On the Determinants of the Implied Default Barrier*

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Abstract

Many structural models specify the default barrier, but few have explored its empirical significance and determinants. The effect of liquidity shortage is not well measured, nor is the effect of strategic default well identified. We use the maximum likelihood (ML) approach to estimate the default barrier model and the Merton-KMV model using market values of equities in a sample of 762 public industrial firms. The estimated barrier is below leverage in our sample. The default probability from the two structural models provides similar in-sample fits, but the default barrier framework achieves better out-of-sample forecasts. Our analysis also focuses on the factors that influence the level of the implied default barrier when leverage is endogenous, and shows that endogenous leverage is not the only determinant of the default barrier as predicted by the standard structural credit model. The implied default threshold is positively related to financing costs, and negatively related to liquidity, asset volatility, and firm size. Three strategic default variables (liquidation costs, renegotiation frictions and equity holders' bargaining power) increase the implied default barrier level. This evidence supports strategic default models.

Keywords: Barrier option; default barrier; bankruptcy prediction; maximum likelihood estimation; strategic default; liquidity shortage.

JEL classification: G32, G33.

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1. Introduction

One of the most important assumptions in structural models of credit risk is that the firm defaults when its value reaches a minimal threshold. This threshold is often called the default barrier or the default boundary. All the structural models in the credit risk literature make assumptions about the default barrier and calibrate the level of asset value below which the firm defaults. Most of the empirical tests of these models compare the credit risk premia generated by the structural models with those actually observed on the credit derivatives or debt contracts. Although numerous, these studies have paid little attention to the underlying assumptions regarding the level of the default barrier. One exception is Davydenko (2007), who studies whether default is triggered by low market asset value or by liquidity shortages.

Structural models often rely on parameters that are not directly observable. One example is the default barrier where the dynamics and the location are not visible. In this case, researchers must specify the default barrier based on indirect information. While many structural models specify the analytic default barrier, the relation between the asset value at default and firm-specific characteristics has rarely been explored empirically. A better understanding of the determinants of the default threshold could be valuable for modeling and predicting the decision to default. For instance, value-based endogenous structural models identify asset volatility as an input in a default threshold setting. In contrast, cash-based models underscore the importance of external financing costs and liquidity. The strategic default models predict a positive relationship between the barrier level and both variables: liquidation costs and the firm’s bargaining power. Other models focus on private debt holders’ ability to set the default threshold. Knowing which factors are significant in evaluating default probabilities is important for debt holders and financial institutions.

In this paper, we first revisit the estimation of the default barrier to provide empirical insights into the assumptions regarding default barriers provided in the structural model literature. Using a new sample of public companies, we compare the default prediction obtained from the Merton-KMV model, in which the default barrier is set as a given fraction of the firm's debt, with that generated by the Down-and-Out European Call option model (hereafter DOC option model) introduced by Brockman and Turtle (2003), in which the firm defaults whenever its asset value reaches the estimated default barrier. Our results show that the estimated default barrier is below leverage for our sample of public firms. We also find that our implementation of the Merton-KMV approach and the DOC option model have similar in-sample fits in explaining default occurrence. However, the DOC option model provides superior out-of-sample bankruptcy forecasts in our sample. For the subsample of defaulted firms, the estimated asset values with the DOC option model are much closer to the model-implied default barrier than those of the surviving firms. On average, the estimated default barrier accurately measures the value of the assets at the time of default.

We then proceed with the main contribution of this paper and perform a statistical analysis of the default barrier level on a set of firm characteristics. No contributions in the literature consider both liquidity problems and strategic factors to explain the variation in the implied default barrier in a model where leverage is endogenous. The results indicate that the DOC implied default barrier is affected not only by the level of endogenous leverage, but also by the liquidity of the firm and its debt cost. This underlines the importance of the liquidity shortage and external financing cost concerns. The implied default barrier location is also influenced by liquidation costs, renegotiation frictions, and equity holders' bargaining power, which supports the strategic default models.

The rest of the paper is organized as follows. Section 2 presents a brief review of the literature on structural models. Section 3 describes the methodology used to estimate the models' parameters and presents the data. Section 4 analyzes the results of the estimated barrier and compares the default prediction capacity of the DOC option model and the Merton-KMV model. Section 5 discusses the choice of explanatory variables that affect the default barrier, together with the regression results. Section 6 analyzes the simultaneous relation between default barrier and indebtedness. Section 7 concludes the paper. Appendixes are available at http://neumann.hec.ca/gestiondesrisques/09-02.pdf.

2. Literature review

Several structural models propose default triggers. Most of them are first-passage-time approaches in the sense that they extend the seminal framework of Black and Scholes (1973) and Merton (1974) (hereafter BSM) by allowing default whenever the value of a firm's assets crosses a pre-specified barrier (Black and Cox, 1976), rather than solely at the debt's maturity. This default trigger can be given either endogenously or exogenously. For the endogenous default trigger, the equity holders choose to default or reorganize to maximize the value of their claims (e.g. Leland, 1994; Leland and Toft, 1996; Acharya and Carpenter, 2002). Exogenous default trigger models, in contrast, impose a pre-specified default barrier and extend the basic framework to include characteristics of bond markets, such as stochastic risk-free interest rates (Longstaff and Schwartz, 1995), stochastic default barrier (Hsu, Saá-Requejo, and Santa-Clara, 2002), and mean-reverting leverage (Collin-Dufresne and Goldstein, 2001).

The practical implementation of the structural models must specify assumptions regarding the level of the default barrier. In exogenous structural models, the default barrier is usually expressed as a fraction of the face value of the debt. These models therefore assume that the default barrier depends solely on the level of the face value of debt. For instance, Longstaff and

Schwartz (1995) consider a default barrier that equals the total principal value of the debt. Nonetheless, such a default barrier seems unrealistic because many firms continue to operate with a negative net worth. To deal with this, Huang and Huang (2003) suppose that the default barrier equals 60% of the face value of debt, while Leland (1994) calibrates the default barrier to match the observed recovery rates. This leads to a default barrier of 73% of the face value of the debt. Alternatively, the Merton-KMV model assumes that default occurs only at debt maturity and that the default point is set to the short-term debt plus half of the long-term debt (Crosbie and Bohn, 2003; Vassalou and Xing, 2004; Dionne et al., 2008).

The endogenous models pioneered by Black and Cox (1976) and extended by Leland and Toft (1996) and Acharya and Carpenter (2002), among others, offer a richer specification of the default barrier in which the equity holders or managers decide whether or not to default depending on the continuation value of the firm relative to current debt service payment. The default barrier corresponds to the cut-off point of the asset value below which equity holders would benefit more from defaulting on the firm's debt. This setting makes the default barrier sensitive to factors other than the debt's principal value. For example, in Leland and Toft's (1996) model, the optimal barrier level decreases in debt maturity, asset volatility and the risk-free rate, whereas it increases in default costs and the book value of debt.

As mentioned by Davydenko (2007), this kind of model assumes the absence of either minimum cash-flow covenants or market frictions, which could limit the firm's ability to raise sufficient external financing. As a result, the firm will never fall into default for cash shortage reasons: If the firm faces a liquidity crisis and its value is above the default barrier, the equity holders will always be able to avoid default by raising new funds. Few models presented in the literature relax these assumptions (see Kim, Ramaswamy, and Sundaresan, 1993; and Anderson and Sundaresan, 1996). Instead of setting the asset value as the default trigger, these models assume that default occurs when the firm's cash-flow fails to meet the debt service payment. Given the unavailability or limitations of external financing, default becomes exogenous and happens only in the event of a cash crisis. Fan and Sundaresan (2000) combine the endogenous value-based and exogenous liquidity-based defaults by assuming an exogenous covenant on the minimum cash flow for the former, and costly external fund raising for the latter.

Another trend in the literature considers the possibility of debt contract renegotiation and deviation from the absolute priority rule, allowing strategic debt service. Indeed, Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) stipulate that the liquidation costs and the bargaining power of the equity holders may lead the firm's creditors to accept a partial payment of the debt, which in turn may encourage opportunistic default by the equity holders. In addition, many firm-specific strategic factors were found to have an effect on default and recovery decisions. Asquith, Gertner, and Scharfstein (1994), Frank and Torous (1994), and Betker (1995) posit that the complexity of debt structure, managerial share ownership, and asset tangibility influence the occurrence of the formal and informal reorganization and deviations from the absolute rule. Therefore, they can expect the default barrier to be conditioned by both strategic factors and other firm-specific factors.

François and Morellec (2004) distinguish between default and liquidation of the firm, and explicitly account for the possibility of debt renegotiation under Chapter 11. In this setting, the firm is liquidated if its assets stay below the default barrier for a given period. Thus, the firm's equity is modeled as a Parisian down-and-out call option on the firm's assets. Moraux (2004), considers financial distress, or the cumulative time spent below the barrier. Galai, Raviv, and Wiener (2005) go further by considering the severity of the distress as well. Finally, Carey and Gordy (2007) develop a model in which the default barrier is set mainly by private debt holders. They present evidence that the recovery rate increases sharply with the pre-bankruptcy share of private debt in the firm's total debt.

To our knowledge, only Davydenko (2007) has explicitly studied the value of assets at default. He investigates whether default is triggered by low asset values or by liquidity shortages. He uses a sample of low-grade US firms with observed market value of both debt and equity, which allows him to observe the asset value at default. He finds that the asset value at default varies largely in cross-section, and depends on balance sheet liquidity, asset volatility, and tangibility. If on average, a barrier of 72% of the face value of the debt correctly predicts the probability of default, the large cross-section variability regarding the default barrier of defaulted firms leads him to conclude that structural models based on a well-defined default trigger have a limited ability to predict defaults in cross-section.

Brockman and Turtle (2003) pioneered empirical studies employing the barrier option framework. Using the equity's market value and the debt's book value as a proxy for the firm's market asset value, they find evidence of a significant default barrier for the majority of the firms in their sample. However, Wong and Choi (2009) find a limitation of the market value of this asset proxy, which leads to default barrier overestimation. They use an alternative estimation method and obtain support for a significant default barrier on average. Reisz and Perlich (2007) obtain similar results. They find that the DOC option framework predicts defaults better than the Merton-KMV model does.

In contrast with Davydenko (2007), we estimate the default barrier implied by Brockman and Turtle's (2003) model using a maximum likelihood estimation procedure for all the firms in our sample, and we do not limit our investigation to firms with directly observable asset value. We focus on the barrier level perceived by the market participants, because it is derived from the common equity price. Indeed, the default announcement could convey additional information about the defaulting firm's financial situation. Recovery rates may underestimate the asset value at which the firm defaults, due to the presence of bankruptcy costs and departures from the absolute priority rule. In other words, we focus on equity market prices, which is the set of information prior to default.

In addition to verifying the performance of the DOC model in forecasting defaults, we investigate whether the market perception of the default threshold accounts for a firm's characteristics, which purportedly influence its decision to default.

3. Estimation of the implied default barrier

3.1 Estimation method

The Brockman and Turtle (2003) model is an extension of the basic BSM framework, in which the firm's equities are viewed as a down-and-out call option and default is triggered when the value of the assets crosses the barrier level such that bond holders can receive the remaining value of the firm before it deteriorates further. In this setup, the default barrier can be seen as a debt covenant. We propose an estimation of the barrier level implied by the traded equity prices. The methodology relies on calibrating the barrier level such that the Down-and-Out European call option price formula matches the observed equity prices. To do so, we need to know the value of assets, the instantaneous drift and volatility of the assets' return, and the face value of the debt. Because the actual value of the firm's assets is not observable, we can approximate the total value of the assets by adding the market value of equity to the book value of debt.

The literature provides several ways of calibrating the firm's asset value, V_t, and the standard deviation of the asset volatility, σ_V. In the framework of BSM, the first method, referred to here as the variance restriction method, uses Ito's lemma to obtain a system of two equations linking the unknown asset values and the asset volatility to the observed equity values and volatility (Jones, Mason, and Rosenfeld, 1984; Ronn and Verma, 1986). However, several drawbacks of this method have been identified. Crosbie and Bohn (2003) argue that the equation relating equity volatility to asset volatility holds only instantaneously. Duan (1994) criticizes the implicit assumption in the variance restriction method of constant equity volatility and its independence from corporate asset value and time. He also underlines the lack of statistical inference for the estimates of V_t and σ_V with the variance restriction method.

Duan (1994) develops a transformed data maximum likelihood estimation method to estimate V_t and σ_V from equity prices, which views the observed equity time series as a transformed data set in which the theoretical equity pricing formula is used as a transformation. In addition to the statistical inference provided by maximum likelihood estimation, Ericsson and Reneby (2005) compare the estimation methods described above, and find that the transformed data maximum likelihood estimation method is superior. Wong and Choi (2009) also use this method in the DOC option framework. KMV developed an iterative method based on the variance restriction method, described in Crosbie and Bohn (2003). For the standard call approach, Duan, Gauthier, and Simonato (2004) show that the KMV method estimates are identical to the maximum likelihood estimates for the Black-Scholes-Merton model. However, when a more complex structural model involving an unknown capital structure parameter is considered, the KMV method is unable to estimate the additional parameter involved because only two equations are used. This contrasts with the transformed MLE method, which can estimate the capital structure parameter as the default barrier in the DOC option model. These features led us to choose the MLE estimation method.

3.2 Down-and-Out Call Option

As mentioned earlier, the DOC option model hinges on viewing the firm's equity as a DOC option on the firm's assets. We assume a geometric Brownian motion for the asset value:

$$d\ln V^t = (\mu - \sigma^2/2)dt + \sigma dW_t,$$

where V^t is the market value of the firm's assets at time t, σ is the asset value volatility, μ is the expected return on assets, and W_t is a Wiener process. The down-and-out call option price without rebate[1] is given by:

$$\begin{aligned} E^t_{DOC} &= V^t N(a) - Xe^{-rT}N(a-\sigma\sqrt{T}) - V^t(H/V)^{2\eta}N(b) + Xe^{-rT}(H/V^t)^{2\eta-2}N(b-\sigma\sqrt{T}) \\ &= \beta(V^t,\sigma,H) \end{aligned} \tag{1}$$

where T is the time to maturity of the option, H is the default barrier, $N(.)$ is the CDF of the standard normal distribution. The values of a, b, and η are given in Appendix I. The DOC framework includes the standard call option framework as a special case. Indeed, if we set the barrier H to zero in equation (1), we obtain the pricing formula of a European call option. The function relating the equity price to the asset value, $\beta(V^t,\sigma,H)$, is inverted to express V^t as a function of E^t_{DOC}, σ, and H; that is, $V^t = \beta^{-1}(E^t_{DOC},\sigma,H)$. We apply the Wong and Choi (2009) likelihood function in the DOC framework.[2] This likelihood function is given by[3]:

$$L^E = \sum_{t=2}^{T} \ln\left[f(\ln\hat{V}^t \mid \ln\hat{V}^{t-1};\mu,\sigma,H) \times \left(\hat{V}^t \frac{\partial\beta(V,\sigma)}{\partial V}\right)^{-1} \right] \tag{2}$$

where $\hat{V}^t = \beta^{-1}\left(E^t_{DOC},\sigma,H\right)$ and $f\left(\ln\hat{V}^t \middle| \ln\hat{V}^{t-1};\mu,\sigma,H\right)$ is the conditional density function of $\ln V^t$. We conducted simulations to check for the estimation's ability to retrieve the model parameters.[4] We also estimate the Merton-KMV model to compare the performance of the DOC

[1] In our setting, we assume that the Absolute Priority Rule holds, and that the equity holders receive nothing if the firm defaults, which implies no rebate for the DOC option.

[2] See Duan, Gauthier, and Simonato (2004) for an alternative likelihood function to estimate the Brockman and Turtle model with the maximum likelihood method. The difference resides in the conditional density function.

[3] Please refer to Appendix I for more details on the derivation of the likelihood function.

[4] To verify the estimation performance, we run a Monte Carlo study comparable to that of Wong and Choi (2009) and Duan, Gauthier, and Simonato (2004). We find similar results; the MLE procedure produces accurate estimates of the model parameters. The simulation procedure and results are available in Appendix II.

option model in predicting default probabilities with the standard European call framework. In this setting, the pricing equation becomes:

$$E_{SC} = V^t N(d) - Xe^{-rT} N(d - \sigma\sqrt{T}) \text{ where } d = \frac{\ln\left(V^t / X\right) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}. \quad (3)$$

The corresponding transformed maximum log-likelihood function, as derived by Duan et al., (2004), is given by:

$$L^E = -\frac{n}{2}\ln(2\pi\sigma^2\Delta t) - \frac{1}{2}\sum_{t=2}^{n}\left[\frac{\ln(\hat{V}^t / \hat{V}^{t-1}) - (\mu - \sigma^2 / 2)\Delta t}{\sigma\sqrt{\Delta t}}\right]^2 - \sum_{t=1}^{n}\ln\hat{V}^t - \sum_{k=1}^{n}\ln(\Phi(\hat{d})). \quad (4)$$

Here, $\hat{V}^t$ is obtained by inverting (3). Given the high non-linearity, the likelihood function in both cases is maximized using the Nelder-Mead Simplex Algorithm (fminsearch in MATLAB). Once the models' parameters are estimated, the default probability is given by:

$$DP_{Barrier} = N\left(\frac{-(\ln(V_0 / H) - (\mu - \sigma^2 / 2)\tau)}{\sigma\sqrt{\tau}}\right) + e^{-\frac{2\mu\ln(V_0 / H)}{\sigma^2}} N\left(\frac{-(\ln(V_0 / H) + (\mu - \sigma^2 / 2)\tau)}{\sigma\sqrt{\tau}}\right) \quad (5)$$

for the DOC option model, while for the standard call option this probability takes the form $DP_{SC} = N(-d)$ where *d* is defined in (3).

3.3 Data

The study covers public Canadian industrial firms listed on the Toronto Stock Exchange from January 1988 to December 2004. To compute default probabilities for the first year, we needed market and accounting data on the previous year to obtain one year of daily observations. Thus, to estimate the structural models, we gathered data starting from January 1987.

Firms that went bankrupt or reorganized were identified using Financial Post Predecessors & Defunct, CanCorp Financials (Corporate Retriever), and Stock Guide. Between 1988 and 2004, 130 firms were identified as being in default; 112 were bankrupt and 18 were undergoing reorganization. After merging the accounting data with the daily market data, 77 firms remained in the intermediary database of defaults. The main reason for this attrition is that we had incomplete market data for some firms and only one year of accounting data for others. In both cases, the data consequently became unusable for our study, because application of the structural model requires at least 200 consecutive daily market prices coupled with available accounting data on the book value of debt for defaulted firms.

Consistent with Vassalou and Xing (2004), we use the book value of debt for the new fiscal year starting only four months after the end of the previous fiscal year. This ensures that we use only the data available to investors at the time of calculation. We therefore needed at least two successive financial statements to obtain the 200 estimation observations required.

We examined the lags between the default dates from the last financial statements of some defaulted firms in greater detail. Many firms do not publish financial statements in the years prior to their bankruptcy. We decided to withdraw defaults for which these lags exceeded 18 months. For those that had defaulted between 12 and 18 months after their final financial statement, we moved up the date of the default to reconcile it with the last available year of financial statements. This reduced the number of defaulted companies in our sample to 60.

The data on daily market prices of equities for both defaulting and surviving firms were obtained from DataStream. The accounting data for the non-default sample came from the Stock Guide database, whereas accounting data for the defaulted firms were gathered from various sources, including Stock Guide, CanCorp Financials, and the companies' financial statements on SEDAR. We end up with 4916 observations (year-firm), representing 762 single firms, of which 56 are defaults. Of these 56 defaults, 11 are reorganizations.

4. Analysis of the results

4.1 Estimated default barriers

For the estimation of both models considered here, we use a one-year window, which is equivalent to an average of 261 daily market value observations. Apart from the market value of equities and the risk-free rate, which are the same for the two models, the parameter choice differs mainly because the two models define variables differently. In the standard call option, the default point retained is the same as in Crosbie and Bohn (2002) and Vassalou and Xing (2004): the sum of current liability and 50% of the long-term debt. The time to maturity is therefore adjusted and set to one year for the standard call option, because this debt is supposed to mature one year later. The parameters of the DOC option model differ according to the underlying assumptions. Here, the level of debt retained is the total liability because the option time to maturity is set at 20 years, which represents the life interval of the firm's equity. Brockman and Turtle (2003) use a 10-year interval. Reisz and Perlich (2007) try different time horizons between 5 and 20 years and find that this does not affect the default barrier. Brockman and Turtle (2003) contend that varying the option maturity from 3 to 100 years has a minor effect on the barrier level estimates. We assume a time to maturity of 20 years for the European DOC option. We test in Appendix III the sensitivity of the results with respect to this assumption.

<Insert Table 1 here>

The barrier estimates are presented in Table 1.[5] The estimated barrier to implied assets for the pooled sample in Panel B is about 29% and has a standard deviation of 27%, which is significant at all the usual confidence levels. The median of this ratio is 25%. Thus, our first

[5] Given that our sample of defaulting firms contains both liquidated and reorganized firms, we compare the default barrier estimates for the two categories. We also compare other characteristics such as firm size, liquidity, volatility, and debt cost between the bankrupted and reorganized firms. We find no significant differences between the two groups.

finding is that a public firm's equities can be seen as a down-and-out call option on their assets because on average, the implied default barrier is not nil. A closer look at the barrier estimates shows that the percentage of observations with barriers greater than zero is 77%. The average leverage ratio in our sample is 54% of the book value of assets, as shown in Panel A of Table 1. Compared with 29.38% for the implied barrier/estimated asset value, the default barrier is much lower than the face value of debt.

Our estimates of average barrier and median are in line with Reisz and Perlich (2007), who report an average barrier to implied asset value of 30.53% (median of 27.58%). This result contrasts with the average barrier of 69.2% found by Brockman and Turtle (2003). This discrepancy comes primarily from using the sum of the market value of equity and the book value of debt as proxies for the market value of assets. This approximation overstates the default barrier estimate (Wong and Choi, 2009; Reisz and Perlich, 2007).

When we compare the ratio of the default barrier to the implied asset value between defaulting and surviving firms, we observe an obvious difference between the two sub-samples. Indeed, while the median is as low as 25% of the asset value for non-defaulting firms, the median for defaulting firms is 76% of the implied asset value, and the third quartile reaches 91%. Therefore, we can conclude that for defaulting firms, the barrier level is much closer to the estimated asset value. We also obtain an estimated asset drift of 5% for the non-defaulted firms while it is equal to -18% for defaulted firms (Panel C). The asset volatility is higher for defaulted firms (Panel D).

4.2 Comparison of models' capacity to predict defaults

In this section, we compare the estimates from the DOC option model and the Merton-KMV model. We also compare the ability of these models to predict default occurrence in our sample of public Canadian firms. The aim here is to see whether the DOC option can predict defaults more accurately than the Merton-KMV does.

To compare the ability of these models to forecast bankruptcy one year in advance, we perform two probit regressions where the dependent variable is a dummy equal to 1 if the firm defaulted in a given year, and 0 otherwise. The only independent variable is the estimated default probability using the DOC option model for the first regression, and the Merton-KMV default probability for the second one.

Sobehart, Keenan, and Stein (2000) suggest that quantitative models of credit risk should be developed and validated using an out-of-sample test, which would avoid embedding undesirable sample dependency. For the out-of-sample validation, we split the full data set into a training sample, which is used to estimate the coefficient on the structural models' default probabilities in the probit model. The resulting estimates are used to compute the scores for the remaining data. Out-of-sample validation allows us to evaluate the ability of these scores to predict future defaults. We ran these regressions using two estimation/validation subsample definitions. The fit and associated statistics for the out-of-sample regressions are reported in Table 2. We present the ROC curves for the two competing models in Figure 1. This figure shows that the DOC option model ROC curve dominates that of Merton-KMV for most of the cut-off points.

<Insert Table 2 here>

<Insert Figure 1 here>

We retain the AUC measure as our primary statistic to compare the two models. DeLong, DeLong, and Clarke-Pearson (1988) offer a nonparametric test for the difference of AUC for correlated ROC curves. This statistic follows a Chi-square distribution with 1 degree of freedom. We report the out-of sample estimation results in Panels A and B of Table 2. In Panel A, the training sample runs from 1988 to 1995, for a total of 1,310 observations containing 23 defaults.

The out-of-sample contains 3,606 observations, of which 33 are defaults. The probit estimation on the first subsample shows a higher AUC for the Merton-KMV, 0.883 versus 0.845. However, the difference between these two performance measures is not significant at all the usual confidence levels. The opposite figure appears in the out-of-sample validation; the AUC measure in out-of-sample supports the DOC option model. Indeed, the Chi-square statistic value is 3.224, which is significant at the 10% confidence level. Thus, it seems that the DOC option model achieves better bankruptcy prediction in out-of-sample data.

In Panel B of Table 2, we test the robustness of the previous result to the choice of cutoff year for the separation of the training and out-of-sample data. We perform another out-of-sample test. The training sample expands from 1988 to 1996 for a total of 1,567 observations, including 26 bankruptcies and reorganizations. The remaining sample contains 3,349 firm-year observations, of which 30 are defaults. Here again, while the in-sample estimation shows no statistically significant difference in the AUC between the two models, the out-of-sample validation shows that the area under the curve for the DOC option is significantly higher than that of the Merton-KMV. Indeed, the Chi-square statistic, testing for no difference between the areas under the two ROC curves, is 6.907 and rejects the null with a *p-value* below 1%.[6]

5. Barrier determinants

5.1 Independent variables

The theoretical financial literature identifies several firm-specific factors that can explain both the decision to default and the output of the reorganization process. These factors therefore account for the barrier level. They fall into two broad categories: strategic and non-strategic factors. We discuss these factors and justify their choice below.

[6] The implemented Merton-KMV model is not the model currently used to estimate the distance to default by the Moody's-KMV corporation. The proprietary model adds steps and adjustments to those described in Crosbie and Bohn (2003) that cannot be applied in this paper.

5.1.1 Non-strategic factors

Most of the exogenous default models, including the basic Merton (1974) and Longstaff and Schwartz (1995) models, specify the barrier level as a fraction of the debt. It is therefore natural that our first barrier determinant is the firm's leverage. We expect a positive relationship between the firm's leverage and our implied barrier measure. We measure the leverage as the ratio of total liabilities to book asset value.

Unlike value-based models, where default depends on the value of the assets, we use cash-based models where default is assumed to happen whenever the firm's cash flows are insufficient to cover the firm's debt payments.[7] Cash-based models include Ross (2005), Anderson and Sundaresan (1996), and Kim, Ramaswamy, and Sundaresan (1993). These models assume that the firm has no access to external financing, implying that default can occur due to a cash shortage even if the company has a positive net worth. However, this assumption is restrictive because external financing could be accessible at a given cost, depending on the firm's financial soundness and debt capacity. The existence of financing costs raises the issue of liquidity management. Indeed, firms may accumulate a cash cushion to avoid having to procure external financing during downturns. Asvanunt et al. (2007), Acharya et al. (2006), and Anderson and Carverhill (2007) account for liquidity management and financing costs. If a cash shortage can cause default or at least accelerate default occurrence, we could expect an adjustment of the implied barrier level to the firm's cash holdings. More cash in the firm's assets should be associated with a lower barrier to the estimated asset market value. We measure liquidity as the ratio of cash and equivalents to the book value of assets. The same reasoning applies to external financial constraints. If the firm can contract new debt at low cost to avoid cash shortage defaults, its implied default barrier should account for this effect, and we should observe a lower barrier

[7] Davydenko (2007) compares the value-based and cash-based models and their assumptions.

level when the debt costs are low. We therefore expect a positive relation between the debt cost, measured by the ratio of interest expenses to total liabilities, and the implied barrier.

Although credit risk models are scale-free, we include the size, as measured by the logarithm of assets, to account for information availability. Indeed, large firms are generally more diversified and should therefore default at lower asset value, ceteris paribus. Thus, we expect a negative relation between the firm's size and the default barrier.[8] We also control for asset volatility because it is related to firm risk, which is measured by the estimated asset volatility. Finally, we control for the state of the economy as measured by the growth rate of the real GDP of the Canadian economy. GDP can also capture the effect of the interest rate on the default barrier (Leland, 1994).

5.1.2 Strategic factors

Strategic factors are specific to the endogenous default models. They fall into three categories: 1) costs of liquidation, 2) relative bargaining power, and 3) renegotiation friction. Betker (1995) and Franks and Torous (1994), among others, find that these factors affect the occurrence and outcome of reorganizations. In addition, models put forth by Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Hart and Moore (1998), and Fan and Sundaresan (2000) allow for strategic defaults. In contrast with liquidity default, which is due to insufficient cash flows, strategic default happens when equity holders decide to forgo debt payment even if they can afford to pay. Indeed, if deadweight costs associated with liquidation of the firm's assets are high, debt holders could benefit from conceding some of their debt to help the firm survive. Therefore, equity holders may be interested in defaulting opportunistically to

[8] The size of the firm can be seen as a strategic factor. Large or mature firms are often associated with high bargaining power in debt renegotiation with debt holders, whereas small or young firms are considered weak in renegotiation. For instance, Hackbarth et al. (2007), Houston and James (1996), Johnson (1997), Krishnaswami et al. (1999), and Denis and Mihov (2003) report evidence that the proportion of public debt to total debt increases with the size and the age of the firm. In addition, Carey and Gordy (2007) observe a higher recovery rate for firms with more bank debt. A combination of the two effects could also lead a negative effect for the size variable.

benefit from such debt cutbacks. Given that strategic default is more likely when the liquidation costs and equity holders' bargaining powers are more pronounced, we can anticipate a positive relation between these variables and the default barrier level. Davydenko and Strebulaev (2007) find that the credit spread is positively related to liquidation costs and bargaining power, and negatively related to renegotiation friction.

As a proxy for the liquidation costs, we use the percentage of fixed assets. Fixed assets are measured by the total value of capital assets, including land, buildings, computers, factories, office equipment, leasehold improvements, and assets under capital leases, net of accumulated depreciation and amortization. Fixed assets are the physical assets that are easiest to sell in case of liquidation. Accordingly, we can expect a negative relation between the proportion of fixed assets and the default barrier. As an additional measure of asset specificity, we use the R&D expenditures to the book value of assets. Indeed, research expenditures could be a good proxy for the firm's asset specificity. We anticipate a positive relation between these asset specificity measures and the level of the default barrier. More specific assets are generally harder to liquidate in bankruptcy and imply higher liquidation costs. To account for the equity holders' bargaining power, we use the percentage of votes attached to the voting shares held by the directors and other individuals or companies that own more than 10% of all voting rights. Here, we retain the percentage of votes instead of shareholding because we believe that shareholding better reflects the manager's and major equity holders' control over the firm's assets.

Finally, renegotiation frictions could impede debt renegotiation, and reduce recovery rates ex-post. Hart and Moore (1998) and Fan and Sundaresan (2000) argue that renegotiation frictions could prevent strategic defaults, but they also render the liquidation costs harder to avoid in liquidity default and thus decrease recovery rates. Our proxy for renegotiation friction is the portion of current liabilities to total liabilities. Berglöf and von Thadden (1994) contend that for financially distressed firms, short-term creditors rarely forgive debt, yet concessions are often made by subordinated long-term claim holders. Thus, if the strategic default effect prevails, and higher short-term debt indicates more renegotiation friction, the latter could prevent strategic defaults and therefore lower the default barrier level. However, when the liquidity default risk effect is more pronounced, short-term debt holders may deter debt renegotiation and force bankruptcy, with the associated liquidation costs, which may increase the ex-ante default threshold. Therefore, the overall effect of the renegotiation friction is ambiguous, as measured by the current to total debt ratio on the default barrier location.

Davydenko and Strebulaev (2007) observe that a larger proportion of current debt to long-term debt increases the liquidity shortage risk, because more cash flows are used for day-to-day debt service. Hence, we infer that more short-term debt could push the default barrier upward. We use the ratio of short-term liabilities to total liabilities as a proxy for renegotiation frictions. Because the short-term debt proxy for renegotiation friction can be contaminated by liquidity risk, we use the proportion of the outstanding public debt to the book value of total debt as an alternative renegotiation friction proxy. Davydenko and Strebulaev (2007) find a negative relation between credit spread and the proportion of public debt. Therefore, public debt seems to deter strategic default by equity holders because it is more difficult for firms with multiple dispersed creditors to renegotiate their debt, as argued by Hege and Mella-Barral (2004), Gertner and Scharfstein (1991), and Berglöf and von Thadden (1994), among others. Carey and Gordy (2007) contend that private debt holders (banks) endogenously set the asset value threshold below which firms declare bankruptcy, and find evidence of a strongly increasing recovery rate in the share of private debt. This could also explain the negative relation between the proportion of public debt and the default barrier.

To compute the outstanding amount of public debt for companies in our sample, we start by scanning the new Canadian bond issue lists of the FISD and the SDC Platinum databases to compile a list of companies in our dataset that are active on the bond market. For each identified issuer, we manually collected information on outstanding public debt for each fiscal year from the long-term debt section of printed Moody's/Mergent international manuals. Because the FISD and SDC Platinum databases are not exhaustive for Canadian issuers, we also look for the remaining Canadian firms in the Moody's/Mergent international manuals to check if they have public debt in their capital structures. We end up with 104 unique bond issuers out of 575 single firms in our dataset, for a total of 867 firm-year observations between 1988 and 2004. The remaining firms are assumed to have only private debt in their capital structure.

5.2 Regression analysis results

5.2.1 Descriptive statistics

Our dependent variable is the ratio of the implied barrier estimated from market price to the estimated asset value. We standardize the implied barrier using the estimated asset value instead of the book value of debt or the book value of assets, because these accounting measures can diverge substantially from their corresponding market values. We believe that the estimated market prices measure the value of the assets under management more accurately. For our regression analysis, we drop observations with nil implied barrier estimates. This reduces our initial sample of 4,916 to 3,609 firm-year observations.[9] After eliminating observations with insufficient data on independent variables, we end up with 3,232 firm-year observations for 575 single firms, covering 17 years from 1988 to 2004. The average number of years per firm is therefore 5.6.

<Insert Table 3 here >

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[9] We also performed the regression analysis on the whole sample; the results are similar to those of the restricted sample.

Our final default sample contains 127 firm-year observations from the default database, and includes 50 observations of default or reorganization. The descriptive statistics for both dependent and independent variables are reported in Table 3. The average default barrier is 40.1% of the estimated asset value, while the average leverage is 48.2% of the book value of assets. Thus, on average, the ratio of barrier to estimated assets is below the leverage ratio in our sample.

5.2.2 Non-strategic factors

The regression analysis results of the implied barrier on the non-strategic factors are presented in Table 4. The objective here is to test whether market participants adjust the implied default barrier estimated from the equity price, viewed as a down-and-out call option, to account for the possibility of cash shortages and the impossibility of contracting new debt.

<Insert Table 4 here>

Table 4 shows the result of the regression of the default barrier on the non-strategic factors. As expected, the leverage ratio is positively associated with the implied default barrier. The coefficients on the leverage ratio are positive and significant at the 1% level in all the regressions. One question that we address in this study is whether indebtedness is the only driver of the ex-ante perceived default barrier. Our regression results clearly demonstrate that this is not the case. The liquidity measure is negatively related to the implied barrier. This coefficient is also highly significant in all regressions. Thus, the implied default barrier associated with firms with more cash holdings suggests that they may eventually default at a lower asset value, because they can handle debt payments and avoid default due to liquidity constraints. This supports the underlying assumptions of cash-based models such as Ross (2005), Anderson and Sundaresan (1996), and Kim, Ramaswamy, and Sundaresan (1993). The debt cost also has a significant positive impact on the location of the default threshold. Financing frictions seem to greatly influence the value at which the firm is expected to default. Higher ability to contract new debt at a low cost lowers a firm's default threshold. This result, together with the liquidity concern results, shows that credit risk models with endogenous cash management in the presence of external financing costs better describe the reality of the firm, despite their complexity.

The negative relation of the ex-ante default barrier with volatility is consistent with endogenous barrier models like that of Leland and Toft (1996). Higher volatility makes the option to wait more valuable and lowers the level of the default barrier. The size of the firm has the expected sign. Larger firms benefit from the lower perceived ex-ante default barrier level, probably for informational reasons and bargaining power. Large firms have greater visibility and are followed more closely by analysts. This reduces the uncertainty regarding the asset level below which firms default. The result can also be explained by the bargaining power of large firms. Consequently, the default barrier for large firms is lower than that of smaller companies. Finally, the GDP growth rate is positively and significantly related to the default barrier. This result may seem surprising because one could expect lower barrier levels in economic expansion. However, a possible explanation lies in investors' expectations regarding future economic conditions. If firms set the default barrier according to these expectations instead of actual economic conditions, and given the cyclical aspect of the real GDP growth rate, a positive relation between GDP growth and barrier level may result. Another explanation is that GDP growth can approximate the interest rate; the endogenous default model of Leland (1994) predicts a positive relation between default barrier and interest rate.

To check the robustness of our results, we perform a panel regression with random and fixed effects using the same specifications in regressions (2) and (3) of Table 4. Our results are robust to the inclusion of both firm-specific constants and random error terms. The R square of the model is 29.9% for the random effect regression and 26.8% for the fixed effect panel regression.

We also test whether our liquidity and debt cost results are driven by the presence of outliers in our data. We applied the method of Hadi (1992, 1994) for outlier detection in multivariate data to these variables, leading to the exclusion of 62 observations. Columns (4) and (5) in Table 4 report the random and fixed effect panel regression results for the remaining observations. Here again, our results are unaffected by the sign or statistical significance of the coefficient estimates. Indeed, the debt cost becomes significant at the 1% level in the fixed-effect panel regression.

A closer look at the liquidity variable shows that in 78 observations, the ratio of cash and equivalents to total assets is above 50%. Only two of the observations come from our defaulted firms' data set. These observations may represent firms in the asset liquidation process. To ensure that liquidation or reorganization of assets do not influence our results, we drop such observations from the sample in regression (6), and find that our results are not driven by these observations. To further check the liquidity effect on the barrier level, we apply another measure of asset liquidity, the current ratio. The current ratio is measured as current assets to current liabilities, and is a proxy for the ability of the firm to meet its short-term obligations with its short-term assets. In unreported results, this alternative measure of liquidity also has a negative and significant coefficient estimate, both in random and fixed effect panel regressions.

On the debt cost side, we observe that a fairly high proportion (around 14%) of firms in our sample do not use long-term debt, and rely solely on accounts payable to suppliers. Because suppliers generally allow a 90–day grace period for payments, these firms have zero or low interest expenses. We test whether the positive relation between debt cost and the default barrier level is driven by these observations. Regression (7) in Table 4 indicates the contrary. The coefficient on the debt cost variable is higher for the remaining sample and is significant at all the usual confidence levels. The overall results show that liquidity shortages and costs of external finance are important drivers of the default barrier level. The regression analysis demonstrates that through equity prices, market participants adjust the level of assets at which the firm is expected to default due to liquidity shortage concerns and to difficulty obtaining new financing. This result holds even after controlling for leverage, asset volatility, firm size, and economic conditions, and is robust to the presence of potential outliers.

5.2.3 Strategic factors

We now turn to the results of the regression analysis of the implied default barrier on strategic factors. We report these results in Table 5. All the regressions include the non-strategic factors, but we do not report them in the table for the sake of brevity. All the non-strategic factors are significant and have the same signs as in Table 4. Regressions are restricted to observations where we were able to find data on the voting rights of the directors and major shareholders. The final sample contains 3,085 observations for 509 single firms. Our estimates in Table 4 are robust to the exclusion of firms without data on voting rights.

Liquidation costs are an important strategic factor in endogenous default models. Creditors should be more willing to forgive part of a firm's debt when the asset value for going concerns is much higher than its liquidation value, and when the liquidation costs are high. This gives equity holders a greater incentive for strategic default, to benefit from these debt concessions. If equity market participants are aware of such effects, the implied barrier should increase with default costs. Regression (1) of Table 5 supports this hypothesis. As expected, the coefficient on the fixed assets is negative and significant at the 1% confidence level. This liquidation cost effect is also supported by Davydenko and Strebulaev (2007), who find that the proportion of non-fixed assets is positively correlated with credit spread. The results are robust to the introduction of random effects, as indicated in regression (2).

<Insert Table 5 here>

As an alternative liquidation cost proxy, we use the ratio of research and development expenses to asset book value. Regression (3) of Table 5 shows that R&D is not relevant and does not have the expected sign, with a *t-statistic* of –0.33. We obtain the same figure after including random effects to account for the panel pattern in our data. Given that a large number of firms in our data set do not undertake R&D programs, in unreported results we used a dummy variable set to 1 if the R&D expenses are non-nil, and 0 otherwise. We found that this dummy is positive as expected but not significant, with a *t-statistic* of 1.44. These results suggest that the proportion of fixed assets is more effective than R&D at approximating liquidation costs in our data.

Regarding renegotiation friction, we use two alternative proxies: the short-term debt to total debt ratio and the public debt to total debt ratio. Regressions (5) and (6) show that the short-term debt coefficient is positive and statistically significant at all the usual confidence levels. This supports the higher liquidity default risk in the presence of more short-term debt rather than the strategic default explanation. Indeed, the effect of liquidity risk seems to dominate the strategic default effect on the ex-ante default barrier in our data. Further, because short-term debt is related to liquidity default risk, it is not an effective measure of renegotiation friction.

To better isolate the effect of renegotiation friction on the estimated default barrier, we use the public debt variable in specifications (1) to (4) of Table 5. In all four regressions, the estimated default barrier decreases significantly with the proportion of public debt in the firm's capital structure. The coefficient on the renegotiation friction proxy is negative and significant at the 1% level in these regressions. This result underlines the role of renegotiation frictions in discouraging potentially opportunistic shareholders from defaulting, despite the liquidation costs that renegotiation could avoid.

Finally, our proxy for the bargaining power of CEOs and major shareholders is positively and significantly related to the implied default barrier. This result indicates that greater shareholder bargaining power implies a higher default barrier. This is consistent with the strategic default effect--greater bargaining power of equity holders encourages strategic defaults--because equity holders could gain more from renegotiation. Once the firm defaults, greater shareholder bargaining power implies greater deviation from the absolute priority rule. This result is consistent with Betker (1995), who finds that deviations from APR in Chapter 11 increase sharply with CEO shareholding. In addition, Davydenko and Strebulaev (2007) assert that CEO shareholding increases the credit spread. The overall regression results lend strong support to the strategic default effect on the implied default threshold. This evidence is in line with endogenous default models such as those in Leland (1994) and Fan and Sundaresan (2000), where the shareholders deliberately choose to default to benefit from debt cutbacks.

6. Endogenous leverage

Our non-strategic factor regressions support the presumption that more leverage implies a higher implied default barrier level. However, one could argue that the default threshold could also influence borrowing decisions. Therefore, we can reasonably anticipate a simultaneous relation between default barrier and indebtedness, where the optimal leverage ratio would be the result of equilibrium.

To account for the endogenous relationship between leverage and the implied default barrier, we need to specify the rest of the leverage determinants. The financial literature identifies several determinants of capital structure choice. Undoubtedly, the first motive for companies to contract debt is to benefit from the debt tax shield. We therefore include the actual tax rate, defined as the ratio of tax payment to earnings before tax, as an explanatory variable for the debt equation. Because firms also benefit from a non-debt tax shield, we add the depreciation and amortization scaled by the book value of total assets as an additional independent variable. We expect a positive sign for both *tax rate* and *depreciation and amortization* variables.

Titman and Wessels (1988) argue that the firm’s collateral value increases its ability to contract debt. Our proxy for collateral value is the book-to-market ratio. We infer that value companies (those with high book-to-market ratio) have more productive assets in place than do those with low book-to-market ratios, whose value is primarily driven by less pledgeable growth options. Thus, we expect a positive sign for the coefficient on the book-to-market ratio. Similarly, we added the ratio of *R&D* expenses to the book value of assets to account for the fact that firms operating in technology-intensive sectors have more specific assets. Because these assets are less valuable in liquidation, we could expect a negative relation with leverage. R&D expenses can be interpreted as a measure of the firm's uniqueness. As an additional proxy for firm uniqueness, we use selling, general, and administrative expenses scaled by net sales; the same logic leads us to anticipate a negative sign. Finally, the profits generated by the firm's operations decrease its need for external financing, and should be associated with less debt. Our measure for profitability is EBITDA divided by net sales; here again, we expect a negative relation with leverage.

<Insert Table 6 here>

We report the results of the three-stage least-square estimation in Table 6. The dependent variables are the estimated default barrier to estimated asset value and the leverage ratio (book value of debt to book value of assets). We notice that all the independent variables in the leverage equation have the expected signs and are significant at the 1% confidence level. Regarding the default barrier equation, the previous results hold for both strategic and non-strategic factors. The endogeneity between the leverage and default barrier does not affect our regression results.

7. Conclusion

This article proposes a new estimate of the default barrier and analyzes its determinants by focusing on liquidity shortage and strategic default. We use a new data set of 762 public industrial firms listed on the Toronto Stock Exchange.

In structural models of credit risk, default is often assumed to happen when the market value of assets falls below a given barrier. The financial theory stipulates different assumptions regarding this barrier, ranging from Merton (1974), whose threshold is debt value at maturity, to more sophisticated settings in which the default barrier is determined endogenously by stakeholders, as in Leland and Toft (1996). However, due to the unobservability of the firm's asset value, these assumptions cannot be tested directly. The overall model performance can be assessed by predicting either defaults or credit spreads. In this paper, we use the maximum likelihood estimation method of Duan (1994) to infer the implied default barrier of the DOC option model from equity prices. We analyze a sample of public firms to compare the Merton-KMV model with the DOC option model in terms of default prediction accuracy. The Merton-KMV and the DOC option models perform equally well for in-sample fitting. However, the DOC option default probability estimate is more accurate in out-of-sample default forecasting. Further, the implied barrier for defaulting firms is close to their estimated asset market value at default.

We also perform regression analysis of the implied default barrier against firm-specific and macro-economic factors, and find that both capital structure and firm-specific factors influence the default barrier location. In addition, the ex-ante implied default barrier is adjusted to both non-strategic and strategic factors. It is sensitive to asset liquidity, debt cost, liquidation cost, renegotiation friction, and equity holders' bargaining power. Therefore, the market seems to adjust the implied default threshold, through equity prices, to account for firm-specific determinants that go beyond the debt level. Our results provide new insights into modeling and predicting the decision to default that are important for the management of default risk.

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Figure 1: Comparison of ROC curves of DOC option and Merton-KMV models

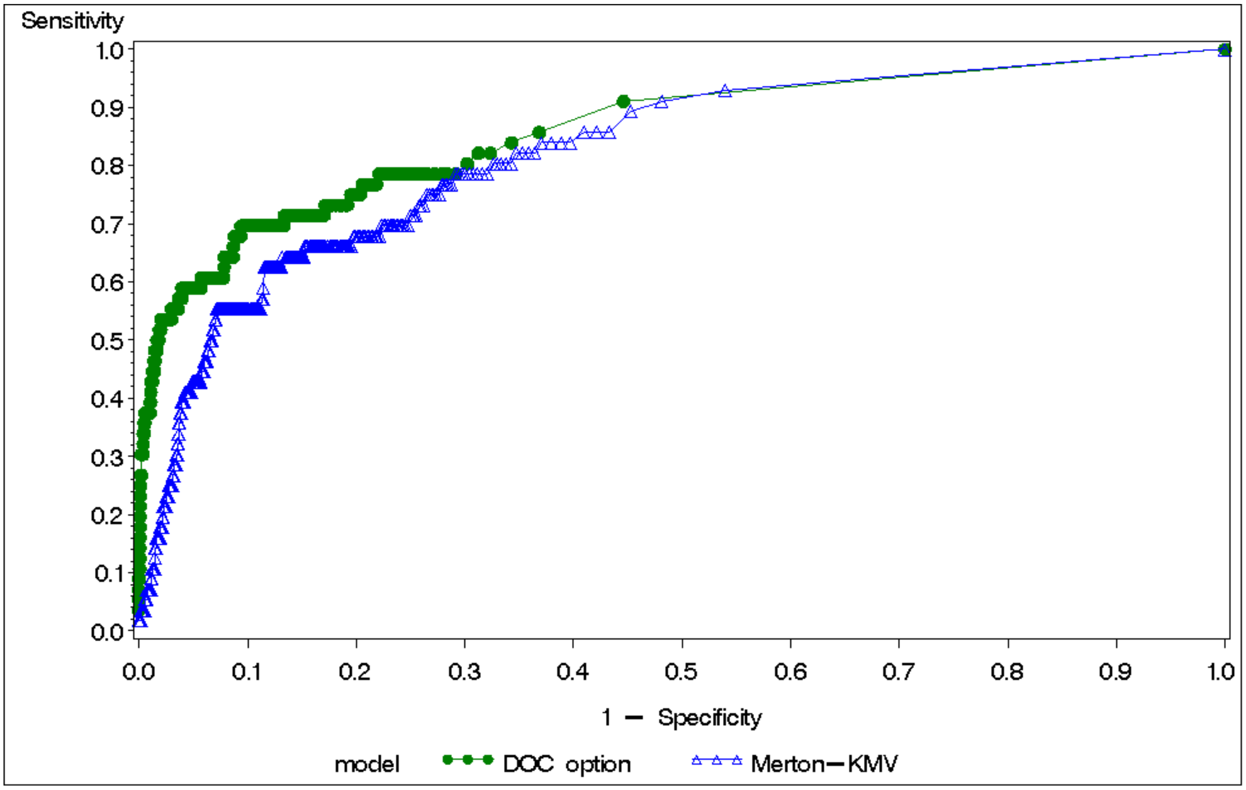

Table 1: Estimated barrier, asset drift and volatility

This table presents the MLE estimates of the default barrier. The results refer to firm-year observations. Panel A presents the leverage ratio for the sample. The ratio is the book value of total liabilities divided by the book value of total assets. Panel B presents the barrier as a fraction of the estimated asset market value. Panels C and D present the estimated asset drift and volatility respectively using the DOC option model.

	N.	Mean	Std	Min	Q1	Median	Q3	Max
Panel A: Leverage ratio (total liabilities/ total assets)								
Overall sample	4916	0.54	0.70	0	0.30	0.49	0.66	19.5
Non-defaulted firms	4860	0.53	0.69	0	0.30	0.49	0.66	19.5
Defaulted firms	56	1.12	1.16	0.26	0.62	0.85	1.23	8.36
Panel B: Barrier to implied asset value								
Overall sample	4916	0.29	0.27	0	0	0.25	0.5	0.99
Non-defaulted firms	4860	0.29	0.27	0	0	0.25	0.5	0.99
Defaulted firms	56	0.65	0.31	0	0.47	0.76	0.91	0.99
*Panel C: Estimated asset drift**								
Overall sample	4916	0.05	0.31	-0.43	-0.13	0.02	0.19	1.02
Non-defaulted firms	4860	0.05	0.31	-0.43	-0.13	0.03	0.2	1.02
Defaulted firms	56	-0.18	0.28	-0.43	-0.32	-0.2	0	0.9
Panel D: Estimated asset volatility								
Overall sample	4916	0.53	0.40	0.00	0.24	0.41	0.70	3.98
Non-defaulted firms	4860	0.53	0.40	0.00	0.24	0.41	0.70	3.98
Defaulted firms	56	0.57	0.36	0.05	0.26	0.58	0.82	1.77

*To avoid the effect of outliers, the asset drift estimates were limited to the interval between the 1st and 99th percentiles.

Table 2: Comparative performance in predicting bankruptcy

This table reports the comparative performance in predicting bankruptcy one year ahead. The estimated default probability from structural models is used as a regressor in probit regression including an intercept. In-sample estimation uses the full data set to estimate probabilities, while out-of-sample estimation uses the coefficient estimated from the in-sample estimation to evaluate the predictive performance of the resulting probabilities on the unused out-of-sample data. For each probit regression considered, we report the maximum rescaled R^2 of Nagelkerke (1991), the percentage of bankruptcy concordant with the model, the area under the receiver operating characteristic curve (AUC) measures of the ability of the model to rank observations correctly. In the last column, we add the nonparametric test of difference between areas under correlated ROC curves of Delong et al. (1988).

Model	DOC option	Merton-KMV	$\chi^2_{(1)}$ Statistic *(p-value)*
Panel A: Out-of-sample validation; estimation period 1988-1995; evaluation period 1996-2004			
# obs.	3606	3606	
# Defaults	33	33	
Max rescaled R square	0.283	0.107	
Percent concordant	73.5	62.2	
Area under ROC	0.871	0.787	3.224 (0.072)
Panel B: Out-of-sample validation; estimation period 1988-1996; evaluation period 1997-2004			
Out-of-sample			
# obs.	3349	3349	
# defaults	30	30	
Max rescaled R square	0.274	0.095	
Percent concordant	74.1	61.4	
Area under ROC	0.883	0.780	6.907 (0.009)

Table 3: Descriptive statistics of dependent and independent variables

Default barrier is the implied default barrier divided by the estimated asset value. *Leverage* is the book value of total liabilities divided by the book value of total assets. *Liquidity* is the cash and cash equivalents divided by the book value of total assets. *Debt Cost* is the ratio of interest expenses to the book value of total liabilities. *Asset volatility* is the estimated asset volatility from the DOC option model. *Size* is the logarithm of the total book assets in millions of dollars. *GDP Growth* is the real GDP growth rate. *Fixed assets* is the total value of capital assets, including land, buildings, computers, factories, office equipment, leasehold improvements and assets under capital leases, and net of accumulated depreciation and amortization to the book value of total assets. *R&D* is the ratio of research and development expenses to total assets. *Short-term debt* is the ratio of current liabilities to total liabilities. *Voting* is the percentage of votes attached to the voting shares of a company held by the directors and other individuals or companies that own more than 10% of all voting rights. *Public Debt* is the ratio of the amount of outstanding public debt to the book value of total debt.

Variable	N	Mean	Median	Std. Dev.	Minimum	Maximum
Default barrier	3232	0.401	0.383	0.242	0	0.992
Leverage	3232	0.482	0.488	0.293	0.002	8.361
Liquidity	3232	0.093	0.034	0.137	0	0.988
Debt Cost	3232	0.034	0.031	0.031	0	0.284
Volatility	3232	0.458	0.359	0.334	0.007	3.983
Size	3232	11.67	11.38	2.18	4.63	17.54
GDP Growth	17	0.031	0.031	0.018	-0.021	0.055
Fixed Assets	3232	0.426	0.401	0.278	0	1
R&D	3232	0.029	0.000	0.09	0	1.855
Short-term debt	3232	0.565	0.553	0.284	0	1
Voting	3085	0.304	0.242	0.257	0	0.961
Public Debt	3232	0.15	0	0.311	0	1

Table 4: Regression analysis of the implied default barrier on non-strategic factors

This table reports the results of the regression analysis of the implied default barrier on non-strategic variables. The dependent variable is the implied default barrier divided by the estimated asset value. The sample consists of all firm-year observations with non-nil estimated barriers. *Leverage* is the book value of total liabilities divided by the book value of total assets. *Asset volatility* is the estimated asset volatility from the DOC option model. *Liquidity* is the cash and cash equivalents divided by the book value of total assets. *Debt Cost* is the ratio of interest expenses to the book value of total liabilities. *Size* is the logarithm of the total book assets in millions of dollars. *GDP growth* is the annual growth rate of the real GDP. Values of *t-statistics* are reported in parentheses. Coefficients marked ***, **, and * are significant at the 1%, 5%, and 10% significance levels respectively. *Hausman test* is the Hausman test for random effect following a chi square distribution with *k* degrees of freedom, where *k* is the number of independent variables. The corresponding *p value* is reported below. We do not reject the random effect model.

	All			Without outliers			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage	0.106***	0.103***	0.07***	0.102***	0.069***	0.069***	0.08***
	(8.16)	(6.84)	(3.71)	(6.69)	(3.50)	(3.49)	(3.91)
Liquidity	-0.112***	-0.142***	-0.188***	-0.135***	-0.202***	-0.242***	-0.189***
	(-3.97)	(-4.47)	(-5.16)	(-3.66)	(-4.69)	(-5.03)	(-3.76)
Debt Cost	0.277**	0.237*	0.3**	0.399**	0.472***	0.316**	0.444***
	(2.26)	(1.81)	(2.06)	(2.48)	(2.57)	(2.12)	(2.75)
Volatility	-0.438***	-0.522***	-0.611***	-0.522***	-0.613***	-0.612***	-0.595***
	(-34.88)	(-37.98)	(-38.94)	(-37.64)	(-38.41)	(-38.47)	(-35.44)
Size	-0.044***	-0.05***	-0.036***	-0.051***	-0.036***	-0.036***	-0.032***
	(-22.45)	(-17.16)	(-6.71)	(-17.24)	(-6.49)	(-6.43)	(-5.20)
GDP Growth	1.44***	1.38***	1.3***	1.466***	1.38***	1.36***	1.25***
	(7.06)	(7.36)	(6.75)	(7.34)	(7.10)	(6.99)	(6.01)
Const.	1.02***	1.15***	1.04***	1.14***	1.03***	1.04***	0.967***
	(35.43)	(29.59)	(15.76)	(29.37)	(15.09)	(15.07)	(12.48)
Fixed effect	No	No	Yes	No	Yes	Yes	Yes
Random effect	No	Yes	No	Yes	No	No	No
R^2	0.299	0.299	0.268	0.298	0.269	0.269	0.257
Observations	3232	3232	3232	3170	3170	3154	2716
Single firms	575	575	575	574	574	571	508
Hausman test	-	-	147.3	-	141.91	144.74	106.74
p-value	-	-	0.00	-	0.00	0.00	0.00

Table 5: Regression analysis of the implied default barrier on strategic factors

This table reports the results of the regression analysis of implied default barrier on strategic variables. The dependent variable is the implied default barrier divided by the estimated asset value. The sample consists of all firm-year observations with non-nil estimated barriers. *Fixed assets* is the total value of capital assets, including land, buildings, computers, factories, office equipment, leasehold improvements and assets under capital leases, and net of accumulated depreciation and amortization to the book value of total assets. *R&D* is the ratio of research and development expenses to total assets. *Short-term debt* is the ratio of current liabilities to total liabilities. *Voting* is the percentage of votes attached to the voting shares of a company held by the directors and other individuals or companies that own more than 10% of all voting rights. *Public Debt* is the ratio of the amount of outstanding public debt to the book value of total debt. *Leverage, Liquidity, Debt cost, Volatility, Size, GDP growth,* and the constant are included in all specifications. Values of *t-statistics* and *z-statistics* are reported in parentheses. Coefficients marked ***, **, and * are significant at the 1%, 5%, and 10% significance level respectively.

		(1)	(2)	(3)	(4)	(5)	(6)
Liquidation costs	*Fixed assets*	-0.044***	-0.035*				
		(-3.09)	(-1.91)				
	R&D			-0.137	0.001	-0.053	0.001
				(-0.33)	(0.02)	(-1.27)	(0.02)
Renegotiation frictions	*Short- term debt*					0.088***	.082***
						(6.16)	(4.95)
	Public debt	-0.048***	-0.064***	-0.057***	-0.068***		
		(-3.44)	(-3.58)	(-4.10)	(-3.84)		
Bargaining power	*Voting*	0.079***	0.08***	0.086***	0.086***	0.076***	0.079***
		(7.05)	(3.86)	(6.00)	(4.15)	(5.22)	(3.78)
Random effect		No	Yes	No	Yes	No	Yes
R^2		0.30	0.30	0.31	0.33	0.30	0.30
Observations		3085	3085	3085	3085	3085	3085
Single firms		509	509	509	509	509	509

Table 6: Three-stage least square estimation of barrier and leverage equations

This table reports the results of the three-stage least square regressions for panel data with fixed assets where the endogenous variables are the implied default barrier divided by the estimated asset value and the leverage ratio, defined as the book value of total liabilities divided by the book value of total assets. The sample covers all firm-year observations with non-nil estimated barriers and sufficient data. *Asset volatility* is the estimated asset volatility from the DOC option model. *Liquidity* is the cash and cash equivalents divided by the book value of total assets. *Debt Cost* is the ratio of interest expenses to the book value of total liabilities. *GDP growth* is the annual growth rate of the real GDP. *R&D* is the research and development expenses scaled by the book value of assets. Profitability is the EBITDA to net sales ratio. *Tax rate* is the tax payment of the year divided by the earnings before taxes. *Dep & Amt* is the depreciation and amortization scaled by the book value of total assets at the end of the year. *Selling & Adm* is the selling and corporate expenses divided by the net sales. Values of *z-statistics* are reported in parentheses. Coefficients marked ***, **, and * are significant at the 1%, 5%, and 10% significance level, respectively.

	Barrier equation	Leverage equation
Constant	0.426***	0.250***
	(8.86)	(17.59)
Default barrier		0.26***
		(8.86)
Leverage	0.191**	
	(2.53)	
Liquidity	-0.187***	
	(-4.35)	
Debt Cost	0.823**	
	(5.21)	
Volatility	-0.334***	
	(-27.01)	
GDP Growth	1.252***	
	(6.51)	
R&D	0.132***	-0.235***
	(2.86)	(-4.99)
Fixed assets	-0.075***	
	(-4.94)	
Public debt	-0.06***	
	(-4.03)	
Voting	0.07***	
	(4.80)	
Profitability		-0.22**
		(-2.18)
Book-to-Market		0.028***
		(16.42)
Tax rate		0.099***
		(5.37)
Dep & Amt		1.158***
		(10.01)
Selling & Adm		-0.062***
		(-2.50)
N	3085	3085
R^2	0.21	0.08
Chi2 stat	1160.33***	549.75***

Supplementary material, not for publication

Appendix I

We assume a geometric Brownian motion for the asset value; that is:

$$d \ln V^t = (\mu - \sigma^2 / 2)dt + \sigma dW_t$$

where V^t is the market value of the firm's assets at time t, σ is the asset value volatility, μ is the expected return on assets, and W_t is a Wiener process. In general, the down-and-out call option price is given by:

$$\begin{aligned} E_{DOC}^t = {} & V^t N(a) - Xe^{-rT} N(a - \sigma\sqrt{T}) - V^t (H/V)^{2\eta} N(b) + Xe^{-rT} (H/V^t)^{2\eta-2} N(b - \sigma\sqrt{T}) \\ & + R(H/V)^{2\eta-1} N(c) + R(V^t/H) N(c - 2\eta\sigma\sqrt{T}) \end{aligned} \tag{A1}$$

where T is the time to maturity of the option, H is the default barrier, and R is the rebate of the barrier option; that is, the payment made to the equity holders if the value of the firm's assets breaches the barrier. $N(.)$ is the cumulative distribution function for the standard normal distribution, and

$$a = \begin{cases} \dfrac{\ln(V^t / X) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \text{ if } X \geq H, \\ \dfrac{\ln(V^t / H) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \text{ if } X < H, \end{cases}$$

$$b = \begin{cases} \dfrac{\ln(H^2 / V^t X) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \text{ if } X \geq H, \\ \dfrac{\ln(H / V^t) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \text{ if } X < H, \end{cases}$$

$$\frac{\ln(H / V^t) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}, \ \eta = \frac{r}{\sigma^2} + \frac{1}{2},$$

where r is the risk-free rate and X is the nominal value of debt.

The DOC framework includes the standard call option framework as a special case. Indeed, if we set the barrier H to zero in equation (A1), we obtain the pricing formula of a European call option. In our setting, we assume that the Absolute Priority Rule holds and the equity holders receive nothing if the firm defaults. Thus, the last two terms in equation (A1) become null, and equation (A1) is reduced to:

$$\begin{aligned} E_{DOC}^{t} &= V^{t} N(a) - Xe^{-rT} N(a - \sigma\sqrt{T}) - V^{t}(H / V)^{2\eta} N(b) + Xe^{-rT}(H / V^{t})^{2\eta-2} N(b - \sigma\sqrt{T}) \\ &= \beta(V^{t}, \sigma, H) \end{aligned} \quad \text{(A2)}$$

The function relating the equity price to the asset value, $\beta(V^{t}, \sigma, H)$, is inverted to express V^{t} as a function of E_{DOC}^{t}, σ, and H; that is, $V^{t} = \beta^{-1}(E_{DOC}^{t}, \sigma, H)$.

We apply the Wong and Choi (2009) likelihood function in the DOC framework. In the first specification, V^{t} is the asset value at time t, and $f(\ln V^{t} | \ln V^{t-1}; \mu, \sigma, H)$ is the conditional density function of $\ln V^{t}$. Because the asset value should remain above the barrier between two successive observation dates (respectively t and $t-1$), the density function should account for this feature. The corresponding density function is given by:

$$f(\ln V^{t} | \ln V^{t-1}; \mu, \sigma, H) = \varphi(\ln(V^{t} / V^{t-1})) - e^{2\eta(\ln H - \ln V^{t-1})} \varphi(\ln(V^{t} V^{t-1}) - 2\ln H) \quad \text{(A3)}$$

where $\varphi(x) = \frac{1}{\sigma\sqrt{2\pi\Delta t}} \exp\left(-\frac{(x - (\mu - \sigma^{2}/2)\Delta t)^{2}}{2\sigma^{2}\Delta t} \right)$ and Δt is the time interval between two successive observation dates.

When the asset value is observable, the log-likelihood function is:

$$L^{V} = \sum_{i=2}^{n} \ln f(\ln V^{t} | \ln V^{t-1}; \mu, \sigma, H).$$

However, because we observe E^t_{DOC} rather than V^t, we modify the model as follows:

$$\begin{aligned} L^E &= \sum_{t=2}^{T} \ln\left[f(\ln \hat{V}^t \mid \ln \hat{V}^{t-1}; \mu, \sigma, H) \times \left(\frac{\partial \beta(V,\sigma)}{\partial \ln V} \right)^{-1} \right] \\ &= \sum_{t=2}^{T} \ln\left[f(\ln \hat{V}^t \mid \ln \hat{V}^{t-1}; \mu, \sigma, H) \times \left(\hat{V}^t \frac{\partial \beta(V,\sigma)}{\partial V} \right)^{-1} \right] \end{aligned} \tag{A4}$$

where $\hat{V}^t = \beta^{-1}(E^t_{DOC}, \sigma, H)$ and

$$\begin{aligned} \frac{\partial \beta(V^t,\sigma)}{\partial V^t} &= N(a) + \frac{1}{\sigma\sqrt{T}} n(a)(1-\frac{X}{H}) + \frac{X}{V^t} e^{-rT} (\frac{H}{V^t})^{2\eta-2} N(b-\sigma\sqrt{T})(2-2\eta) \\ &\quad - (1-2\eta)(\frac{H}{V^t})^{2\eta} N(b) + \frac{1}{\sigma\sqrt{T}} n(b)(1-\frac{X}{H})(\frac{H}{V^t})^{2\eta} \end{aligned} \qquad \text{if } H \geq X,$$

and

$$\begin{aligned} \frac{\partial \beta(V^t,\sigma)}{\partial V^t} &= N(a) - (\frac{H}{V^t})^{2\eta} N(b)(1-2\eta) \\ &\quad + (2-2\eta)\frac{X}{V^t} e^{-rT} (\frac{H}{V^t})^{2\eta-2} N(b-\sigma\sqrt{T}) + \frac{1}{\sigma\sqrt{T}} n(b)(1-\frac{X}{H})(\frac{H}{V^t})^{2\eta} \end{aligned} \qquad \text{if } H <.$$

Appendix II

Monte Carlo study of the MLE estimation

The true values used in the Monte Carlo simulation for μ, σ, and α are 0.1, 0.3, and 0.833 respectively. Mean, Median, Standard-deviation, Min, and Max are the sample statistics of the estimates from 1,000 simulations. The values used in the simulations are V_0=10,000, F=6,000, α =0.833 (H=5000), r = 0.05, and Δt =1/250, and N =200 is the number of daily observations. We use the Wong and Choi (2009) maximum likelihood function as given in equation (2).

	$\hat{\mu}$ True=0.1	$\hat{\sigma}$ True=0.3	$\hat{\alpha}$ True=0.833
Mean	0.103	0.301	0.824
Median	0.095	0.296	0.849
Std	0.321	0.068	0.350
Min	-0.866	0.109	0
Max	1.144	0.548	1.840
t-stat	0.34	0.42	-0.86
p-value	0.73	0.68	0.39

Appendix III

We also have to assume the firm's lifespans in the DOC framework. In other words, to model the firm's equity as a DOC option, we have to set the option maturity, which represents the lifespan of the company. In the previous analysis, we assumed a lifespan of 20 years. To ensure that our findings are not affected by the choice of this input parameter, we estimated the default barrier assuming lifespans of 5 and 10 years. The results in the table below show that the average estimated barrier is not particularly sensitive to the firm's assumed lifespan; changing the option maturity from 20 years to 5 years only moves the mean of the estimated barrier from 0.29 to 0.26. Thus, the economic importance of the default barrier estimates is robust to the alternative maturity choice. The correlation of the default barriers estimated with 20-year maturity and those estimated with 10-year maturity is 0.99, whereas the correlation between the 20-year and the 5-year default barriers is 0.96. Finally, the estimates of the implied barrier on the non-strategic and strategic factors keep the same signs and significance when the firm's assumed lifespans are changed. We can therefore conclude that our findings are robust to the choice of maturity parameter.

	N.	Mean	Std	Min	Q1	Median	Q3	Max
5 years	4916	0.26	0.26	0	0	0.19	0.42	0.99
10 years	4916	0.27	0.26	0	0	0.22	0.46	0.99
20 years	4916	0.29	0.27	0	0	0.25	0.5	0.99