (4), we obtain:

$$\alpha_{SR}\, p_P x_P = CCE_P \text{ when } p_P \in H \quad (6)$$

and

$$\alpha_{IR}\alpha_{SR}\, p_\$ x_\$ = CCE_\$ \text{ when } p_\$ \in L. \quad (7)$$

So since $\alpha_{IR} > 1$ the lottery with the low probability that was not selected in the choice task now has a higher $CCE_\$$ than the one with the high probability, provided both have the same mathematical expectation. This explains the standard preference reversal. One can note that the MV task is a choice certainty equivalent. Had we used a judged certainty equivalent, the evaluation of the second lottery would have been $\alpha_{IR}\, p_\$ x_\$$ and the number of reversals would have been larger (Tversky et al., 1990).

Since $p^* \in ].3,.5[$ it could happen that both probabilities .5 and .94 would belong to H for some tests (like Test 3 in CMS, 2004). So both lotteries are priced in the same way and we now have $\alpha_{SR}\, p_P x_P = CCE_P$ and $\alpha_{SR}\, p_\$ x_\$ = CCE_\$$. So a constant $\alpha_{SR}$, as in Alarie and Dionne (2004), is not sufficient to explain this test and a more refined model is needed. One avenue is to use non-constant parameters $\alpha_{IR}$ and $\alpha_{SR}$. For example, the distortion of the probabilities is larger near the points p=0 and p=1 for the Inverse S-shape probability function discussed in Tversky and Kahneman (1992) and in Prelec (1998). An interesting way to introduce non-constant $\alpha$ parameters is to consider that, when judged with boundary 1, the higher the probabilities in H the smaller the $\alpha_{SR} < 1$ parameter. Similarly, the lower the probabilities in L the larger the $\alpha_{IR} > 1$ parameter, when judged with boundary 0. So, for the pricing of lotteries, we obtain a parameter such as $\alpha_{SR}(p_H,1)$ when the probability belongs to H and a parameter such as $\alpha_{IR}(p_L,0)$ when the probability belongs to L. When both $\partial\alpha_{SR}(p_H,1)/\partial p_H$ and $\partial\alpha_{IR}(p_L,0)/\partial p_L$ are negative, the SPR can exist for the lotteries where both $p_P$ and $p_\$$ belong to H. For example $\alpha_{SR}(.94,1) < \alpha_{SR}(.5,1)$ and then $CCE_P < CCE_\$$. In conclusion, use of the non-constant parameters $\alpha_{SR}$ and $\alpha_{IR}$ explains the occurrence of the SPRs for the MV group. Non-constant parameters $\alpha$ do not change the conclusion for the cases where one probability is in H and the other is in L.

***PV task***

Based on the pricing of lotteries where test results show an Inverse S-shape curve and on the previous discussion, it is clear that the parameters $\alpha_{SR}$ and $\alpha_{IR}$ cannot be constant. So we could expect that the other parameters — $\alpha_{HH}$, $\alpha_{LL}$ and $\alpha_{HL}$ — not to be constant either. We now show that the tests in CMS confirm this hypothesis.

For the PV task the monetary amount used in the CMS study is \$10.00 and the decision maker must select a probability equivalent $PE_\$$ for the \$-bet and $PE_p$ for the p-bet. This leads by using (2) for the \$-lotteries to:

$$PE_\$ = p_\$ x_\$ / 10\alpha_{HH}(p_\$, PE_\$). \qquad (8)$$

For the P-lottery we have:

$$PE_P = \alpha_{HH}(p_P,PE_P)p_Px_P/10. \qquad (9)$$

One notes that for some tests we must use $\alpha_{HL}$ depending on the $p^* \in ].3,.5[$ we observed. The comparison of lotteries (equation (10) below) and the difference ($PE_P$-$PE_\$$) between the two probability equivalents (equation (11) below) are for the PV group:

$$\alpha_{HH}(p_P,p_\$)p_Px_P - p_\$x_\$ \qquad (10)$$

$$[\alpha_{HH}(p_P,P_{EP})p_Px_P - p_\$x_\$/\alpha_{HH}(p_\$,PE_\$)]/10. \qquad (11)$$

If the parameters are constant the model allows CPR. However, for the PV group, we also observe standard preference reversal which, in our model, calls for the use of an $\alpha$ parameter as a function of the probabilities. A necessary condition to obtain a SPR is:

$$\alpha_{HH}(p_P,p_\$) > \alpha_{HH}(p_P,PE_P)\alpha_{HH}(p_\$,PE_\$). \qquad (12)$$

Even though the CPR seems more likely to occur, the inequality (12) is quite possible. For most tests we have $p_P > PE_P > p_\$$ and $p_P > PE_\$ > p_\$$ and the difference between the probabilities to the left of (12) is larger than the ones to the right of (12). The tests in CMS seem to indicate that the parameter $\alpha$ must vary according to these differences in probabilities. So the subjects can exhibit CPR or SPR depending on their parameter $\alpha$.

## Conclusion

The model presented explains the occurrence of standard preference reversal for the MV group. For the PV group it explains the higher number of counter reversals and the presence of the standard reversals. These tests (PVs, MVs and the direct comparisons) confirm the need for non-constant parameters $\alpha$. Furthermore they give some clues as to how these parameters vary.

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