Structural Credit Risk Models: A Review[§]

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Abstract

In this paper, we review the literature on credit structural models. Contingent claim analysis offers an appealing theoretical framework allowing not only evaluating firm's claims and default risk, but also financing and investment decisions, as well as determining the impact of policy changes on the firm value and decisions. First, we present the major structural models and their underlying assumptions, beginning with exogenous default models and the following development leading to endogenous default models as well as other models that accounts for bankruptcy procedures, capital structure decisions and strategic defaults among others. The second part of the paper covers the empirical works related to the structural credit models. These works could be classified into three major categories; the first group examines the ability of structural models to explain the credit spread, the second one evaluate their performance to forecast default occurrence, the last group uses the structural models to study the relationship between credit risk and stock returns.

Résumé

Dans ce papier, on revoit la littérature sur les modèles structurels de crédit. Du coté théorique, les modèles d'analyse d'actifs contingents procurent un cadre qui permet non seulement la valorisation des actifs de l'entreprise et son risque de défaut, mais aussi les décisions d'investissement et de financement ainsi que leur impact sur la valeur de l'entreprise et ses décisions. Dans la première partie, nous présentons les principaux modèles structurels, leurs hypothèses sous-jacentes, à commencer par les modèles à défaut exogènes et les développements subséquents qui ont mené aux modèles de défaut endogène, ainsi que l'intégration des procédures de faillite, les décisions de structure de capital et les décisions de défaut stratégiques ainsi que d'autres développements. La deuxième partie est consacrée aux recherches empiriques. Ces travaux empiriques peuvent être classés en trois groupes : le premier examine la capacité de différents modèles structurels à expliquer les écarts de crédit ; le deuxième groupe évalue la performance de ces modèles dans la prévision des défauts ; finalement, certains travaux utilisent les modèles structurels pour étudier la relation entre le risque de crédit de l'entreprise et la performance boursière de ses actions.

[§] This literature review corresponds to the first chapter of the author's PhD thesis at HEC Montréal, supervised by Georges Dionne.

Introduction

In this paper, we seek to provide a summary of recent developments in structural credit risk models literature. In recent years credit risk modeling and measures knew increasing interest from both financial institutions and academics. This is due mainly to two reasons. First, the Capital Accord of 2006, or Basel II, allows large banks to use their internal models to assess their capital requirement instead of the more constraining standardized model. Second, the huge increase of off-balance-sheet derivatives and the rising use of the securitization of loans necessitate more developed credit analysis methods.

The last decades showed a growing number of studies modeling the decision to default, or endogenous default models. Our primary goal is to present a taxonomy of these models and a comparison between their underlying assumptions, their results and the related empirical evidence. We also, briefly cover the evolution of the credit risk methodology and distinguish the different categories of models. We point out the forces and limitation of each category. Here, we focus mainly on structural models. Previous reviews covering structural models include Bielecki and Rutkowski (2002), Uhrig-Homburg (2002), Lando (2004) and François (2005).[1]

Despite the appealing theoretical underpinning of structural models, they lack accuracy in explaining the cross-section of credit spreads measured by the yield difference between risky corporate bonds and riskless bonds. The default spread obtained through structural models is far below the credit spread (Eom, Helwege, and Huang, 2004). Moreover, structural models underpredict short-term default probabilities (Leland, 2004).

To overcome these limitations, a first trend of the literature propose several extensions to account for more realistic features of financial markets and firm's financing and investment decisions. These developments include specifying stochastic models of risk-free interest rate (Longstaff and Schwartz, 1995; Kim, Ramaswamy, and Sundaresan, 1993; Briys and de Varenne, 1997). Another trend of the literature accounts for the

[1] Reduced-form models are outside the scope of this review. For reviews on reduced-form models please refer to Duffie and Singleton (1999) or Bielecki and Rutkowski (2002) for instance.

possibility of strategic debt service and debt renegotiation (Hart and Moore, 1994; 1998; Anderson and Sundaresan, 1996; Mella-Barral and Perraudin, 1997; Acharya and Carpenter, 2002). Recent studies points out the difference between private (bank) and public debt in renegotiation (Hackbarth, Henessy and Leland, 2007; Carey and Gordy, 2007). Other researches account for departure from absolute priority rule and renegotiation under Chapter 11 (François and Morellec, 2004; Broadie, Chernov, & Sundaresan, 2007). Another approach considers a dynamic capital structure (Collin-Dufresne & Goldstein, 2001; Goldstein, Ju, & Leland, 2001; Ju, Parrino, Poteshman & Weisbach, 2005), while Mauer and Triantis (1994), Childs, Mauer and Ott (2005) and Sundaresan and Wang (2007) consider endogenous investment. The cash holding management policy is accounted for in Acharya, Huang, Subrahmanyam and Sundaram (2006), Anderson and Carverhill (2007), and Asvanunt, Broadie, and Sundaresan (2007). Leland (1998) allows for optimal dynamic risk management and Sarkar and Zapatero (2003) consider mean reverting cash flows. Zhou (2001), Duffie and Lando (2001) and Giesecke & Goldberg (2004) add a jump component to the value process of assets allowing for "surprise" default at the cost of closed-form solution. Alernatively, Hackbarth, Miao & Morellec (2006) consider jumps in the cash flow process with regime change. Finally, Longstaff (1996), Morellec (2001) and Ericsson and Renault (2006) include a liquidity premia to price corporate debt. We seek to provide a synthesis of the assumptions and the major results of these structural models as well as the related empirical evidence.

As an alternative explanation to the credit spread puzzle, several factors beside default risk explain corporate credit spreads. Indeed, variables that in theory determine credit spreads have limited explanatory power as documented by Collin-Dufresne, Goldstein and Martin (2001) and Campbell and Taksler (2003) among others. Firm specific factors and systematic market risk have substantial explanatory power of credit spread differential. Liquidity is also found to be an important determinant of the credit spread: both bond-specific illiquidity and macroeconomic measures of bond market liquidity explain variations in the observed credit spread (Longstaff, Mithal, and Neis, 2005 and Chen et al., 2007). These evidences suggest that the limited ability of structural models to replicate the observed credit spread is more due to the presence of non-default factors in credit spread rather than their failure to capture the default risk of corporate debt.

Structural models share a common theoretical foundation, namely the classical Merton (1974) model. In this setting, and for a particular diffusion process of asset's value, the firm defaults when its assets reach an exogenous level. Given the central role of the Merton model for all the subsequent structural models, an obvious starting point is to present a short description of this model.

1. The Merton approach

The Merton model relies on the assumption that default is triggered by the value of the assets, therefore, the starting point is to set the diffusion process of the assets. The value of assets *V* is assumed to follow a log-normal diffusion process that is under the physical probability measure:

$$dV_t / V_t = (r - \delta)dt + \sigma dW_t \qquad (1)$$

where W_t is the a standard Brownian motion, μ is the instantaneous expected return on assets, σ is the constant proportional volatility of the return on the firm value, δ is the firm's total dividend payout to shareholders. Moreover, the additional assumption of simple capital structure is made. The firm liabilities are represented by a single zero-coupon paying bond maturing at *T*. The value of the firm is the sum of equity, *E*, and the debt value with face value *D*.

The value of the equity represents a call option on the assets of the firm with maturity *T* and strike price of *D*. The risky zero-coupon bond is equal to its corresponding risk-free zero-coupon bond minus the value of an European put option on the firm's assets *V*, a strike of *D*, and maturity *T*. If the asset value at the maturity of the zero-coupon bond is sufficient to make the necessary payment then the firm remains the property of the shareholders. Otherwise, the firm defaults and the bondholders take possession of the firm's assets and the shareholders receive nothing.

The Merton model assumes that the assets of the firm are traded and the market is sufficiently complete, this allows using risk neutral probability measure, and replaces the expected return in equation (1) by the risk-free rate *r*. Hence, the Black and Scholes (1973) formula can be applied to value the equities of the firm as an European call option:

$$E = V\,N(d_1) - De^{-r(T-t)}N(d_2),$$

$$d_1 = \frac{\ln(V/D) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2 = d_1 - \sigma\,\sqrt{T-t}\,,$$

where E is the value of equities, and N is the Normal (0,1) cumulative distribution function.

The simplicity of the Merton model relies on applying the Black and Scholes formula of pricing the European options to value firm's equity and debt. However, this comes at the cost of too simplistic assumptions regarding the asset value process, interest rate, and the capital structure.

The assumption of a single zero-coupon bond for the liabilities of the firm is far from being realistic. Geske (1977) relaxes this assumption and considers the firm's liabilities as a coupon-paying bond, where the equity holders make the needed coupon payment through issuance of new equities. The coupon payments can cause the default of the firm. At each coupon date, the shareholders have the choice either to make the payment to bondholders or to forego the coupon payment causing the default of the firm. In this setting, the coupon bonds are valued as compound options.

The subsequent contributions in the structural models literature are mainly extensions of the Merton basic framework. One conventional way to regroup these pricing models is their assumptions regarding the default trigger. While the exogenous models assume a default trigger determined solely by the capital structure of the firm, endogenous models assume that equity holders/managers decide to default whenever it is optimal for them to stop paying the firm's debt service. Depending on the default trigger, we could classify a model as endogenous or exogenous. In addition to the exogenous/endogenous default classification, we refine these categories by distinguishing the default event assumed in the different models. Indeed, three possible default triggers where identified in the literature. First, the most commonly used is the zero net worth trigger. That is the firm default whenever its asset's value falls below the nominal of debt or some other exogenous trigger. This category includes Merton (1974), Brennan and Schwartz (1978), Longstaff and Schwartz (1995) and Briys and de Varenne (1997) models. These models are thus value-based. The second set of structural models considers that the firm defaults as soon as its cash flow is insufficient to face the debt service requirement. Thus, the default can be triggered by a liquidity shortage in this setting. We refer to these models as cash-based models. This category is represented by the contributions of Kim, Ramaswamy, and Sundaresan (1993), Anderson and Sundaresan (1996) and Ross (2005). The major drawback of this approach is that external financing is assumed unavailable. In addition to the unrealistic feature of this assumption, in presence of external financing costs, cash management becomes possible. Acharya, Huang, Subrahmanyam and Sundaram (2006), Anderson and Carverhill (2007), and Asvanunt, Broadie, and Sundaresan (2007) account for financing costs and cash management. These cash-based and value-based models are exogenous models in the sense that default is a result of breaching an exogenous covenant.

Endogenous default models, pioneered by Black and Cox (1976), derive the minimum asset level under which the shareholders maximize their own claim by stopping debt service payment. The default in this setting become a result of a decision making process by the firm's stakeholders. The basic Black and Cox model was extended along several dimensions. For instance, Leland (1994) and Leland and Toft (1996) include tax advantage of debt and bankruptcy costs. Hart and Moore (1994, 1998), Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) and Mella-Barral (1999) include the possibility of strategic defaults of the equity holders in order to obtain debt concessions from creditors. The interest of the endogenous default model resides in their ability to offer a richer modeling of the default decision and to account for stylized facts regarding firm's default and reorganization decision and outcome. Moreover, the contingent claims analysis provides a general framework allowing not only evaluating firm's claims and default risk, but also financing and investment decisions as well as determining the impact of policy changes on the firm's value and decision. In the next sections we present the different categories of structural models.

2. Exogenous default

The first exogenous default model is the Merton model in which the default barrier is equal to the nominal value of the debt. Kim, Ramaswamy and Sundaresan (1993) extend the Merton model to incorporate both default risk and interest rate risk. The model is cash-based in the sense that the default is triggered by a cash-flow shortage. The asset value of the firms follows a geometric Brownian motion with a proportional dollar payout ratio δV to security holders. Moreover, the firm's debt is constituted of a single coupon-paying bond, with a continuous coupon flow, c, until the maturity. The asset value model is given by $dV = V(\mu - \delta)dt + \sigma V dz$ .The firm defaults the first time its cash flow falls below the coupon payment. This implies that the default barrier is given by $V_B = c / \delta$.

The short-term interest rate is given by the Cox, Ingersoll Ross (CIR) process, that is

$$dr = a(b - r)dt + \sigma_r \sqrt{r}\, dw.$$

The two Wiener process dw and dz are correlated with an instantaneous correlation coefficient ρ_{Vr} The assumption of a CIR short-term interest rate process comes at the cost of a numerical solution for bond price.

The Longstaff and Schwartz (1995) model is similar to the Kim, Ramaswamy and Sundaresan (1993) model in the sense that it considers both the default risk and the interest rate risk to price the corporate debt. A major difference however, is that the short-term interest rate is assumed to follow a Vasicek model, that is:

$$dr_t = a(b - r_t)dt + \sigma_r dw_t$$

where a and b are constant, while the dynamic of the total value of assets is given by:

$$dV = \mu V dt + \sigma V dz \, .$$

Here again the two standard Wiener processes dw and dz are correlated with an instantaneous correlation coefficient ρ_{Vr}.

Longstaff and Schwartz (1995) assume moreover that the value of the firm is independent of its capital structure and assumes a threshold value V_B for the firm at which the financial distress occurs. As soon as the value of the firm value falls below V_B, the firm immediately enters financial distress and defaults on all of its obligations. Longstaff and Schwartz argue that this definition of financial distress is consistent with both the cases where the generated cash flows are insufficient to pay its current obligations as well as the violation of the net worth covenant[2]. Briys and de Varenne (1997) criticize this default definition and argue that when the corporate bond reaches maturity the firm can be in a solvent position, i.e, with the value of assets above the default threshold, but with no sufficient assets to pay the face value of the bond at maturity. This is equivalent to a situation where $V_B < V_T < F$ where V_T is the value of assets at maturity T, F is the face value of the debt and V_B is the default threshold.

Longstaff and Schwartz assumes also that when a reorganization occurs the security holders receives $1 - w$ times the face value of the security at maturity, where w represents the percentage writedown of security in reorganization. The recovery in their setting is on the treasury value of the security and is assumed to be a fixed constant.

[2] Wruck (1990) and Kim, Ramaswamy, and Sundaresan (1992) discuss the difference between the flow-based and the cash-based insolvency.

The value of the riskless bond with nominal 1$ and maturity T is given by the Vasicek (1977) model and is central in the derivation of the valuation expressions for risky corporate securities and is denoted by $D(r,T)$[3].

The price of a contingent claim, that pays 1$ if default doesn't occur during the life of the bond and $1-w$ otherwise, is given by the quasi closed-form solution:

$$P(X,r,T) = D(r,T) - wD(r,T)Q(X,r,T)\,, \tag{2}$$

where $X = V / V_B$ and $Q(X,r,T)$ represents the risk neutral probability of default.

The price of risky discount bond is a function of V and V_B through their ratio X only, which can be viewed as a summary measure of the default risk of the firm. By consequence the specification of V and V_B separately is no longer necessary which simplifies the implementation of the model.

The first term in equation (2) corresponds to the price of the bond in absence of default risk, while the second term represents a discount for the default risk of the bond. This discount is the product of two components: the first component, $wD(r,T)$, is the present value of the writedown on the bond in case of default, and the second term, $Q(X,r,T)$, is the risk neutral probability of default. This last term is solved recursively by numeric methods.

Nielsen, Saá-Requejo and Santa-Clara (1993) extend the Longstaff and Schwartz model by assuming a stochastic default threshold V_B and also suppose a Vasicek process for short-term interest rate.

Briys and de Varenne point out that the payment to creditors upon default is independent of the level of the stochastic barrier and the value of assets. This in turn could lead to situations where the bondholders receive more than the assets value at bankruptcy[4].

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[3] See Vasicek (1977) for the closed-form of the discount bond.
[4] See also Collin-Dufresne and Goldstein. 2001

To overcome the limitations of the Nielsen, Saá-Requejo and Santa-Clara and the Longstaff and Schwartz models discussed above, Briys and de Varenne (1997) propose a model with stochastic interest rate, where a generalized Vasicek model drives short-term interest rate:

$$dr_t = a(t)(b(t) - r_t)dt + \sigma_r(t)dw_t$$

where $a(t)$, $b(t)$ and $\sigma_r(t)$ are deterministic functions, and $\sigma_r(t)$ is the instantaneous standard deviation of r_t. They define the exogenous default triggering barrier as

$V_B(t) = \alpha F P(t,T)$ where $0 \le \alpha \le 1$, F is the face value of the corporate bond and $P(t,T)$ is the default-free zero coupon bond maturing at *T*.

Under this specification of the default barrier, a closed-form solution to corporate risky zero-coupon bond is obtained.

3. Endogenous default

For the endogenous models, we just describe in detail the seminal contributions of Black and Cox (1976), Leland (1994) and Leland and Toft (1996). We then review more recent developments and other extensions.

3.1. *The Black and Cox (1976) model*

In the Merton model, the timing of the default event is questionable. Indeed, the default time is restricted to the maturity of debt, independently of the evolution of the asset's value before the maturity. Default cannot occur before the maturity of debt. In response to this shortcoming, Black and Cox (1976) pioneered the first passage models, where the firm defaults as soon as the value of its assets reaches a non-random default barrier V_B. In this case, bondholders get V_B and equity holders get nothing. We now describe in detail the assumptions and the major results of this approach. Black and Cox suppose a perpetual debt, e.g. consol bond, paying constant coupon rate, c, proportional to the firm value. Under these assumptions, the following process drives the firm's value:

$$dV_t / V_t = (r-\delta)dt + \sigma dW_t \tag{3}$$

where, the interest rate r is constant and $\delta \geq 0$ is the payout ratio.

Even if the dividend payments are allowed for in equation (3), the shareholders cannot sell assets in order to pay coupons. The coupon payments are possible only through issuance of new equities. However, under a given asset value, V_B, the stockholders are no longer willing to issue new equities in order to pay coupons. For a given V_B, the optimal default time take the following form: $\tau^* = \inf\{t \geq 0 : V_t \leq V_B\}$. For a fixed default boundary, the price of the consol bond, *D(V),* has to solve the following ordinary differential equation:

$$\frac{1}{2}V^2\sigma^2 D_{VV} + rVD_V - rD + c = 0 \tag{4}$$

subject to the lower boundary condition $D(V_B) = \min(V_B, c/r)$ since the value of the bond does not exceed the default free value of the consol bond, that is *c/r*. The upper boundary condition is given by: $\lim_{V\to\infty} D_V(V) = 0$ since the value of the bond tends to its riskless value as the value of the assets tends to infinity. The solution to this differential equation gives the value of debt:

$$D(V) = \frac{c}{r} + \left(V_B^{\alpha+1} - \frac{c}{r} V_B^{\alpha} \right) V^{-\alpha} \qquad \text{where } \alpha = 2r/\sigma^2.$$

In order to maximize the value of their equities, stockholders chose the optimal default boundary in such a manner that the debt value *D(V)* is minimized. The optimization problem leads to the optimal default boundary[5]:

$$V_B^* = \frac{c}{r + \sigma^2/2}. \tag{5}$$

Note that the optimal level of the barrier is independent of the current value of the firm. However, the optimal barrier increases with the coupon size and decreases in the asset

[5] This holds when there is no dividend payment.

volatility. Moreover, the barrier is decreasing in the asset's volatility. This result can be explained by a higher value of the option to wait for a recovery of asset's value when its volatility is higher.

We should notice here that both the Merton and Black and Cox models do not allow for debt that is coupon-paying and has finite maturity. They also do not allow analysis of optimal capital structure.

3.2. *The Leland (1994) model*

As in the Black and Cox model, Leland (1994) model assumes that the firm issues a consol bond paying a coupon at a rate c and the firm defaults when the process V, as given by equation (3), hits for the first time a lower barrier V_B. The major contributions of the Leland model are the introduction of the tax shield of debt and the bankruptcy costs. When the firm defaults, the bondholders receive a recovery payment of $(1-\lambda)V_{\mathrm{B}}$ and the shareholders receive nothing with $0 \leq \lambda \leq 1$, λ being the cost of bankruptcy. The value of bankruptcy cost BC is a decreasing convex function of V.

Moreover, let τ be the tax rate. The firm benefits from the tax shield τC from debt financing as long as it remains solvent. In case of default, tax benefit cannot be claimed. The tax benefit is modeled as a security that pays a constant coupon τC. The value of this security, TB, is increasing in the value of assets. The total value of the firm, v, is then the sum of the firm's assets, V, and the value the tax shield of the interest payment, $TB(V)$, minus the value of the bankruptcy costs, that is:

$$v(V) = V + TB(V) - BC(V) \tag{6}$$

with

$$TB(V) = \tau \frac{C}{r}\left(1 - \left(\frac{V}{V_B}\right)^{-\alpha}\right)$$

$$BC(V) = \alpha V_B \left(\frac{V}{V_B} \right)^{-\alpha} \text{ where } \alpha = 2r / \sigma^2.$$

Holding the default barrier level constant, and solving the ordinary differential equation similar to equation (4) with the adequate boundary conditions, the debt value is given by:

$$D(V) = \frac{c}{r} + \left((1-\lambda)V_B - \frac{c}{r} \right) \left(\frac{V}{V_B} \right)^{-\alpha}. \tag{7}$$

The effect of the debt issuance has two contrary effects on the value of the firm. The first effect reduces the firm value since more debt implies higher value of bankruptcy costs. On the other hand, increased interest payment implies more tax shield, due to their deductibility, which in turn increases the value of the leveraged firm.

Leland considers, in a first step, the case of unprotected debt, that is, there is no lower bound imposed on the value of the endogenously chosen default barrier. The equity holders set the default barrier with the objective of maximizing their claims without constraints, that is $E(V) = v(V) - D(V)$ where $v(V)$ and $D(V)$ are given by equations (6) and (7) respectively. The optimal default barrier is obtained by solving the equity-holders problem:

$$V_B^* = \frac{(1-\tau)c}{r + \sigma^2 / 2}. \tag{8}$$

When the tax benefits are neglected, the default boundary is equal to the one derived in the Black and Cox model. However, we note that the optimal default boundary is insensitive to the bankruptcy costs, even though these costs lower the value of the firm. The reason is that the maximized equity value is independent of the default level. Indeed, all the reduction in the firm's value related to bankruptcy costs comes from the decreased value of debt value.

With the closed form formulas for the debt and equity values, Leland derives the optimal capital structure of the firm. In addition, the firm determines the optimal coupon rate that maximizes the value of the leveraged firm. By considering the tradeoff between the tax advantage and the bankruptcy costs, a relation is established between bond prices and the optimal leverage to the value of assets, the firm risk, taxes, bankruptcy costs and interest rates.

3.3. *The Leland and Toft (1996) model*

The Leland (1994) model relies on the extreme assumption of a perpetual debt in order to obtain a closed form formula for debt, equities and firm value. Leland and Toft (1996) relax this assumption. Instead, they assume that debt is continuously rolled over. That is, the same amount of principal is issued each time an already outstanding bond matures. This modeling of the firm's debt guarantees that, at any time, the outstanding principal, coupons payments and average debt maturity are independent of time, despite the fact that each individual bond has a finite maturity.

More specifically, Leland and Toft begin by considering a single bond with maturity *t*, paying a continuous coupon flow *c(t)* and principal *p(t).* In case of default the bondholders receives a fraction $\rho(t)$ of the default-triggering asset value V_B. In a risk neutral valuation framework, and for a given exogeneous V_B, the value of the bond is given by:

$$d(V;V_B,t)=\frac{c(t)}{r}+e^{-rt}\left[p(t)-\frac{c(t)}{r}\right]\left[1-F(t)\right]+\left[\rho(t)V_B-\frac{c(t)}{r}\right]G(t), \qquad (9)$$

where *F(s)* is the cumulative distribution function of the first passage time to bankruptcy,

$$G(t)=\int_{s=0}^{t}e^{-rs}f(s;V,V_B)ds,$$

and *f(s)* is the density function of the first passage time to bankruptcy.

Leland and Toft also assume that the firm issue new bond at par with maturity *T* at a rate $p=P/T$ per year, where *P* is the total principal value of all outstanding bonds. Thus, previously issued bond principal that matures each year is replaced. This allows keeping the total principal of outstanding bonds, *P*, and the coupon payment per year, *C,* constant until T, if the firm remains solvent. The total debt service is then equal to $C + P/T$ per year, and is independent of time. Moreover, they assume that the fraction of assets received by bondholders in case of default is independent of the bond maturity in such a way that whenever the default occurs bondholders always receive $(1-\alpha)V_B$.

The value of all outstanding bonds can then be expressed as:

$$
\begin{aligned}
D(V;V_B,T) &= \int_{t=0}^{T} d(V;V_B,t)dt \\
&= \frac{C}{r} + \left(P - \frac{C}{r}\right)\left(\frac{1-e^{-rT}}{rT} - I(T)\right) + \left((1-\alpha)V_B - \frac{C}{r}\right)J(T)
\end{aligned},
$$

where

$$I(T) = \frac{1}{T}\int_0^T e^{-rt}F(t)dt$$

and

$$J(T) = \frac{1}{T}\int_0^T G(t)dt.$$

This stationary capital structure allows Leland and Toft (1996) to find an explicit formula for the optimal value of the default barrier, which depends on the maturity of debt. The value of equity is maximized for the optimal default barrier V_B:

$$V_B^* = \frac{(C/r)\left(A/(rT) - B\right) - AP/(rT) - \tau Cx/r}{1 + \alpha x - (1-\alpha)B}, \tag{10}$$

where

$$A = 2ae^{-rT}N(a\sigma\sqrt{T}) - 2zN(z\sigma\sqrt{T}) - \frac{2}{\sigma\sqrt{T}}n(z\sigma\sqrt{T}) + \frac{2e^{-rT}}{\sigma\sqrt{T}}n(a\sigma\sqrt{T}) + (z-a),$$

$$B = -\left(2z + \frac{2}{z\sigma^2 T}\right)N(z\sigma\sqrt{T}) - \frac{2}{\sigma\sqrt{T}}n(z\sigma\sqrt{T}) + (z-a) + \frac{1}{z\sigma^2 T},$$

$$a = \frac{(r - \delta - (\sigma^2 / 2))}{\sigma^2},$$

$$z = \frac{\sqrt{a^2\sigma^4 + 2r\sigma^2}}{\sigma^2} \quad and \quad x = a + z.$$

Equation (10) shows that the bankruptcy triggering barrier depends on the debt maturity *T*. As the maturity of debt tends to infinity the barrier tends to the one defined by equation (8). Moreover, LT note that for long term debt structures the bankruptcy threshold is inferior to the principal value of debt.

3.4. *Strategic default models*

The existence of bankruptcy costs may lead to situations where it is optimal to debt holders to concede a part of coupon payment to equity holders through renegotiation of debt. However, such concessions can induce the equity holders to opportunistic default in order to profit from such concessions. Indeed, Asquith, Gertner, and Scharfstein (1994), Franks and Torous (1989, 1993), and Weiss (1990) report evidence of opportunistic behavior of stakeholders due to the bankruptcy procedure as well as deviations from absolute priority rules. However, renegotiation is not always possible and inefficiency due to bankruptcy and liquidation could not be avoided.

Hart and Moore (1998) consider a two period discrete model. They also assume that there is no asymmetry of information between the debtor and creditor. The returns on the project at the end of the first and second period, R_1 and R_2, are specific to the debtor /entrepreneur who promises a stream of payment to the debt holder. As long as he makes these payments, the creditor continues to run the project. Otherwise, the creditor can seize the firm and liquidate the project assets. In this case, there is room for renegotiation of the contract because the borrower can extract debt concessions by threatening to withdraw his human capital from the project.

Anderson and Sundaresan (1996) use a discrete time model where all the bargaining power belongs to shareholders. They posit a binomial process for the value of the firm and assume that the firm generates a cash flow proportional to its assets value, at each time point. Moreover, all the involved parties have full information on the state of the nature. The terms of the contract require a constant coupon payment of CS_t out of the generated cash flows at each time point until the maturity of debt T. However, if this generated cash flow is not sufficient to make the necessary payment, the firm is not automatically thrown into bankruptcy. The equity holders make a take-it-or-leave-it offer that do not exceed the generated cash flows. In this case, the creditors face a decision node where they have to choose between two options: (1) liquidate the firm and receives the liquidation value less the liquidation costs, or (2) accept the proposed payment. The presence of liquidation costs is an incentive for the creditors to accept the offered payment. In a game theory setting, the equity holders determine the minimum coupon payment above which the creditors are not willing to force liquidation. Thus, Anderson and Sundaresan show that recursive equilibrium is possible and is unique when the liquidation costs are strictly positive. They also demonstrate that accounting for bankruptcy costs leads to credit spreads that are closer to the observed ones, relative to models that do not account for strategic debt service.

While Anderson and Sundaresan (1996) give all the bargaining power to shareholders, Fan and Sundaresan (2000) propose a bilateral bargaining in a game-theoretic setting that can accommodate varying bargaining powers between debt holders and equity holders. They develop continuous-time model that extends Anderson and Sundaresan (1996) approach along several dimensions. The main extension of Fan and Sundaresan (2000) is the inclusion of a tax advantage of debt. In the presence of such advantage, a bargaining on the firm value becomes possible and its value becomes endogenous, since it depends on the optimal reorganization policies.

Indeed, when corporate taxes are considered, the value of assets could differ from the value of the firm. Two bargaining formulations by claimants are then possible. In the first, the borrower and the lender bargain over the value of the assets of the firm. The future tax benefits are assumed to be lost making the value of the assets coincide with the value of the firm. They also consider that the liquidation of assets implies fixed and proportional costs, α and K, respectively. Debtors settle for a debt-equity swap in which the lenders exchange their claims for equity for an endogenously determined barrier, which can be seen as a distressed exchange where the absolute priority rule is violated. The firm becomes an all-equity firm in this case, which in turns avoids costly liquidation. The sharing rule, θ, is subject to the Nash bargaining formulation, and its optimal value depends on the relative bargaining power of equity holders, η:

$$\theta^* = \min\left(\eta \frac{\alpha V_S + K}{V_S}, \eta\right), \tag{11}$$

where V_S is the trigger point of the debt equity swap.

The value of equity satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 V^2 E_{VV} + (r-\delta)VE_V - rE + \delta V - c(1-\tau) = 0, \tag{12}$$

where V is the asset's value, δ is the cash payout ratio,

subject to following boundary conditions:

$$\lim_{V\to\infty} E(V) = V - \frac{c(1-\tau)}{r},\ \lim_{V\to V_S} E(V) = \eta\alpha V_S \text{ and } \lim_{V\to V_S} E_V(V) = \eta\alpha$$

where the first boundary condition comes from the fact that the debt becomes risk free as the value of asset approaches infinity, and the two last conditions are implied from the bargaining game. Solving for the equity value in equation (12) gives the debt-equity swap triggering point:

$$V_S = \frac{c(1-\tau)}{r}\frac{-\lambda_-}{1-\lambda_-}\frac{1}{1-\eta\alpha} = \frac{c(1-\tau)}{r+\frac{\sigma^2}{2}}\frac{1}{1-\eta\alpha}, \tag{13}$$

where

$$\lambda_- = \left[0.5 - \frac{(r-\delta)}{\sigma^2}\right] - \sqrt{\left[0.5 - \frac{(r-\delta)}{\sigma^2}\right]^2 + \frac{2r}{\sigma^2}}.$$

We can see from equation (13) that the triggering asset value found by Leland (1994), is a special case of the Fan and Sundaresan distress exchange triggering asset value. In the Leland framework there is no possibility of renegotiation, that is $\eta = 0$, making the default occurs at a lower level of asset's value. In this framework, stronger equity holders bargaining power, η, and superior liquidation costs α implies higher default triggering barrier.

In the second bargaining formulation, the borrower and the lender bargain over the value of the firm, $v(V)$, instead of the value of its assets. When an endogenously determined trigger point is reached, V_S, borrowers offer a debt service that is less than the contractual amount as an equilibrium outcome of the bargaining process. This allows them to get potential tax benefits in the future when the firm recovers from distress and the present value of these tax benefits is included in the bargaining process.

Fan and Sundaresan derive the value of the firm, $v(V)$, given a trigger point of strategic debt service, $\tilde{V}_S$. The value of the firm is always greater than the value of the assets because of the present value of the tax shield.

The optimal sharing rule that satisfies the Nash bargaining game in this case, is given by:

$$\theta^* = \arg\max \left\{\theta v(V)\right\}^{\eta} \left\{(1-\theta)v(V) - \max\left[(1-\alpha)V - K, 0\right]\right\}^{1-\eta},$$

which is solved by

$$\theta^* = \min\left(\eta\left(1 - \frac{(1-\alpha)V - K}{V}\right), \eta\right). \tag{14}$$

Both the strategic debt servicing amount, $S(V)$, and the trigger level $\tilde{V}_S$ are determined endogenously. Solving the differential equations for the equity value with adequate boundaries, in the same vain than the equity-debt swap case, gives the following strategic debt service trigger point:

$$\tilde{V}_S = \frac{c(1-\tau+\eta\tau)}{r}\frac{-\lambda_-}{1-\lambda_-}\frac{1}{1-\eta\alpha}, \tag{15}$$

and the strategic debt service when the value of the assets is lower the trigger point is given by $S(V)=(1-\eta\alpha)\delta V$. Note here that this strategic debt servicing is decreasing in equity holders bargaining power and liquidation costs.

In summary, the basic difference between the two bargaining formulations is that, within the debt-equity swap, claimants bargain over the value of the assets of the firm, but in the second bargaining formulation, the claimants bargain over the whole firm value, that is asset value plus future tax benefits.

The Fan and Sundaresan model shows that debt renegotiation encourage early default and increases credit spreads on corporate debt, given that shareholders can renegotiate in distress to avoid inefficient and costly liquidation. It might be in the interest of debt holders to forgive part of the debt service payments if it can avoid the wasteful liquidations, which can be shared by the two claimants. If shareholders have no bargaining power, no strategic debt service takes place. Furthermore, by introducing the possibility of renegotiating the debt contract, the default can occur at positive equity value. This is in contrast to the Leland's (1994) model in that the default occurs when the equity value reaches zero as a consequence of issuing new equity is costless and the APR is respected.

Mella-Barral and Perraudin (1997) also incorporate strategic debt service by equityholders in a standard, contingent claims asset pricing model. The state variable here is no longer the firm's value, but rather the output price of the firm product. They also assume that there is no informational asymmetry and that agents are risk neutral. They consider a firm that produces a unit of output sold at a price, p_t. This output price follows a geometric Brownian motion

$$dp_t = \mu p_t + \sigma p_t dB_t$$

where μ and σ are constant and B_t is a standard Brownian motion. The firms also incur a fixed cost of production w per period in such a manner that its net earning flow is equal to $p_t - w$.

Both direct and indirect bankruptcy costs are included. For the direct cost of bankruptcy, whenever the bankruptcy occurs the new owners can only generate lower earnings, where $\xi_1 \leq 1$ and $\xi_0 \geq 1$. Moreover, the liquidation value of the firm is constant and is equal to γ. The indirect costs of bankruptcy comes from the fact the investment decision can be distorted.

Mella-Barra and Perraudin consider first a case of a firm financed only by equities, and show that even in absence of debt, liquidation may be optimal. This fact is due to the presence of bankruptcy costs described above. Introducing debt financing creates inefficiencies because of the direct bankruptcy costs it entails and because liquidation ultimately occurs at a lower level of earnings. Indeed, new owners of the firm are assumed unable to maintain the same profitability of the firm's assets compared to initial holders. The authors consider the case where the equity holders can make a take-it-or-leave-it offer to bondholders, that is, all bargaining power belongs to the equity holders, the optimal service debt proposed in this case is below the promised debt service. Thus, equity holders continue to operate the firm despite the lowered debt service payments. Inefficient liquidation is avoided in this context, at least until the liquidation threshold of a purely equity financed firm is reached, which is the efficient liquidation threshold.

When bondholders have all bargaining power, similar results are obtained. Here, bondholders cover operating losses for output prices below the optimal bankruptcy point that would occur without renegotiations. By injecting cash, bondholders keep the firm alive in hands of the equity-holders until liquidation is efficient.

Mella-Barral (1999) extends the previous cited works by allowing for departure from absolute priority rule (APR) in liquidation. This is achieved by dissociating the events of default and liquidation. Moreover, the liquidation price depends on the state variable of the model and liquidation costs are related to the inalienable human capital of the investor. In the first case, when the leverage is high, then liquidation can occur early in an inefficient manner, while for lower leverage the liquidation can occur inefficiently late. In case of low leverage, the creditors have interest in avoiding or postponing an inefficient liquidation by conceding interest payment. In the case of high leverage the investors may have interest in accelerating the default and avoiding inefficient late liquidation by offering to equity holders some of their proceeds from the liquidation, which explains the departure from the absolute priority rule.

3.5. *Bankruptcy procedures*

The models discussed above suppose a private workout for renegotiation. Nevertheless, the U.S. bankruptcy laws allow for a Court supervised debt renegotiation under Chapter 11 filing. Francois and Morellec (2004) extend the Fan and Sundaresan (2000) model to incorporate the possibility of Chapter 11 filings. Under this supervised renegotiation, the court grants the survival of the defaulting firm for an observation period. To incorporate this feature, equities are modeled as a Parisian down-and-out option on the firm's asset. The firm is liquidated, i.e. the equity holders' option to repurchase the firm's asset dies, when the value of the firm's assets reaches the default threshold and stays below that threshold for the observation period, denoted by d.

The majority of firms in financial distress that fills for the Chapter 11 emerge from the renegotiation process as an ongoing concern. In fact, Gilson, John and Lang (1990) and Weiss (1990) report evidence of low percentage of firms liquidated under Chapter 7 (Liquidation) after filing for Chapter 11. Thus, two categories of firms can be distinguished: Those that are profitable in general but default in reason of temporary financial distress and which recover under Chapter 11 and firms that continue to have losses during the reorganization process and will be liquidated by the end of the reorganization process.

Similar to Fan and Sundaresan's approach, Francois and Morellec consider a Nash bargaining game between shareholders and equity holders, where their bargaining power is denoted by η and $1-\eta$ respectively. They also suppose the firm renegotiates its debt obligations whenever the asset value falls below a constant threshold, V_B. However, Francois and Morellec model differs from Fan and Sundaresan's approach regarding the renegotiation costs. They assume that proportional costs ϕ are incurred by the company during the renegotiation process under Chapter 11, while the renegotiation costs are ignored in the Fan and Sundaresan's approach. Indeed, the financial costs of financial distress are higher in Chapter 11 filing compared to private workouts.

In this framework, the sharing rule upon default, denoted by θ, satisfies the following relation:

$$\theta^* = \arg\max \left\{ \left[\theta v(V_B)\right]^{\eta} \left[(1-\theta)v(V_B)-(1-\alpha)V_B\right]^{1-\eta} \right\}, \tag{16}$$

where $v(V_B)$ is the firm value under renegotiation that is shared between both parties and $(1-\alpha)V_B$ is the value of bondholder's claims in case of default.

$\theta v(V_B)$ and $(1-\theta)v(V_B)-(1-\alpha)V_B$ represents the renegotiation surplus for equity holders and bond holders respectively.

The solution to equation (16) is given by the following optimal sharing rule:

$$\theta^* = \eta\left(1-\frac{(1-\alpha)V_B}{v(V_B)}\right). \tag{17}$$

Francois and Morellec gives closed-form solutions to the corporate equities and debt for a given renegotiation boundary, and then assess endogenously this renegotiation threshold by maximizing the equity value. The optimal renegotiation threshold is given by:

$$V_{\mathrm{B}} = \frac{\xi}{1-\xi}\frac{c\left[1-\tau+\eta\tau(1-B(d)\right]}{r\left\{1-\eta\left[\alpha(1-C(d))-\frac{\phi}{\delta}(\delta A(d)-C(d))\right]\right\}}, \tag{18}$$

where

$$A(d)=\frac{1}{\lambda}\left(\frac{1}{\lambda+b+\sigma}+\frac{1}{\lambda-b-\sigma}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})}\right),$$

$$B(d)=\frac{\lambda-b}{2\lambda}+\frac{\lambda+b}{2\lambda}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})},$$

$$C(d)=\frac{\Phi(-(\sigma+b)\sqrt{d})}{\Phi(\lambda\sqrt{d})},$$

$\Phi(x)=1+x\sqrt{2\pi}\exp\left(\frac{x^2}{2}\right)N(x)$ where N is the standard Normal cumulative distribution function,

$$b=\frac{1}{\sigma}(r-\delta-\frac{\sigma^2}{2}),\ \lambda=\sqrt{2r+b^2}\text{ and }\xi=\frac{1}{\sigma}(b+\lambda).$$

The authors show that the default boundary in equation (18) extends both the Leland (1994) and Fan and Sundaresan (2000) models. For the Leland model the liquidation is automatic in case of default. This corresponds to the case where there is no observation period ($d=0$). On the other hand, Fan and Sundaresan allow only for private workout. This corresponds to the case where liquidation never occurs and renegotiation is costless ($d\to\infty$ and $\phi=0$). They also note that for optimal leverage level, the default threshold is increasing with the tax rate, and decreasing with shareholders' bargaining power, liquidation costs, costs of financial distress, firm risk and payout ratio.

The model implies that the introduction of possibility of renegotiation under Chapter 11 increases the credit spread on corporate debt and encourages early default, while its impact on the optimal leverage level is ambiguous.

Moraux (2004) extends the Francois and Morellec framework to account for the total time spent by the state variable, i.e. the firm's asset value, below the default level. He assumes that liquidation is triggered when the accumulated excursion time of the asset's value below the distress threshold exceeds a pre-determined grace period. Thus, the liquidation becomes a result of the entire history of the firm's financial distress, instead of only the last episode of default.

Galai, Raviv and Wiener (2007) point out two additional bankruptcy procedures characteristics:

1. Recent distress events may have greater impact on the decision to liquidate the firm compared to older financial distress episodes.
2. The impact of a financial distress on the decision to liquidate the firm is proportional to its severity.

To account for these bankruptcy procedure features, they introduce the notion of a dynamic grace period, which depends on the severity of the distress period, on its length as well as on its distance from the present. Thus, more severe and more recent distress periods are more likely to cause liquidation compared to older and less profound financial troubles.

Broadie, Chernov, and Sundaresan (2007) develop a model that also distinguishes between default and liquidation. In their model, the optimal debt and equity values are determined in the presence of both Chapter 7 and Chapter 11 under the U.S. bankruptcy code. They explicitly consider two distinct barriers for default and liquidation and consider the optimal choice of these two boundaries.

The authors extend the model of Leland (1994), where only liquidation under Chapter 7 is allowed, by accounting for the key characteristics of the reorganization procedure under chapter 11, such as automatic stay of assets during the grace period, absolute priority, and transfer of control rights from equity holders to debt holders in bad states. The state variable considered in their work is the earning before interest and taxes

(EBIT), denoted by δ_t. They assume a geometric Brownian motion for the EBIT under a risk-neutral measure. This in turn implies a geometric Brownian motion for the value of assets of an unlevered firm V_t, since $V_t = \delta_t / (r - \mu)$. Moreover, the firm issues a single consol bond to finance its projects. The bankruptcy in their model has no effect on the EBIT process. They model financial rather than economical distress since bankruptcy by itself does not cause poor performance. Therefore, when its earnings are insufficient to make the necessary coupon payment, c, the firm leaves the liquid state and enter financial distress.

If the firm's EBIT deteriorates further to reach the bankruptcy boundary, δ^B, the firm stop paying dividends to equity holders and bears a proportional distress cost as long as it remains in the default state. Moreover, the total EBIT is accumulated in a separate account S_t during bankruptcy, while A_t represents the accumulated unpaid coupons plus interest in arrears.

Depending on the evolution of the firm's EBIT after default, three scenarios are possible. First, when the firm recovers from Chapter 11, the debt holders will forgive a fraction of $1-\theta$ arrears, where $0 \le \theta \le 1$, and receive an amount θA_t. If S_t is not sufficient to repay the arrears, equity holders must raise the remaining at the cost of diluting equity. In contrary, if $S_t > \theta A_t$, the amount of θA_t is paid to creditors and the remaining is distributed to shareholders.

The second scenario is when the firm remains in bankruptcy, for a time longer than the grace period. In this case, the automatic stay provision is no longer granted and the firm is liquidated at a cost α. Finally, if the firm's earning continues to deteriorate during the grace period, in such a manner to breach a lower liquidation barrier δ^B, then the firm is liquidated.

The main contribution of Broadie, Chernov, and Sundaresan (2007) compared to previous models that distinguishes between default and liquidation, is the possibility of liquidation whenever assets value become too low during the observation period. Thus, liquidation could happen as the firm value either reaches the liquidation barrier or stays under the bankruptcy barrier for longer than the grace period.

In their paper, they focus on the issues of bankruptcy proceedings and the optimal choice of these two boundaries driven by different objectives. They show that the first-best outcome, the total firm value maximization ex-ante upon filing Chapter 11, is different from the equity value maximization outcome. They also show that the first-best outcome can be restored in large measure by giving creditors either the control to declare Chapter 11 or the right to liquidate the firm once it is taken to Chapter 7 by the equity holders. This serves as the threat from debtholders to prevent equity holders from filing for Chapter 11 too soon to get debt relief. Finally, they also find that on average the firms are more likely to default and are less likely to liquidate relative to the benchmark model of Leland (1994).

3.6. *Dynamic capital structure*

The models described above assume a static capital structure. The optimal leverage remains constant during the life of the firm. Fisher, Heinkel and Zechner (1989) propose a model where shareholders choose optimal recapitalization in a continuous-time framework. They assume that the firm's investment decisions are exogenous and independent from financing decision. They also assume a geometric Brownian motion for the firm's assets, A. Therefore, for a given face value of debt, B, the value-to-debt ratio, $y = A / B$, also follows a geometric Brownian motion. The firm issues new debt if its value-to-asset ratio, y, increases to an upper boundary, $\overline{y}$, in order to benefit from debt-related tax shields. When y reaches a lower boundary, $\underline{y}$, the firm reduces its debt this time to avoid bankruptcy costs or to be compliant with equity holders limited liability.

In addition, the model relies on two assumptions. First, the value of an optimally levered firm can only exceed its unlevered value by the amount of transactions costs incurred in order to lever it up. This hypothesis is aimed to avoid the possibility of purchasing the sub optimally levered firm, issue additional debt and then sell it for a riskless profit (no arbitrage possibility). Second, a firm that follows an optimal financing policy offers a fair risk adjusted rate of return. Therefore, if leverage is advantageous, then it follows that unlevered firms offer a below-fair expected rate of return.

Fisher, Heinkel and Zechner (1989) characterize the advantage of leverage as:

$$\delta = r(1-\tau_p) - \hat{\mu}$$

where r is the risk free rate, τ_p is the personal tax rate and $\hat{\mu}$ is the risk-adjusted expected growth rate of the market value of the firm's unlevered assets.

The capital structure equilibrium is defined by the upper and lower recapitalization boundaries, respectively $\overline{y}$ and $\underline{y}$, the face value of debt, B, the advantage of leverage, δ and the coupon rate, i, that maximize the value of firm net of recapitalization costs. The maximisation problem can be expressed as:

$$\underset{\underline{y},\overline{y},B,i}{Max}\, V(y_0, B, \underline{y}, \overline{y}) - kB$$

subject to

$$V(y_0, B, \underline{y}, \overline{y}) = By_0 + Bk:$$

$$E_y(y = \underline{y}, B, \underline{y}, \overline{y}) \geq 0$$

$$D(y_0, B, \underline{y}, \overline{y}) = B,$$

where V is the value of the firm, E is the value of equity, D is the value of debt[6], y_0 is the initial value-to-debt ratio and k is the recapitalization proportional cost. The first condition is a no arbitrage condition. Indeed recall that according to Fisher, Heinkel and Zechner (1989), in absence of arbitrage the value of the firm must be equal to the value of

[6] The value of equity and debt is obtained by solving a PDE similar to equation (11). The interested reader is referred to the original paper for further details.

its unlevered assets, $A_0 = By_0$, plus the transaction costs Bk. The second constraint grants that the equity value is positive[7] and the last one state that the debt is issued at par.

Numerical solutions for different parameters values show that the resulting optimal dynamic capital structure policy depends on the tax advantage, the bankruptcy costs, the assets volatility, the riskless interest rate and the costs of recapitalization.

Goldstein, Ju, and Leland (2001) argue that the two assumptions advanced by Fisher, Heinkel and Zechner (1989) do not hold in practice. First, the necessary premium to gain control of the firm may deter arbitrage possibility for under-levered firms and second, the market price adjustment allows obtaining fair expected return for firms with publicly traded assets, even if they are unlevered. Goldstein, Ju, and Leland (2001) choose to model the dynamics of EBIT as state variable, instead of the usually used unlevered firm value. They justify this choice by the invariance of the EBIT generating mechanism to the capital structure decision. They notice that using the generated cash flows to pay dividends, taxes or debt services have the same effect on the firm. The advantage of taking the claim on future EBIT is that all contingent claimants to future EBIT flows, including the tax payment, are treated in a consistent fashion. Especially, the tax shelter is no longer treated as a cash inflow in a form of tax benefit, but as a cash outflow in the form of tax. The authors argue that the invariance feature makes the claims on EBIT a well suited framework for investigating multiple capital structure changes and. hence, optimal dynamic capital strategy.

Goldstein, Ju, and Leland (2001) assume a geometric Brownian motion for the EBIT, δ, under a risk-neutral measure with drift μ and volatility σ. This in turn implies a geometric Brownian motion for the value of assets of an unlevered firm V_t, since $V_t = \delta_t / (r - \mu)$. They also assume a single consol bond issuance to have time independence of the payout. This grants that any claimant satisfies the following ordinary differential equation:

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7 The authors consider the case of riskless debt. In this case the second constraint becomes $\underline{y} = 1$.

$$\frac{\sigma^2}{2}V^2F_{VV}+\mu VF_V-rF+P=0, \tag{19}$$

where P is the payout flow.

They define $p_B(V)$ as the present value of a claim that pays 1$ when the firm's value reaches V_B, the default boundary. This claim satisfies Equation (19) with $P=0$ because there is no intermediate payout. The solution takes the following form:

$$p_B(V)=A_1V^{-y}+A_2V^{-x} \tag{20}$$

$$\text{where } x=\left[\left(\mu-\frac{\sigma^2}{2}\right)+\sqrt{\left(\mu-\frac{\sigma^2}{2}\right)^2+2r\sigma^2}\right]\Bigg/\sigma^2,$$

$$y=\left[\left(\mu-\frac{\sigma^2}{2}\right)-\sqrt{\left(\mu-\frac{\sigma^2}{2}\right)^2+2r\sigma^2}\right]\Bigg/\sigma^2.$$

where x is positive, while y is negative. The boundary conditions are defined by:

$$\lim_{V\to\infty}p_B(V)=0 \text{ and } \lim_{V\to\infty}p_B(V)=0.$$

Therefore

$$p_B(V)=\left(\frac{V}{V_B}\right)^{-x}.$$

Define $V_{solv}(V)$ a claim entitled to the entire payout δ as long as the firm remains solvent, i.e., firm value remains above V_B. The solution for equation (19) takes the form:

$$V_{solv}(V)=V+A_1V^{-y}+A_2V^{-x}$$

with the following boundary conditions:

$$\lim_{V\to\infty}V_{solv}=V\ (A_1=0) \text{ and } V_{solv}=0 \text{ as } V=V_B$$

which gives

$$V_{solv} = V - V_B p_B(V) = V - V_B \left(\frac{V}{V_B}\right)^{-x}.$$

For the claim on the interest payment while the firm is solvent, the solution is in the form of $V_{\text{int}}(V) = C/r + A_1 V^{-y} + A_2 V^{-x}$ where C is the coupon payment. When V tends to infinity this claim tends to C/r, thus $A_1 = 0$ here again and $V_{\text{int}} = 0$ when $V = V_B$. The claim on interest is then given by $V_{\text{int}} = \frac{C}{r}[1 - p_B(V)]$.

The separation of value of the continuing operation between debt, equity and government gives:

$$E_{solv}(V) = (1-\tau_{eff})(V_{solv} - V_{\text{int}}) = (1-\tau_{eff})\left[(V - \frac{C}{r}) - (V_B - \frac{C}{r}) p_B(V)\right],$$

$$D_{solv}(V) = (1-\tau_i) V_{\text{int}} = (1-\tau_i)\frac{C}{r}[1 - p_B(V)],$$

$$G_{solv}(V) = \tau_{eff}(V_{solv} - V_{\text{int}}) + \tau_i V_{\text{int}}.$$

τ_{eff} is the effective tax rate, and τ_i is the tax rate on interest payments.

Both the coupon level C and the bankruptcy level V_B are chosen by management to maximize the equity wealth. The optimal bankruptcy level is obtained by the smooth-pasting condition $\left.\frac{\partial E}{\partial V}\right|_{V=V_B} = 0$ which yields:

$$V_B^* = \left(\frac{x}{x+1}\right)\frac{C^*}{r}.$$

The optimal coupon C^* is obtained by maximizing the shareholder wealth, i.e. the value of equity and debt:

$$\max_c \left\{(1-q)D(V, V_B, C) + E(V, V_B, C)\right\}.$$

This yields to

$$C^{*}=V\left(\frac{xr}{x+1}\right)\left[\left(\frac{1}{1+x}\right)\left(\frac{A}{A+B}\right)\right]^{\frac{1}{x}}$$

where

$$A=(1-q)(1-\alpha)-(1-\tau_{eff})$$

$$B=\frac{x}{x+1}(1-\tau_{eff})\left[1-(1-q)(1-\alpha)\right],$$

q denotes the restructuring costs and α the bankruptcy costs.

In contradiction with models that use the unlevered firm value as state variable, e.g. Leland (1994), the comparative statics shows that the value of equity is decreasing in the effective tax rate. This is due to the fact that a rise of tax rate increases the government claim at the expense of equity, instead of considering the tax benefit as a cash inflow.

Goldstein, Ju, and Leland extend the static model to allow for a dynamic capital structure where the management can adjust the firm leverage upward. As in Fisher, Heinkel and Zechner (1989), they assume that in addition to the threshold V_B where the firm optimally chooses to default, there will be a threshold V_U where the management call the outstanding debt and sell a larger issue. They show by backward induction, that if the EBIT increases by a scale γ at each period, then the optimal restructuring and bankruptcy thresholds will increase by the same factor. They find that the optimal initial leverage level with dynamic capital structure is much lower than the one found with static capital structure. This is explained by the option to increase leverage in the future. Also, the bankruptcy threshold decreases when the capital structure is dynamic, the intuition behind this result is that the firm with the option to adjust its capital structure is more valuable, and therefore has more incentive to avoid bankruptcy.

4. Other extensions

Ju, Parrino, Poteshman and Weisbach (2005) consider a dynamic model of optimal capital structure where the firm financing decision is determined by a balancing between corporate taxes advantage and bankruptcy costs (trade-off theory). The value of the unlevered assets has an exogenous process. The authors specify a model in which new debt is reissued when old debt matures to keep a given leverage ratio. However, the default boundary is exogenous and has an exponential form. Collin-Dufresne and Goldstein (2001) also consider a dynamic capital structure by modeling a mean-reverting leverage ratio and stochastic interest rate.

Acharya and Carpenter (2002) develop a model with both stochastic interest rate and endogenous defaults. The interest rate is modeled as one-factor diffusion process and the issuer follows optimal call and default rules. Thus, they bridge the gap between endogenous default and stochastic interest rate literatures. They model call and default options as American options written on a non callable, default free bond with fixed continuous coupons. The authors characterize the default region for both callable and non callable bonds and find that this default region is smaller for the callable bond relative to the non callable one. They show that the existence of the call option can encourage the firm to continue servicing its debt when it would otherwise default.

Most of the structural models assume that firm’s risk remains constant. Leland (1998) allows the firm to choose its risk strategy and examine the agency problem between equity holders and debt holders related to asset substitution. The model also permits to examine the interaction between capital structure and risk strategy.

Leland (1998) assumes that risk choices are made after the debt is in place, and these choices cannot be constrained through debt covenants or other precommitments. However, he presumes rational expectations, in that both equity holders and the debt holders will correctly anticipate the effect of debt structure on the chosen risk strategy, and the effect of this strategy on security pricing. Thus, he assumes that there is no information asymmetry. In this setting, once the financing decision is set, the stockholders choose the investment policy that maximizes the equity value ex post, but reduce the value of other claimants such as tax, external claimants in default and especially debtholders, creating agency costs due to asset substitution. The initial optimal capital structure made ex ante will balance these agency costs with the tax benefits of debt less default costs.

To measure these agency costs, the firm value with ex post investment decision is contrasted with the situation where both risk strategy and debt structure are made simultaneously ex ante to maximize the firm value. The difference in optimal firm value between ex post and ex ante situations represents the loss in value due to maximization of equity value instead of firm value.

A similar approach is adopted for risk hedging strategy. The firm can decrease its risk level through hedging, and cease hedging at any time. Two environments are considered. In the first, both capital structure and hedging strategy are determined ex ante to maximize market value (ex ante hedging strategy), while in the second, the hedging strategy is established to maximize equity value ex post, i.e. after financing decision is made (ex post hedging strategy). The optimal firm values is compared under ex ante hedging and ex post hedging strategies with the situation where the firm can never hedge and the situation where the firm always hedge. The difference between the value of a firm using optimal hedging strategies and the value of the same firm when hedging is not allowed, represent the benefit of hedging.

Hackbarth, Henessey and Leland (2007) distinguish between bank and public debt. They assume that renegotiation through private workout is only possible for bank debt. This renegotiation possibility makes bank debt more attractive, but limits bank debt capacity for strong firms, e.g. firms with high bargaining power. When the strong firm reaches its bank debt capacity, the firm complements bank debt by public debt to benefit from more tax shield. The model therefore propose an explanation to the seniority of bank debt, and to the fact that small/weak firms relies exclusively on bank debt while mature/strong firms uses a mix of public and bank debt. Bourgeon and Dionne (2007) extend the Hackbarth, Henessey and Leland (2007) model to allow banks to adopt a mixed strategy in which renegotiation is sometimes refused ex-post in order to raise debt capacity ex-ante. Carey and Gordy (2007) suppose that holders of private debt, e.g. banks, with strong covenants control the choice of the bankruptcy threshold. Since the private debt is senior, the bank triggers bankruptcy only when the asset's value falls below the face value of the bank debt. In accordance with their model, they find empirical evidence indicating that the recovery rate is sensitive to debt composition.

Other extensions include Mauer and Triantis (1994), Childs, Mauer and Ott (2005) and Sundaresan and Wang (2007) who consider endogenous investment. The cash holding management policy is accounted for in Acharya, Huang, Subrahmanyam and Sundaram (2006), Anderson and Carverhill (2007), and Asvanunt, Broadie, and Sundaresan (2007). Sarkar & Zapatero (2003) consider mean reverting cash flows. Zhou (2001), Duffie and Lando (2001) and Giesecke & Goldberg (2004) add a jump component to the value process of assets allowing for "surprise" default at the cost of closed-form solution. Alernatively, Hackbarth, Miao & Morellec (2006) consider jumps in the cash flow process with regime change. Finally, Longstaff (1996), Morellec (2001) and Ericsson and Renault (2006) include a liquidity premia to price corporate debt, while Duffie and Lando (2001) consider accounting information uncertainty.

5. Empirical Evidence on Corporate Credit Risk

The empirical literature on structural models assesses the ability of different models to predict the credit spread on bonds and CDS. Another trend of the literature assesses the ability of different credit risk models, including the structural models, to predict defaults and the relation between the default risk and equity return.

5.1. *Corporate Credit Risk, Yield Spread and Default Frequency*

Jones, Mason, and Rosenfeld (1984), compare the spread predicted by the Merton (1974) model and the empirically observed spreads and find that the credit yield spreads generated by the Merton model are too low. Franks and Torous (1989), find similar results with realistic parameter. Moreover, Anderson and Sundaresan (2000), Lyden and

Saraniti (2000) show mixed results on the ability of structural models to explain observed corporate yield spreads.

Elton, Gruber, Agrawal, and Mann (2001) using Fixed Income Database on US corporate and financial institutions bonds from 1987 to 1996, find that default risk accounts for a low portion of the yield spread. Indeed, depending on credit quality and industry, default risk accounts for between 7% and 35% of the yield spread while the tax differential is found to be a major factor in the overall credit spread. Elton et al. argue that the rest of the corporate bond yield spread represents compensation for systematic risk in corporate bonds. Using linear regressions of bond returns on empirically identified Fama-French factors, the authors show that a large proportion of the yield spread unexplained by default risk and taxes is explained by the three factors of Fama and French (1993) (67% for financial institutions and 85% for industrial). They conclude therefore that the credit risk and tax premium can only partly explain for the difference in corporate spread.

Huang and Huang (2003) use a variety of structural models to examine how much of the historically observed corporate-Treasury yield spread is due to default risk. To explore whether this spread can be explained by implied default probabilities from the structural models. The structural models studied include Longstaff and Schwartz (1995) with stochastic interest rate, Leland and Toft (1996) for endogenous default boundary, Anderson and Sundaresan (1996), Anderson, Sundaresan, and Tychon (1996) and Mella-Barral and Perraudin (1997) for strategic default, and Collin-Dufresne and Goldstein (2001) for mean reverting leverage ratio.

Huang and Huang calibrate each model's parameters to match the observed expected default frequency and the average loss given default for each broad rating category. The average empirical leverage by rating grade is also used as input in the calibration. Since the structural models predict not only bond prices but also equity prices, the authors use equity premium to assess the assets risk (volatility) premium. Thus, the target quantities to calibrate the models are the leverage ratio, the equity premium, the default probability and recovery rate. The time horizons considered are respectively 10 and 4 years.

They find that the calibrated structural models generate similar credit spreads. Moreover they find that the credit risk explains between 20% and 30% of the investment grade treasury yield, while this proportion increases for riskier bonds and accounts for a large portion of the yield spread. However, this fraction decreases as the bond maturity shortens. Indeed, the fact that structural models rely on diffusion process of the value of the firm's assets, makes the credit spread converge to zero for short maturities, which contradicts the empirical observation. The authors conclude that additional factors such as illiquidity and taxes must be important in explaining market yield spreads.

Collin-Dufresne, Goldstein, and Martin (2001) focus on changes in corporate credit spreads. They use the theoretical inputs of structural models as explanatory variables in credit spreads regression. They find a limited explanatory power of these variables, and that a significant part of the residuals is driven by a common systematic factor that is not captured by the theoretical variables. They also find that credit spreads decrease as the market becomes more liquid as measured by the relative frequency of quotes versus matrix prices in the Fixed Income Database (FID). Thus, Collin-Dufresne, Goldstein, and Martin show that the credit spreads of individual bonds react to changes in aggregate liquidity, but do not address changes in liquidity at the individual bond level.

Similar analysis is performed by Campbell and Taksler (2003) using regressions for levels of the corporate bond spread. They conclude that firm specific equity volatility is an important determinant of the bond spread, and that the economic effects of volatility are large. Cremers, Driessen, Maenhout, and Weinbaum (2004) give support to this result and argue that option-based volatility contains useful information for this type of analysis that is different from historical volatility.

These evidences suggest that the observed yield spread contains a large proportion due to liquidity and tax differential. This could explain the weak performance of structural models to reproduce yield spread without a larger jump sizes or larger credit risk premia than in typical calibration.

To circumvent the problem of liquidity and taxes differential in yield spreads, Leland (2004) focus on the ability of exogenous and endogenous structural credit risk models to capture the observed default frequencies across bonds with different ratings. He calibrates the exogenous default models as represented by the Longstaff, Schwartz (1995) model, and the Moody's-KMV variant of the Merton model with common inputs and examines how well these models match the observed default frequencies as reported by Moody's over the period 1970-2000. Leland finds that both models achieve good performance in predicting the shape and the level of default probabilities for horizons exceeding 5 years, but under-predict the default frequencies for shorter time horizons. Since the default frequencies are not affected by bonds liquidity, he concludes that the addition of jumps in the asset value process, as proposed by Zhou (2001) for instance, can solve both the underestimation of the default probabilities and the yield spread.

Eom, Helwege and Huang (2004) also test the ability of five structural models to predict the yield spread of firms with simple capital structure. They find that the Merton (1974) and Geske (1977) models generate spreads that are far below the observed ones on the bond market, in accordance with the previous literature. However, the Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001) models overestimate spreads for riskier bonds (high volatility and leverage) while they underestimate the spreads for less risky bonds.

Longstaff, Mithal, and Neis (2005) make use of the Credit Default Swaps premia to separate the corporate bond yield spread into a default component and a non-default component. They find that the default component increases from an average of 51% of the spread for AA bonds up to 83% for BB bonds. The non-default component in their sample varies substantially with a range of 18.8 to 104.5 basis points and a mean of 65 basis points. Longstaff et al. (2005) find that the non-default component is related to both the degree of asymmetric tax treatment and a proxy for bond liquidity. The non-default component is positively related to the coupon rate of the bond, indicating the market is pricing the differential tax treatment of corporate bonds.

Houweling and Vorst (2005) implement a set of simple reduced form models on market swap quotes and corporate bond quotes. Their paper focuses on the pricing performance of the model and the choice of benchmark yield curve.

Regarding the calibration of structural models, their implementation requires the knowledge of the assets value and volatility. However, these inputs are not observable since only equities are priced by stock markets. Most of the implementations of structural models approximate the value of assets by the market value of equities plus the book value of debt and the assets' volatility using equities' volatility and adjustment for debt in capital structure (Eom, Helwege and Huang, 2004, for instance). Beside this approximation, several methods were proposed in the literature. Jones, Mason and Rosenfeld (1984), Ronn and Verma (1986) use an alternative method that makes use of Itô's lemma to obtain a system of two equations linking the unknown asset values and the asset volatility to the observed equity values and volatility. However, this method was criticized due to the assumption of constant volatility and lack of statistical inference. Crosbie and Bohn (2003) develop an iterative proprietary method based on variance restriction method of Moody's KMV. Duan (1994) and Duan, Gauthier and Simonato (2004) propose a maximum likelihood estimation method, based on equity prices to estimate asset value and volatility. Ericsson and Reneby (2002) conduct a simulation study for different structural models and demonstrate the higher performance of the maximum likelihood estimation compared to the variance restriction method. Li and Wong (2008) empirically examine the proxy, volatility-restriction and maximum likelihood approaches to implement structural corporate bond pricing models, and find also that ML estimation is superior to the other considered methods. Bruche (2005) propose a method that combines different priced assets to estimate asset value and volatility.

Hull, Nelken and White (2004) present an alternative approach to estimate the unobservable asset volatility. Considering the implied volatility of options on the company's stocks, the authors propose a different approach than the variance restriction method, to measure assets volatility. The method is based on Geske (1979) model, which suggests that since the equity of a company can be considered as an option on the firm's assets, an option on the firm's stock is a compound option, and further provides a valuation formula for such compound option. Using Geske (1979) formulation, the authors present a two-equation system that can be solved with two implied volatilities, sampled from stock options.

While testing the proposed alternative with credit default swaps (CDS) spread data, the authors find that this implementation of the Merton model outperforms the traditional methodology.

Schaefer and Strebulaev (2008) study the sensitivity of the corporate bond returns to changes in the hedge ratios and find that structural models provide accurate estimates of hedge ratio. The authors conclude that the limited ability of structural models to accurately predict bond prices is due to non-credit factors.

Davydenko and Strebulaev (2007) identify firm specific strategic factors that affect credit spread. In fact, strategic default models predict lower bond prices when the threat of strategic default is more likely. They proxy for renegotiation frictions, bargaining power in renegotiation and liquidation costs by using debt complexity measure, equity ownership and asset tangibility respectively. They find a significant relationship between these factors and the credit spread, although the economic effect is limited and could not be the reason of the limited performance of structural models to match the levels of credit spreads.

Several studies investigate the sensitivity of credit spread to macro-economic factors. Bakshi, Madan and Zhang (2006) and Elton, Gruber, Agrawal and Mann (2001) show that an important part of corporate bond credit spreads is explained by factors commonly used to model risk premiums for common stocks. Fama and French (1989) find wider credit spreads when economic conditions deteriorate. Similar results are achieved by Duffie, Saita and Wang (2007) who show that macroeconomic variables explain a large portion of yield spread changes and default rates.

Tang and Yan (2006) model relates the firm credit spreads to macroeconomic conditions through the sensitivity of its cash flows to economic factors. A link between market and credit risk is established in their framework. They show that accounting for the macro-economic effect improves fitting the default probabilities and credit spread. David (2008) and Chen (2007) models also predict a decrease of the default probability and credit spreads in macro-economic expansion.

Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn and Strebulaev (2007), Chen (2007), and David (2008) use regime switching models to link credit spread dynamics to macroeconomic conditions and/or the equity risk premium which allows detecting higher impact of economic aggregates on credit spreads.

Both Fama and French (1989) and Koopman and Lucas (2005) find a countercycle behavior of the credit spread. This evidence suggest a distinction between credit cycle and economic cycle (see Dionne, Maalaoui, François, 2009).

Overall, several factors beside the default risk seem to drive the corporate credit spread, including liquidity, volatility, firm specific factors and market conditions (see Dionne et al., 2008, and Dionne and Laajimi, 2012, for recent estimations of default risk using data from the Toronto Stock Exchange).

5.2. *Structural Models and Default Forecast*

Moody's KMV developed a commercial model derived from the Merton approach, and adjusted to agency ratings and other bond characteristics. The distance-to-default, that is, the normalized distance, measured in standard deviations, of a firm's asset value from its default threshold. Distance-to-default plays a central role in calculating the expected default frequency (EDF) in the Moody's KMV model. Sobehart, Keenan and Stein (2000), and Stein (2002), among others studies, examine the accuracy of the Moody's KMV model. Both studies find the Moody's KMV model to be incomplete. Kealhofer and Kurbat (2002) find opposite results, namely that Moody's KMV model captures all the information contained in agency ratings migration and accounting ratios. Crosbie and Bohn (2003) find that combining market prices and financial statements gives more effective default measurement. The authors empirically test the EDF, derived from the

KMV methodology, versus the credit rating analysis, and show that the EDF obtains a better power curve.

The accuracy of default forecasting of the KMV model is studied in Bharath and Shumway (2004). The authors compare the KMV model accuracy with simpler alternative. They find that implied default probabilities from credit default swaps and corporate bond yield spreads are only weakly correlated with KMV-Merton default probabilities. The authors conclude that the KMV-Merton model does not provide a sufficient statistic for default, which can be obtained using relatively naïve hazard models. Hillegeist, Keating, Cram and Lundstedt (2004) and Du and Sou (2005) compare the KMV model to other models, and conclude that the KMV model does not provide adequate predictive power.

However, Duffie, Saita and Wang (2007) discover a significant predictive strength over time within the KMV model. Campbell, Hilscher and Szilagyi (2004) use hazard models to condition the KMV model on other relevant default variables, and find a poor predictive power of the KMV model.

Moody's proposes its own commercial implementations of hybrid models. Indeed, Sobehart, Stein, Mikityanskaya, and Li (2000) use a comprehensive proprietary database of over 1,400 US non-financial defaults to assess the performance of Moody's hybrid model in predicting defaults. They combine the structural distance-to-default with other rating, market, and accounting variables. They conclude that neither the structural model nor the financial statements will contain all the relevant information on the firm's credit worthiness. Thus, combining the two methods seems justifiable, since the hybrid model outperforms both the pure structural model and the pure statistical one. However, when Kealhofer and Kurbat (2002) attempted to replicate these findings, they got opposite results. The KMV implementation of the Merton structural approach based on distance to-default shows that the structural model excels other measures of credit risk. Hillegeist, Keating, Cram, and Lundstedt (2004) have documented that the theoretical probabilities estimated from structural models do not capture all available information about a firm's credit risk. They show that traditional risk measures, such as the updated versions of

Altman’s Z-Score and Ohlson’s O-Score, do add incremental information and that the default probabilities estimated from structural models are therefore not a sufficient statistic of the actual probability of default.

5.3. *Structural Models and Stock Returns*

The distance to default is widely used in the finance literature as a measure of credit-worthiness. On the other hand, the relationship between financial distress and stock returns was studied in several papers. The financial distress is measured either through accounting based measures, agencies ratings or structural model. Dichev (1998) and Griffin and Lemmon (2002) use Altman’s Z-score and Ohlson’s O-score to measure financial distress and find evidence of underperformance of distressed stocks. Avramov et al. (2006), rely on credit ratings to detect distressed firms and find similar results.

On the other hand, Garlappi, Shu, and Yan (2008) using default risk measures from Moody’s KMV, find that stocks with a high risk of failure tend to have anomalously low average returns. However, Vassalou and Xing (2004) measure the distance to default of listed firms and find that financially distressed stocks earns higher returns contradicting the previous results. This higher return is due mainly to small value stocks. Moreover, Da and Gao (2010) attribute this abnormal return to liquidity factors. Indeed, they find that the liquidity risk rise for distressed stocks and the prices recovers in the following month, which explains the high return of stocks with high default likelihood. Indeed, Campbell, Hilscher and Szilagyi (2008) use both distance to default and logit models to detect financial distress and find evidence of price anomaly since distressed stocks earn lower return. They also find that distressed firms have high market betas and high loadings on the HML and SMB factors of Fama and French (1993, 1996).

Conclusion

In this paper, we review the most influential and representative structural models. Structural models offer an intellectually appealing approach to modeling credit risk. They provide a link between the more traditional corporate finance models and the contingent claims analysis. These models study interesting questions of security design, optimal investment and financing decisions, or the incentives resulting from the bankruptcy law.

Most of the structural models provide closed-from expressions of corporate debt as well as the endogenously determined bankruptcy level, which are explicitly linked to taxes, firm risk, bankruptcy costs, risk-free interest rate, payout rates, and other important variables. The behavior of how debt values (and therefore yield spreads) and optimal leverage ratios change with these variables can thus be investigated in detail.

While theoretically elegant, capital structure models do not perform well empirically in risky corporate bond pricing. Researchers have been attempting to resolve the yield spread underestimates by introducing jumps and liquidity premium. On the other hand, the poor performance of structural models may have more to do with the influence of non-credit factors rather than their failure to capture the credit exposure of corporate debt. Growing evidence shows that multiple firm characteristics and market and economic conditions are important determinants of corporate credit spread. Moreover, since recent capital structure models put numerous efforts on the event of bankruptcy, structural models are useful for prediction of default probabilities or default events. Finally, some researchers argue that the past poor performance of capital structure models may come from the estimation approaches traditionally used in the empirical studies and we have seen some innovative estimation methods aiming for solving the estimation problem in models employing structural approach.

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