A Rationale for Borrowing More than Needed[*]

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Abstract

Suppose an entrepreneur wants to borrow funds from a financier to invest in a risky project whose first period cost is fixed (I), and whose second period return may be high or low. Suppose also that the project's realized return is an information that is private to the entrepreneur. If the amount the entrepreneur pays back to the financier depends on the risky project's outcome, if it is costly for the financier to verify the true project's realization, and if the financier cannot commit to an auditing strategy, then it will be optimal for the entrepreneur to mis-report the true state of the world with some probability. In other words, it will be optimal to lie to the financier with some probability. The Perfect Bayesian equilibrium of this game yields the uncommon result that the entrepreneur's final wealth is greater when the project has a low return. The most striking result of the paper is that the entrepreneur's limited liability constraint in any state of the world is a sufficient condition for the entrepreneur to choose an amount of debt (D) that is greater than the cost of the project (D > I).

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Résumé. Supposons qu'un entrepreneur veuille emprunter de l'argent d'un …nancier a…n d'investir dans un projet risqué dont le coût d'investissement de première période est …xe (I), et dont le rendement de seconde période est risqué. Supposons également que le rendement du projet en seconde période est une information connue de l'entrepreneur seul. Si le montant que doit rembourser l'entrepreneur au …nancier dépend du rendement réalisé sur le projet, s'il est coûteux pour le …nancier de véri…er le rendement du projet, et si le …nancier ne peut pas se commettre à une stratégie d'audit, alors il sera optimal pour l'entrepreneur de ne pas rapporter le vrai état de la nature à tout coût. En d'autres mots, il aura intérêt à mentir avec une certaine probabilité. L'équilibre bayésien parfait de ce jeu entre le …nancier et l'entrepreneur a comme résultat peu commun que la richesse ex-post de l'entrepreneur est plus élevé si le rendement du projet est faible. Le résultat le plus surprenant du model reste toutefois que la contrainte de responsabilité limitée de l'entrepreneur dans tous les états de la nature est une condition su¢sante pour que l'entrepreneur emprunte plus que nécessaire; son montant de dette sera plus grand que le coût du projet.

1 Introduction

1.1 Motivation

When an entrepreneur has a risky project to invest in, he may go to a …nancier[1] to borrow the funds that are needed. An entrepreneur who has some private information concerning the investment project may …nd it in his best interest to attempt to extract a rent from the …nancier. For example the entrepreneur may shirk on the job, thus reducing the probability that the project has a high return. This shirking problem is known as ex-ante moral hazard in the literature.

Another type of asymmetric information between the entrepreneur and the …nancier concerns the return on the project. If the transfer between the entrepreneur and the …nancier depends on a message sent by the entrepreneur after he has privately witnessed the state of the world (i.e.: he is the only one to know what the return on the investment is), then he may have an incentive to mis-report this information to the …nancier. The incentive for the entrepreneur to send a false message to the …nancier concerning the state of the world he observed can be viewed as a problem of ex-post moral hazard.[2] It is moral hazard because it is the entrepreneur's action (report to the …nancier) which is sub-optimal, and it is ex-post because his action occurs after Nature has decided what was the project's realization. In this paper, we will only study the ex-post moral hazard problem.[3]

[1] We will call …nancier the player that lends the money, and entrepreneur the player who wants to borrow money to invest in a risky project. The …nancier will be a she, and the entrepreneur will be a he.

[2] Although some refer to this problem as an adverse selection problem, we will use the term ex-post moral hazard. The di¤erence is semantic of course, but we believe that adverse selection should refer to economic problems where the sequence of play is Nature-Agent-Nature while ex-post moral hazard should refer to a Nature-Agent-Principal sequence of play. In the same vein, ex-ante moral hazard refers in our view to a sequence of play of the type Agent-Nature-Principal or Agent-Principal-Nature.

[3] The are many other problems that may occur when there is asymmetric information.

A well known result in the literature [see Townsend (1979) and Gale and Hellwig (1987)] is that if there is ex-post moral hazard on the part of the entrepreneur, and if the …nancier can commit to a deterministic auditing strategy, then the optimal contract between the two players will be what Gale and Hellwig call a standard debt contract. A standard debt contract has the characteristic that the entrepreneur who is able to make his schedule payments (interest on the debt presumably) is not audited. As soon as he misses a payment, however, he is audited and all his assets are seized. In this type of contract it is not optimal for an entrepreneur to miss a payment he is able to make. Since he is always audited when he misses a payment, he will always be found to have cheated. And since the value of his assets seized is greater than the payment he missed, it is not optimal for the entrepreneur to mis-report the true project's realization.

On the other side of the market, there is the …nancier. For the …nancier, the provisions of the standard debt contract seem attractive. If she commits to auditing the entrepreneur every time he misses a payment, then she is guaranteed that no payment that can be made are missed. Unfortunately, when time comes to audit a payment that was missed, both the …nancier and the entrepreneur …nd optimal to renegotiate[4] that part of the contract. If the …nancier knows that the entrepreneur has told the truth (because in a standard debt contract telling the truth is always the best strategy), and

For example, we may have adverse selection in the sense that some entrepreneur are more quali…ed than others, and their quali…cation is known only to them [see Rothschild and Stiglitz (1976) and Spence (1972)]. We could also have that some entrepreneurs have a better work ethic and never shirk, and that this ethic is also known only to them. On the moral hazard side, we could have that instead of investing e¤ort to make sure that the higer return is realized, an entrepreneur may choose to invest his money in a project that yields greater utility to him, but less to the …nancier [see Jensen and Meckling (1976) and Myers (1977)].

[4] Surprisingly Shleifer and Vishny (1997) do not mention this renegotiation problem in their survey of corporate governance. This renegotiation is not the same as the possibility of bribing a manager, a possibility they do mention.

that auditing the entrepreneur is costly, then she will want to renegotiate the contract with the entrepreneur. She want to do so because it saves her the cost of auditing. The entrepreneur also wants to renegotiate even if he still loses all he has. The entrepreneur has nothing to gain by being audited.

On the other hand if the entrepreneur willingly agrees to give everything to the financier (so that the financier does not need to incur the cost of auditing), then he may be rewarded by the financier. The financier will be willing to share with the entrepreneur the savings generated by not conducting the audit. In other words, when there is an audit, the payoff to the entrepreneur is zero, while the payoff to the financier is $W - c$, where W is the residual wealth of the entrepreneur and c is the cost of auditing. If no audits are conducted, and the entrepreneur willingly gives his residual weath to the financier, then the payoffs to the two economic agents are αc for the entepreneur and $W - \alpha c$ for the financier, where $\alpha < 1$ is some percentage of the savings generated by the non-audit that the financier is willing to share with the financier. Thus auditing is Pareto dominated by not auditing.

If the entrepreneur knows that the financier will never want to audit when the time comes, then he will find it in his best interest to always miss a payment. If the financier knows that the entrepreneur always misses a payment, then the financier will not want to never audit. In the end what we would have is that the entrepreneur sometimes misses a payment he is able to make, and the financier audits only a fraction of the payments that are missed.[5] In game theoretic terms, the two players are playing mixed strategies: The entrepreneur misses on purpose a fraction of the payments he is able to make, while the financier audits only a fraction of the payments

[5] Boyer (1997) provides a discussion of the commitment problem for a principal in an insurance fraud context, while Khalil (1997) does the same in a monopoly regulation context.

that were missed.

It should be clear to the reader by now that the optimal contract we o¤er will not be incentive compatible in the sense that truth-telling is not observed always. We do not suggest that our result are …rst or even second best allocations, since we do not apply the Revelation Principle.[6] Rather we want to characterize the optimal **third best contract** given the impossibility to implement the second best contract that would be obtained using the Revelation Principle.[7]

We will thus relax the assumption that the …nancier can commit to an auditing strategy. With no commitment to an auditing strategy, the …-nancier cannot guarantee that the entrepreneur will always reveal to her the true realization of the project. This means that the incentive compatibility constraint of the entrepreneur is substituted by two constraints: A reporting strategy constraint for the entrepreneur and an auditing strategy constraint for the …nancier. The reporting strategy of the entrepreneur and the auditing strategy of the …nancier will yield a Perfect Bayesian Nash equilibrium (PBNE) in mixed strategies. These Nash equilibrium constraints tell us how the two players are behaving after Nature has decided what realization the risky project had. The goal of this paper is therefore to design the optimal contract between an entrepreneur and a …nancier where the Nash behavior of the players is taken into account.

1.2 Our Findings

The main results of this paper are three-fold. First, subject to some conditions, there will exist a PBNE in mixed strategies in what we will call

[6] The Revelation Principle basicaly states that amongst all optimal contracts, there is at least one where truth-telling is always obtained; see Myerson (1979).

[7] The …rst best is acheived only if the entrepreneur never cheats and the …nancier never audits.

the payment game. In our two-point distribution of the returns the PBNE will be such that 1-The entrepreneur always tells the truth if the low return is realized, 2-The entrepreneur plays a mixed strategy between reporting a high return and reporting a low return if the high return is realized, 3-The …nancier never audits the report of a high return, and 4-The …nancier audits the report of a low return a fraction of the time.

The second result of the paper is that the entrepreneur ends up with greater wealth ex-post if the project has a low return than if the project has a high return. This means that the di¤erence between a project's realization and the payment made to the …nancier is smaller when the project has a high return than when it has a low return. In the low return state, it is as if the …nancier was forgiving part of the entrepreneur's debt. This result was hypothesized to occur by Rajan (1992) and Bester (1994).

Finally, our third and perhaps our most interesting result states that the amount borrowed by the entrepreneur is greater than the cost of the project itself. This third result contradicts the result of Grossman and Hart (1986).[8] They …nd that the possibility for an entrepreneur to extract rents from a …nancier should mandate the …nancier to reduce his stake in the project; we obtain the opposite. We explain this over-borrowing in three ways. First, by borrowing more than he needs an entrepreneur gets to consume perquisites. Second, the …nancier acts in a way as an insurer by smoothing marginal utilities across periods. This smoothing makes sense since the second period's expected wealth is greater than …rst period wealth. Therefore a risk averse entrepreneur would want to transfer some of that second period wealth to the …rst period. Third, by lending more than the cost of the project, the …nancier has a greater incentive to make sure that

[8] Under-investment was also a result obtained by Jensen and Meckling (1976) and Williamson (1985).

the true return is reported. In other words, over-lending is an implicit way for the …nancier to signal that she will audit with a greater probability.

1.3 The Literature

There exists a large and extensive literature on agency problems between a …nancier and an entrepreneur. Shleifer and Vishny (1997) provide a survey of the literature on the general subject of corporate governance. We will complement this survey by focusing our attention on the promblem of an entrepreneur who has private information regarding the return on his investment, and who must report that return to the …nancier. Given that his …nal wealth depends on the report he makes, there will be an incentive to mis-report the actual return. This incentive to mis-report the state of the world is known as ex-post moral hazard.[9]

In a world were it is costless to verify the agent's report, the principal should always verify, and thus no mis-reporting should ever occur. Townsend (1979) challenged this costless auditing assumption. He constructed a model where the principal should divide the possible states of the world in two categories: auditing and non-auditing. When there is no auditing, then the agent should pay[10] a …xed amount to the principal, while if there is auditing, then the payment should depend on the observed state of the world. Gale and Hellwig (1985) used a similar approach to characterize the optimal contract between an entrepreneur and a …nancier.[11] They …nd that the optimal contract is a debt contract where the entrepreneur is never

[9] The …rsts to recognize the di¤erence between moral hazard ex-ante and ex-post were Spence and Zeckhauser (1971). They constructed a model where an agent needed to report to a principal the state of world he observes, and then receive a payment depending on that state.

[10] Although Townsend's model was primarily an insurance model (instead of paying, the agent receives), the extension to debt is straightforward.

[11] See also Hellwig (1977) and Diamond (1984).

audited as long as he is able to make his (…xed) interest payment, while he is always audited if he misses a payment (declare bankruptcy and his assets are seized). In both papers it is possible to show that the level of investment is lower than without the moral hazard problem and costly bankruptcy, just as in Grossman and Hart (1986).

Mookherjee and Png (1989) and Bond and Crocker (1997) …ne tuned the Townsend (1979) and Gale and Hellwig (1985) models by presuming that a stochastic auditing of reports would yield greater utility to the agents than the deterministic auditing assumed in the two other papers. They showed that it is not necessary to audit every report in the auditing region to obtain truth-telling, auditing only a fraction of the reports should do the trick. If the probability that the principal audits a given report is such that the agent is indi¤erent between telling the truth and lying, then there is no gain for the agent in lying.[12] This stochastic auditing yields greater utility to the agent because it is less costly to induce truth-telling than a deterministic auditing strategy. This means that not all bankruptcies would be costly since only a fraction would be audited.

In every paper mentioned so far in this literature review, there is the assumption that the principal is able to commit to the exact audit strategy that induces truth-telling. Unfortunately, the credibility of such a commitment is doubtful. If the principal knows that the agent has told the truth, then what is the point of conductiong an audit? Graetz, Reinganum and Wilde (1987), Picard (1996), Boyer (1997) and Khalil (1997) discuss the implication of not being able to commit to an auditing strategy. What this leads to is that the agent will attempt to extract rents with some probability, while the principal audits claims with some probability that is greater than

[12] The assumption that states that an agent who is indi¤erent between lying and telling the truth is called the epsilon-truthfulness assumption. See Rasmussen (1989) for details.

the probability of audit under full commitment.

The commitment problem has not attracted much research for the case of an entrepreneur who needs to …nance a risky project through money borrowed from a …nancier. Beaudry and Poitevin (1995) study a commitment problem, but it is the entrepreneur's commitment not to seek supplemental …nancing from a second …nancier that was of interest to them. Gobert and Poitevin (1997) also relax the commitment assumption of an agent to the contract. Using a multi-period setting where an agent's future income is unknown and where a principal can provide some smoothing of that future income with a contract, they show that allowing agents to save some of their earnings may mitigate some of the agent's commitment problems.

The only two papers that mentions the commitment problem of a …nancier to the auditing provisions of a contract between an entrepreneur and a …nancier are Scheepens (1995) and Khalil and Parigi (1998). The models in these two papers resemble ours in many ways. For example Scheepens uses a two-point distribution of the return on investment, and he has a …nancier who cannot commit to sending the entrepreneur in bankruptcy if he defaults on a payment. His papers di¤ers from ours in very signi…cant ways however. For starters, his entrepreneur is risk neutral, while ours is risk-averse. He also assumes that the detection of a fraudulent bankruptcy is not perfect. Finally, he restricts his analysis to standard debt contracts, and looks at the optimal behavior of the players given that contract. Our model goes further as we reconsider the optimality of the debt contract per se when the …nancier cannot commit to an auditing strategy.

Khalil and Parigi (1998) approach the problem from a di¤erent angle. Both their players are risk neutral, and the entrepreneur chooses the absolute size of the project rather than the proportion he invests in it. The production function of the entrepreneur is chosen to be increasing and concave such that his payo¤ is an increasing funtion of his investment in the project. Another major di¤erence between our paper and that of Khalil and Parigi is that we make an explicit di¤erence between the investement period and the payo¤ period. Khalil and Parigi make no such di¤erence.

The remainder of the paper goes as follows. In the next section we present the basic assumptions of the model, its setup and the parameters we will use. We also develop the payment game played between the entrepreneur and the …nancier. In section three, we develop our model. We …nd the optimal contract between the informed entrepreneur and the uninformed …nancier, and we discuss the implications of such a contract. Finally section four concludes and leaves room for further research.

2 Assumptions and Setup

The economy has two periods. The expected utility of the players is equal to the discounted sum of their expected utilities in each period. We suppose that the entrepreneur is risk averse with a twice di¤erentiable von Neumann-Morgenstern utility function over …nal wealth for each period ($U^0 > 0$, $U^{00} < 0$, $U^0(0) = 1$, $U^0(1) = 0$), while the …nancier is risk neutral. We also assume that the banking industry is perfectly competitive in the sense that a …nancier expects to make zero pro…t on each contract. The entrepreneur receives exogenous wage Y in each period. In the …rst period the entrepreneur may consume this wage or invest some of it in a risky project. The project pays o¤ in the second period which the entrepreneur discounts at rate $\mu < 1$, and the …nancier discounts it at rate $\frac{1}{1+\pm}$, where $\pm > 0$ is the minimum return acceptable to the …nancier. We do not restrict the discounting to be the same for the two players. However, we assume that the entrepreneur does not discount the second period less than the financier, which means that $\theta(1+\delta) \leq 1$.[13]

In the first period, the contract between the two players is signed and the entrepreneur invests in the project. The cost of the project is fixed and equal to $I > 0$. Since the project is assumed unique and indivisible an entrepreneur will not be able to invest in only a fraction of the project. Let α denote the share of the investment contributed by the entrepreneur himself. This means that he needs to borrow $D = (1-\alpha)I$ from the financier. In the second period, Nature decides on the project's realization. There are only two possible returns on investment (ROI), $\frac{W_L}{I}$ and $\frac{W_H}{I}$, with $W_H > W_L$. W_H occurs with probability π. The actual ROI is private information to the entrepreneur. The financier can learn about the return if she incurs an auditing cost, c.

Given that the entrepreneur borrowed $D = (1-\alpha)I$ from the financier in the first period, he will need to reimburse this loan in the second period. We will not restrict the payback to be the same in the two states. Instead, we will assume that if the high return is realized, then the entrepreneur needs to pay back $(1+r_H)(1-\alpha)I$, while if the low return is realized, then he needs to pay back $(1+r_L)(1-\alpha)I$. Therefore the contract will specify a payback schedule contingent on the state of the world. These paybacks are subject to limited liability constraints in each state: $(1+r_i)(1-\alpha)I \leq W_i$, $i \in \{L; H\}$. These limited liability constraints mean that the entrepreneur cannot be forced to pay back more than the total return on the project in each state. We can view r_H and r_L as the interest rate charged in each state of the world, with $r_H > r_L$. The sequence of the game is shown as figure 1.

[13] It is common in the literature to assume that the agent is more myopic than the principal. Another way to say that an agent is more myopic is that he discounts the future periods at a higher rate. This is exactly what we assume here.

[INSERT FIGURE 1 HERE]

In our setup, the entrepreneur is not restricted to tell the truth. In fact, upon learning about the realized ROI, he may mis-report it to the …nancier. The reason why truth-telling may not be the optimal strategy for the entrepreneur is that the …nancier cannot commit credibly to an auditing strategy ex-ante. This means that the …nancier must decide if he audits the entrepreneur or not only after the entrepreneur has made a report to her concerning the state of the world. This game will then yield mixed strategies for the two players such that the entrepreneur sometimes tells the truth, while the …nancier sometimes audits. If the entrepreneur lies and the …nancier audits, then it will be assumed that the entrepreneur is found guilty of attempted rent extraction from the …nancier with probability one. In this event, we let the payment from the entrepreneur to the …nancier be equal to what he would have paid had he not lied, but that there is also some penalty which he must incur. This monetary penalty denoted by k, is assumed to be a deadweight cost to the economy; the penalty is paid by the entrepreneur, but is not collected by the …nancier. A table listing all the possible payo¤s to the …nancier and the entrepreneur contingent in every possible outcome is provided as table 1.

[INSERT TABLE 1 HERE]

The extensive form of the payment game is displayed in …gure 2.

[INSERT FIGURE 2 HERE]

Before presenting the maximization problem per se, we will start be presenting the equilibrium of the payment game. This game gives us only one possible Perfect Bayesian Nash Equilibrium (PBNE) in mixed strategy.

In our game, the definitions of PBNE and sequential equilibrium coincide.[14] We provide a definition of a PBNE below.

Definition 1 A PBNE is defined in this game as

$$
\begin{aligned}
PBNE &= \begin{pmatrix}
\text{Entrepreneur's strategy if Nature chose return } W = W_L, \\
\text{Entrepreneur's strategy if Nature chose return } W = W_H; \\
\text{Financier's strategy if the Entrepreneur reported } W' = W_L, \\
\text{Financier's strategy if the Entrepreneur reported } W' = W_H; \\
\text{Ex-post beliefs for the Financier resulting from } W' = W_L, \\
\text{Ex-post beliefs for the Financier resulting from } W' = W_H.
\end{pmatrix} \\
&= \left(\xi(W_L); \xi(W_H); \zeta\left(W_L'\right); \zeta\left(W_H'\right); \gamma\left(W_L'\right); \gamma\left(W_H'\right)\right):
\end{aligned}
$$

We will denote this equilibrium as

$$\xi : \{W_L; W_H\} \to \Delta\{W_L'; W_H'\}$$

$$\zeta : \{W_L'; W_H'\} \to \Delta\{A; N\}$$

$$\gamma : \{W_L'; W_H'\} \to [0; 1]$$

where the notation $\xi : \{W_L; W_H\} \to \Delta\{W_L'; W_H'\}$ means that ξ is a function of the observed signal $\{W_L; W_H\}$ to a probability distribution Δ of messages $\{W_L'; W_H'\}$.

This allows us to state the first theorem of the paper.

Theorem 1 Provided that $\pi > \frac{1}{2}$,[15] the unique PBNE in mixed strategy[16] of this game is given by

$$
\begin{array}{ll}
\xi(W_L) = W_L' & \xi(W_H) = \eta_{HL} W_L' + (1 - \eta_{HL}) W_H' \\
\zeta(W_L') = \gamma_L A + (1 - \gamma_L) N & \zeta(W_H') = N \\
\gamma(W_L') = \gamma_L \in (0; 1) & \gamma(W_H') = 1
\end{array}
$$

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[14] See Myerson (1991).

[15] $\pi > \frac{1}{2}$ is sufficient to get weights in the entrepreneur's reporting strategy between zero and one, such that the first order conditions make sense. The necessary and sufficient condition is $\pi > \frac{c}{(r_H - r_L)(1-\alpha)I}$ as we see in equation (4).

[16] Since we have a two-player game where each player has only two possible actions, there can be at most one mixed-strategy equilibrium (see Gibbons, 1992).

where η_{ij}, $i, j \in \{L, H\}$ represents the probability that the entrepreneur tells the financier that the project's realization is j when it is i, while ν_j, $j \in \{L, H\}$ represent the probability that the financier audits the entrepreneur's report of realization j. We get that $\eta_{LL} = 1$ and

$$\eta_{HL} = \left(\frac{c}{(r_H - r_L)(1 - \alpha)I - c} \right) \left(\frac{1 - \pi}{\pi} \right) \tag{1}$$

while $\nu_H = 0$ and

$$\nu_L = \frac{U(Y - (1 + r_L)(1 - \alpha)I + W_H) - U(Y - (1 + r_H)(1 - \alpha)I + W_H)}{U(Y - (1 + r_L)(1 - \alpha)I + W_H) - U(Y - (1 + r_H)(1 - \alpha)I + W_H - k)} \tag{2}$$

Proof: See appendix.[2]

What the theorem says is that

1-The entrepreneur always reports a low ROI if $W = W_L$.

2-The entrepreneur plays a mixed strategy between reporting a low ROI (bank fraud) and reporting a high ROI if $W = W_H$.

3-The financier never audits an entrepreneur who reports a high ROI.

4-The financier plays a mixed strategy between auditing and not auditing an entrepreneur who reports a low ROI.

It is clear that the only type of lying that will occur will be for the entrepreneur to say that the true return is lower than reality. This seems logical; if the entrepreneur needs to pay more to the financier when the return on the investment is greater, then he will want to tell her that the return is lower than reality. This means that $\eta_{LH} = 0$. Also, since the financier knows that the entrepreneur will never say that a project's return is high when in fact it is low, she will know for sure that when she hears a report of a high return that the entrepreneur has told the truth. There is therefore no need to audit in this circumstance, which means that $\nu_H = 0$.

In the other cases, the equilibrium of the game is such that the entrepreneur and the financier play mixed strategies.[17] This means that in equilibrium some entrepreneurs are successful at extracting rents from the financier in the sense that some lie and are not audited.

3 The Model

3.1 Optimal Contract

The equilibrium strategies are constraints that the financier needs to consider when he designs the contract he offers to the entrepreneur: She must anticipate rationally the behavior of the two players in the second period. The problem of the financier is then to choose a payment schedule (r_L and r_H), and a proportion of the investment financed by the entrepreneur himself α that maximizes the entrepreneur's expected utility[18] given by

$$\begin{aligned} EU = {} & U(Y - \alpha I) + \theta(1-\pi)U(Y-(1+r_L)(1-\alpha)I + W_L) \\ & +\theta\pi(1-\eta)U(Y-(1+r_H)(1-\alpha)I + W_H) \\ & +\theta\pi\eta\gamma U(Y-(1+r_H)(1-\alpha)I + W_H - k) \\ & +\theta\pi\eta(1-\gamma)U(Y - r_L(1-\alpha)I + W_H) \end{aligned} \tag{3}$$

If we substitute for the values of η and γ found in (1) and (2), the expected utility of the entrepreneur simplifies to

$$\begin{aligned} EU = {} & U(Y - \alpha I) + \theta(1-\pi)U(Y-(1+r_L)(1-\alpha)I + W_L) \\ & +\theta\pi U(Y-(1+r_H)(1-\alpha)I + W_H) \end{aligned} \tag{4}$$

[17] To ease notation, we will let $\eta = \eta_{HL}$ and $\gamma = \gamma_L$ for the remainder of the paper. There is no possible confusion since $\eta_{LH} = 0$ and $\gamma_H = 0$.

[18] Since we will use a perfectly competitive environment, it does not matter who chooses the contract. In other words, formulating the problem as one where the financier designs a contract will yield the exact same result as the formulation we use. This contract chosen by the financier would also stipulate a payment schedule and a proportion of the investement financed that maximizes the agent's utility.

Another constraint we impose is that the financier's expected profit must be zero. This means that the amount of money the entrepreneur borrows in the first period must be paid back entirely in the second period, with interest, minus expenses due to fraud. This zero-profit constraint is then

$$\begin{aligned}(1-\alpha)I \quad = \quad & \frac{1}{1+\delta}[(1-\pi)(1+r_L)(1-\alpha)I + \pi(1-\eta)(1+r_H)(1-\alpha)I \\ & +\pi\eta\nu(1+r_H)(1-\alpha)I + \pi\eta(1-\nu)(1+r_L)(1-\alpha)I] \\ & -\frac{1}{1+\delta}c\nu(1-\pi+\pi\eta)\end{aligned} \tag{5}$$

$(1-\alpha)I$ is the amount of money borrowed by the entrepreneur from the financier in the initial period. On the right hand side all terms are discounted at rate $\frac{1}{1+\delta}$. The term in brackets represents the expected payback of the entrepreneur to the financier. $(1-\pi)$ is the probability that the ROI is low, in which case he only needs to pay $(1+r_L)(1-\alpha)I$. $\pi(1-\eta)$ is the probability that the ROI is high, and that the entrepreneur tells the truth, in which case he needs to pay $(1+r_H)(1-\alpha)I$. With probability η he tells a lie, in which case he is caught with probability ν. If he is caught telling a lie, then he gives the financier $(1+r_H)(1-\alpha)I$. If he is not caught, then he pays only $(1+r_L)(1-\alpha)I$, which means that he was able to extract a rent from the financier. Finally, $c\nu(1-\pi+\pi\eta)$ is the second period expected cost of audits. Since the financier cannot make the difference between a truthful and an untruthful low return report, he will then have to audit all low return reports with the same probability.

By substituting in the zero-profit constraint for η found in (1) we can simplify the zero-profit constraint to

$$\delta - \pi r_H - (1-\pi)r_L = -\pi\left(\frac{c}{(r_H - r_L)(1-\alpha)I - c}\right)\left(\frac{1-\pi}{\pi}\right)(r_H - r_L) \tag{6}$$

It is clear that $\delta - \pi r_H - (1-\pi)r_L < 0$ since the financier will not lend money to the entrepreneur if the expected payback, $1 + \pi r_H + (1-\pi)r_L$, is lower than the minimum she is willing to accept, $1+\delta$. Thus the left hand side of (6) represents the difference between the financier's minimum acceptable return and the expected actual return on the loan. This difference cannot be positive. Rearranging (6), we get

$$1-\alpha=\left(\frac{\delta-r_H}{\delta-\pi r_H-(1-\pi)r_L}\right)\left(\frac{\frac{c}{I}}{r_H-r_L}\right) \tag{7}$$

This last equation gives us an equation for the proportion of the cost of the project that is financed through debt. The simplified problem[19] is then

$$\begin{aligned}\max_{r_L;r_H;\alpha} EU &= U(Y-\alpha I) &\text{(SP)}\\ &\quad+\theta(1-\pi)U(Y-(1+r_L)(1-\alpha)I+W_L)\\ &\quad+\theta\pi U(Y-(1+r_H)(1-\alpha)I+W_H)\end{aligned}$$

[19] The entire non-simplified problem is

$$\begin{aligned}\max_{r_L;r_H;\alpha} EU &= U(Y-\alpha I)+\theta(1-\pi)U(Y-(1+r_L)(1-\alpha)I+W_L)\\ &\quad+\theta\pi(1-\eta)U(Y-(1+r_H)(1-\alpha)I+W_H)\\ &\quad+\theta\pi\eta\kappa U(Y-(1+r_H)(1-\alpha)I+W_H-k)\\ &\quad+\theta\pi\eta(1-\kappa)U(Y-r_L(1-\alpha)I+W_H)\end{aligned}$$

subject to

$$\begin{aligned}(1-\alpha)I &= \frac{1}{1+\delta}[(1-\pi)(1+r_L)(1-\alpha)I+\pi(1-\eta)(1+r_H)(1-\alpha)I\\ &\quad+\pi\eta\kappa(1+r_H)(1-\alpha)I+\pi\eta(1-\kappa)(1+r_L)(1-\alpha)I]\\ &\quad-\frac{1}{1+\delta}c\kappa(1-\pi+\pi\eta)\\ \eta &= \left(\frac{c}{(r_H-r_L)(1-\alpha)I-c}\right)\left(\frac{1-\pi}{\pi}\right)\end{aligned}$$

$$\kappa=\frac{U(Y-(1+r_L)(1-\alpha)I+W_H)-U(Y-(1+r_H)(1-\alpha)I+W_H)}{U(Y-(1+r_L)(1-\alpha)I+W_H)-U(Y-(1+r_H)(1-\alpha)I+W_H-k)}$$

$$\begin{aligned}W_L &\geq (1+r_L)(1-\alpha)I\\ W_H &\geq (1+r_H)(1-\alpha)I\\ EU^* &\geq (1+\theta)U(Y)\end{aligned}$$

subject[20] to

$$\alpha = 1 - \left(\frac{\delta - r_H}{\delta - \pi r_H - (1-\pi) r_L}\right)\left(\frac{\frac{c}{I}}{r_H - r_L}\right) \tag{ZP}$$

$$W_L \geq (1 + r_L)(1 - \alpha)I \tag{LL_L}$$

$$W_H \geq (1 + r_H)(1 - \alpha)I \tag{LL_H}$$

$$EU^* \geq (1 + \theta)U(Y) \tag{PC}$$

ZP represents the zero-profit constraint of the financier given in (7). The second and third constraints are the limited liability constraints. These constraints state that the amount that the entrepreneur needs to payback in a given state, $(1 + r_i)(1 - \alpha)I$, $i \in \{L; H\}$, cannot be greater than the total return on the project in that state, W_i.[21] Finally the fourth constraint is the participation constraint, which means that the entrepreneur has to be better off investing in this project than not investing.

Let's abstract from the last three constraint and concentrate on an interior solution. The first order conditions of this problem are then

$$\begin{aligned}\frac{\partial EU}{\partial r_L} = {} & \theta(1-\pi)U'(Y - (1 + r_L)(1 - \alpha)I + W_L)(1 + r_L)\alpha_L I \\ & -\theta(1-\pi)U'(Y - (1 + r_L)(1 - \alpha)I + W_L)(1 - \alpha)I \qquad (FOC_L) \\ & +\theta\pi U'(Y - (1 + r_H)(1 - \alpha)I + W_H)(1 + r_H)\alpha_L I \\ & -U'(Y - \alpha I)\alpha_L I\end{aligned}$$

[20] The proportion of the project that comes from the entrepreneur's own pocket, α, is a choice variable in our model. However, since its value is constrained explicitely by the assumption of zero expected profits for the financier, we can discard α as a decision variable. In fact, by choosing r_L and r_H, we will obtain a value for α.

[21] We do not include the entrepreneur's labor income, Y, in the limited liability constraints because we assume that this income is inalienable. This assumption makes intuitive sense because when an economic agents who purchases a property right of a firm will not be held responsible for that firm going bankrupt. In other words, the entrepreneur cannot be forced to dig in his labor income if the project needs an extra injection of funds.

$$
\begin{aligned}
\frac{\partial EU}{\partial r_H} = {} & \theta\pi U'(Y - (1+r_H)(1-\alpha)I + W_H)(1+r_H)\alpha_H I \\
& - \theta\pi U'(Y - (1+r_H)(1-\alpha)I + W_H)(1-\alpha)I \qquad (\mathrm{FOC}_H) \\
& + \theta(1-\pi)U'(Y - (1+r_L)(1-\alpha)I + W_L)(1+r_L)\alpha_H I \\
& - U'(Y - \alpha I)\alpha_H I
\end{aligned}
$$

Letting

$$
\alpha_L = \frac{\partial\alpha}{\partial r_L} = (1-\alpha)\left(\frac{(1-\pi)(r_H - r_L)^2 - (1-\pi)(\delta - r_L)^2 - \pi(\delta - r_H)^2}{(\delta - r_H)(r_H - r_L)(\delta - (1-\pi)r_L - \pi r_H)}\right) \tag{8}
$$

$$
\begin{aligned}
\alpha_H &= \frac{\partial\alpha}{\partial r_H} = (1-\alpha)\left(\frac{(1-\pi)(\delta - r_L)^2 + \pi(\delta - r_H)^2}{(\delta - r_H)(r_H - r_L)(\delta - (1-\pi)r_L - \pi r_H)}\right) \qquad (9) \\
&= (1-\alpha)\left(\frac{(1-\pi)(r_H - r_L)^2}{(\delta - r_H)(r_H - r_L)(\delta - (1-\pi)r_L - \pi r_H)}\right) - \frac{\partial\alpha}{\partial r_L}
\end{aligned}
$$

and

$$
\begin{aligned}
V' = {} & \theta(1-\pi)(1+r_L)U'(Y - (1+r_L)(1-\alpha)I + W_L) \qquad (10) \\
& + \theta\pi(1+r_H)U'(Y - (1+r_H)(1-\alpha)I + W_H) - U'(Y - \alpha I)
\end{aligned}
$$

the necessary conditions to obtain an optimum in this problem are

$$
\frac{\theta(1-\pi)U'(Y - (1+r_L)(1-\alpha)I + W_L)}{V'} = \frac{\alpha_L}{1-\alpha} \tag{NC1}
$$

$$
\frac{\theta\pi U'(Y - (1+r_H)(1-\alpha)I + W_H)}{V'} = \frac{\alpha_H}{1-\alpha} \tag{NC2}
$$

What do these necessary conditions tell us? It is clear that the left hand side numerators and the right hand side denominators are positive. This means that the sign of V' must be the same as that of α_L and α_H. It is clear that $\alpha_H > 0$ since $\delta < (1-\pi)r_L + \pi r_H < r_H$. Therefore $V' > 0$, which means that $\alpha_L > 0$.

In this contract the penalty inflicted to the entrepreneurs found to have lied about the ROI has no impact on the optimal contract. When we look at the necessary conditions for an optimum (NC_1 and NC_2), no where do we see the penalty parameter k. This result contrasts with that of Scheepens (1995). He finds that the size of the penalty has a non-trivial impact on the shape of the optimal contract. The reason why we get a penalty that has no impact on the optimal contract is mainly that the financier adjusts her auditing strategy as a function of the penalty. It is easy to show that the financier's probability of auditing decreases as the penalty increases ($\frac{\partial o}{\partial k} < 0$). This also means that the penalty has no impact on the entrepreneur's probability of sending a false message. Since the entrepreneur's probability of lying could depend on the penalty only through its impact on the optimal contract, and since the contract is independent of the penalty, it follows that the entrepreneur's decision to extract a rent from the financier is independent of the penalty. This result would not hold however if the penalty was paid by the entrepreneur to the financier, rather than being a deadweight loss (see Picard, 1996, and Khalil and Parigi, 1998).

3.2 Implications

The first implication of the optimal contract is that the entrepreneur's final wealth is greater in the state of the world where the return on the project is lower. This is presented as proposition 1.

Proposition 2 If the financier cannot commit to an auditing strategy, then the entrepreneur's wealth in the low return state will be greater than in the high return state.

Proof: See appendix. $\square$

It may seem strange to see that the entrepreneur is better off in the low ROI state than in the high ROI state. Greater wealth in the low ROI state raises the possible problem that the entrepreneur may not want to invest all the necessary effort to make sure that the project has a high return. However, the investment of some kind of effort is not modelled here. We are only concerned about the problems of revealing the true state of the world. In other words, our concern is solely with ex-post moral hazard rather than ex-ante.

In essence proposition 1 says that the entrepreneur is penalized if his project is a success. Lewis and Sappington (1997) obtain a similar result, using a totally different framework. They used a dynamic model where an entrepreneur is faced with adverse selection (different ability in production) and ex-ante moral hazard (effort may not be optimal). Their result states that the agent who succeeds in the first period project should receive lower wealth in subsequent period. Penalizing success then creates an incentive for the entrepreneur to reveal his true type; more specifically it forces the entrepreneur not to understate his ability. We obtain a similar result; the entrepreneur is better off in the low ROI state. The reason for this, if we were to follow the Lewis and Sappington argument, would be that it forces the entrepreneur not to understate the true return on the project. This is exactly what is happening. By increasing the difference between the payment in the high ROI state, $(1 + r_H)(1 - \alpha)I$, and the payment in the low ROI state, $(1 + r_L)(1 - \alpha)I$, the entrepreneur has to lower his probability of sending a false message concerning the true return.[22] Since the financier has more to gain by auditing, the entrepreneur must reduce his probability of sending a false message in order for the financier to remain indifferent between auditing and not auditing.

[22] It is easy to show that by increasing the difference between r_H and r_L that the probability of lying, η, decreases. In other words, $\frac{d\eta}{dr_H} - \frac{d\eta}{dr_L} < 0$.

If there was no ex-post moral hazard, it is easily shown - and quite intuitive - that the entrepreneur would choose a contract where his second period final wealth is equal in the two states of the world. With ex-post moral hazard however, we see that the entrepreneur's final wealth is greater if the project has a low return. Put a different way, ex-post moral hazard reduces the financier's final wealth if the project has a low return. This implicitely increases the willingness of the financier to make sure that the entrepreneur's report of a low return is truthful.

The next question is then to wonder what is the stake of the entrepreneur in the project. If α is equal to zero, then the entrepreneur finances all of the project from the outside, while if α is equal to one, then he self-finances. Proposition 2, which is the most striking result of the paper, shows that in fact the entrepreneur will borrow more than the cost of the project.

Proposition 3 In our economy, the entrepreneur's limited liability is a sufficient condition for him to borrow more than he needs. In other words,

$$\text{if } (1-\alpha)(1+r_H)I \quad W_H \text{ then } \alpha < 0.$$

<u>Proof</u>: See appendix. $\square$

A negative α means that the entrepreneur is borrowing more than he needs for the project.[23] In other words, the entrepreneur's debt is greater than the cost of the project: $D > I$. An example where this may happen is in the consumption of perks by the entrepreneur.

We can view the perks as the difference between the cost of the project and the amount borrowed, $-\alpha I$. This means that the financier acknowledges

[23] It is interesting to notice that the entrepreneur's limited liability is but a sufficient constraint for α to be negative. In fact, it is very easy to construct a numerical example where there is no limited liability, and where α is still negative.

that the entrepreneur will use some of the funds he borrowed to increase his consumption. The question that comes to mind is why would the financier do such a thing?

The main reason why the financier agrees to over-finance the project is that it implicitly forces her to audit more often, and thus induces the entrepreneur to tell the truth with greater probability. The reason why there is more auditing is that the financier has more to lose by not auditing if she lends more than what the entrepreneur needs. The amount at stake is given by $(r_H - r_L)(1-\alpha)I$. This means that ceteris paribus a smaller α (a greater share invested by the financier in the project) increases the amount at risk. Since there is more to be lost, the entrepreneur will have to reduce his probability of sending a false message. In other words, we have that $\frac{\partial \eta}{\partial \alpha} > 0$.[24]

A possible explanation for our result is that over-borrowing represents some kind of bribe paid ex-ante to the entrepreneur. Shleifer and Vishny (1997) suggest that it would be possible to solve the ex-ante inefficiency encountered in Jensen and Meckling (1976) if we were to let the entrepreneur accept a bribe. This bribe would allow the optimal level of investment to be obtained. However, the bribes that Shleifer and Vishny talk about are bribes that would induce the entrepreneur to invest in the socially optimal project. This is more a case of adverse selection between projects. In our setup, there is only one type of project, with more than one outcome. This means that the bribe does not necessarily work as there are still entrepreneurs who lie regarding the true outcome of the risky project.

[24] This is straightforward from equation (4):

$$\frac{\partial \eta}{\partial \alpha} = \left(\frac{1-\pi}{\pi}\right)\left(\frac{c(r_H - r_L)I}{[(r_H - r_L)(1-\alpha)I - c]^2}\right) > 0$$

Another possible explanation for over-investing is that the financier also acts as an insurer in smoothing (if not equalizing) marginal utilities across states. We know that the entrepreneur will not invest in the project in the first period if it does not give him greater expected wealth in the second period. It then makes sense that he would want to transfer some of that excess second period wealth to the initial period. A way to do this is to borrow more than he actually needs.

Another interesting feature of the contract is that it allows negative interest rates. This result is presented in the following corollary.

Corollary 4 If $W_L < I$, then $r_L < 0$.

Proof: See appendix. $\square$

A negative interest rate just means that the entrepreneur needs to reimburse less than the face value of the loan itself. This result is not unusual. Debt contracts where an entrepreneur sees all his assets seized when he cannot make a scheduled payment implicitly get that the interest rate in those states is negative. In fact, in debt contracts the interest rate in the case of default can be easily calculated as $r_i = \frac{W_i}{(1-\alpha)I} - 1$. This would be negative provided that $W_i < (1-\alpha)I$. What is more interesting with our contract is that the entrepreneur will not need to give away all his assets in the bad return state. In fact, since the entrepreneur's ex-post wealth is greater in the low return state, it cannot be that all the project's realized return are paid to the financier. In other words, $r_L < \frac{W_L}{(1-\alpha)I} - 1$, whatever the value of W_L. In fact, nothing in this contract prevents the financier from giving money to the entrepreneur if the return on the project is low.[25]

[25] To see how that can happen, suppose that the project is a total bust in the sense that $W_L = 0$. Since r_L is strictly smaller than $\frac{W_L}{(1-\alpha)I} - 1$, it follows that if $W_L = 0$, then $r_L < -1$. This means that the financier would pay the entrepreneur some amount if the ROI is low.

A direct consequence of this corollary is that the entrepreneur is always able to fulfill the provisions of the contract when the ROI is low. In other words, the limited liability constraint in the low ROI state is never binding. It is obvious that the limited liability constraint is more stringent in the state where the entrepreneur's wealth is lower (and his utility is smaller). In our contract this state is the one where the ROI is high. Therefore the only time an entrepreneur may declare bankruptcy[26] is if the ROI is high. This raises the interesting point that if the entrepreneur is bankrupt, then the financier will never audit him.

The reason we obtain this result comes from the reporting and auditing behavior of the players. We know from the Perfect Bayesian Nash Equilibrium that with some probability the entrepreneur will tell the truth if the ROI is low. We also know that if the entrepreneur declares that his project has a high ROI then the financier never audits him. Combined with the fact that bankruptcy can only occur in the high ROI state, we have that an entrepreneur who declares bankruptcy (announces a high return) is never audited. This result is completely the opposite of the one that is predicted through standard debt contracts à la Gale and Hellwig (1985). In a standard debt contract, an entrepreneur who declares bankruptcy (reports a low return on his project) is always audited, and ends up giving all the realized returns to the financier.

Unfortunately, the contract we obtain still allows some inefficiency to remain in the economy. A major inefficiency that exists is that some investments that have a positive net present value (NPV) are not undertaken because of the cost of conducting the audits. This means that there are projects whose NPV is barely positive that will not find any financing. We

[26] In the sense that all the project's return are given to the bank: $W_i = (1 + r_i)(1 - \alpha)I$.

show this as corollary 6.

Corollary 5 Suppose there exists a project whose NPV is given by $\varepsilon = \pi W_H + (1 - \pi)W_L - I(1 + \delta) > 0$. If ε is close to zero, but still positive, then the project will not be undertaken.

Proof: See appendix. $\square$

This corollary shows that the possibility for the entrepreneur to extract a rent from the financier prevents the undertaking of projects that would be beneficial for society. Having positive NPV project be put on ice is a common result when there are agency problems in the economy. In fact, there are positive NPV projects that are not undertaken even when the financier can commit to every provision of the debt contract. This is because the money necessary to conduct audits has to come from somewhere. It will typically come from the entrepreneur paying a higher interest rate when his project has a high return. Therefore the same project could have a positive NPV using the interest rates when there is no possibility of moral hazard, and a negative NPV when interest rates are adjusted to compensate for the audits.

3.3 Endogenous Investment

In our model we let the size of the investment, I, be fixed and strictly greater than zero. What is chosen is the proportion of this investment that the entrepreneur needs to borrow. Suppose that the entrepreneur can also choose the size of his investment. Suppose also that the return on the investment can be high ω_H or low ω_L such that the second period total profit from the project is given by $\omega_i I$. This means that the problem faced by the entrepreneur can be rewritten as

$$\max_{r_L, r_H, \alpha, I} EU = U(Y - \alpha I) \tag{11}$$

$$+\theta(1-\pi)U(Y-(1+r_L)(1-\alpha)I+\omega_L I)$$
$$+\theta\pi U(Y-(1+r_H)(1-\alpha)I+\omega_H I)$$

subject[27] to

$$\alpha = 1-\frac{\delta-r_H}{\delta-\pi r_H-(1-\pi)r_L}\frac{\frac{c}{I}}{r_H-r_L} \tag{12}$$
$$\omega_L \geq (1+r_L)(1-\alpha) \tag{13}$$
$$\omega_H \geq (1+r_H)(1-\alpha) \tag{14}$$
$$EU^* \geq (1+\theta)U(Y) \tag{15}$$

The first order conditions with respect to r_L and r_H will be the same as those presented as FOC_L and FOC_H. We would, however, get a new first order condition, that with respect to the size of the project.

$$\begin{aligned}\frac{\partial EU}{\partial I} &= 0 = U'(Y-\alpha I)[-\alpha-I\alpha_I] \qquad (FOC_I)\\ &+\theta(1-\pi)[(1+r_L)(I\alpha_I-(1-\alpha))+\omega_L]U'(Y-(1+r_L)(1-\alpha)I+\omega_L I)\\ &+\theta\pi[(1+r_H)(I\alpha_I-(1-\alpha))+\omega_H]U'(Y-(1+r_H)(1-\alpha)I+\omega_H I)\end{aligned}$$

where

$$\alpha_I = \frac{\partial\alpha}{\partial I} = \frac{\delta-r_H}{\delta-\pi r_H-(1-\pi)r_L}\frac{c}{r_H-r_L}\left(\frac{1}{I}\right)^2 > 0 \tag{16}$$

Notice that

$$I\alpha_I = \frac{\delta-r_H}{\delta-\pi r_H-(1-\pi)r_L}\frac{\frac{c}{I}}{r_H-r_L} = 1-\alpha \tag{17}$$

Letting $I\alpha_I = 1-\alpha$ in (FOC_I) and simplifying yields

$$\begin{aligned}\frac{\partial EU}{\partial I} &= 0 = -U'(Y-\alpha I) \qquad (18)\\ &+\theta(1-\pi)\omega_L U'(Y-(1+r_L)(1-\alpha)I+\omega_L I)\\ &+\theta\pi\omega_H U'(Y-(1+r_H)(1-\alpha)I+\omega_H I)\end{aligned}$$

[27] The PBNE constraints have been included directly in the zero-profit constraint, in the participation constraint, and in the maximization problem.

We can now state our last result

Proposition 6 *If the level of investment is endogenous, then there will not exist a non-corner solution to the entrepreneur's maximization problem.*

Proof: See appendix. $\square$

This last proposition tells us that if the entrepreneur can choose how much to invest on top of how much to borrow, then there will not exist an equilibrium. This is probably due to the fact that the entrepreneur will want to borrow an infinite amount: $I \to \infty$. By borrowing an infinite amount, while keeping α negative, the entrepreneur is able to find a solution to FOC_I. Notice that as $I \to \infty$ and $\alpha < 0$, $Y - \alpha I$ goes to infinity, while $Y - (1 + r_L)(1 - \alpha)I + \omega_L I$ and $Y - (1 + r_H)(1 - \alpha)I + \omega_H I$ also tend to infinity since $\omega_i \geq (1 + r_i)(1 - \alpha)$ by the limited liability constraints. Therefore $U'(Y - \alpha I) \to 0$, $U'(Y - (1 + r_L)(1 - \alpha)I + \omega_L I) \to 0$ and $U'(Y - (1 + r_H)(1 - \alpha)I + \omega_H I) \to 0$. This solves the entrepreneur's first order condition with respect to the investment size.

It also solves the entrepreneur's first order conditions with respect to r_L and r_H. Looking at equations 65 and 66 in the appendix, we see that $\lim_{X \to \infty} U'(X) = 0$, where X represents the wealth of the entrepreneur in any state of the world. Once again, by letting $I \to \infty$ and α be smaller than zero, we obtain a corner solution.

So we see by letting the size of the investment be endogenous that the entrepreneur will choose a level of investement equal to infinity. He will also borrow more than he needs (if it is possible to borrow more than infinity) as a negative α is necessary for $I \to \infty$ to be a solution. We can thus presume that the over-borrowing of the entrepreneur is not due to his inability to adjust his investment level to the one he desires.

3.4 Discussion

The first important result of our analysis is that an entrepreneur will have more wealth ex-post if the low return on investment is obtained. In other words the entrepreneur will be better off if his project is not a total success.

Our results also suggest that a financier, who cannot commit to an auditing strategy, should over-finance an entrepreneur's investment project even if that entrepreneur has private information concerning the realization of the project. This means that the standard debt contract is not optimal in an ex-post moral environment. This result contradicts most of the literature which says that if an entrepreneur has the incentive to extract a rent from a financier, then the financier should invest a smaller amount of money in the entrepreneur's project.

We explain the greater utility in the low return state and the over-investment by a lack of explicit credible commitment on the part of the financier. When the financier cannot commit explicitly to an auditing strategy, then he must instead use an implicit commitment. A way to do this is by increasing the amount that the financier has at risk by not auditing; in other words she needs to signal that she will need to audit more frequently because she has a lot to lose by not auditing. In our model, the implicit signal is sent in two ways.

First, the financier increases the difference between the money she is owed by the entrepreneur in the high and low return on investment states. We see that the amount of money the financier will collect is greater if the return on investment is high. This explains why the entrepreneur is better off when his project has a low ROI. By being so much better off when the ROI is high, the financier is implicitly saying that she will make sure whenever the entrepreneur reveals a low ROI that he has told the truth.

Compared with the case of a standard debt contract where $r_L = \frac{W_L}{I}$, and $r_H = R$ such that $\pm = ¼R + (1 ¡ ¼)\frac{W_L}{I}$, our contract is much less equitable. We get $r_L < \frac{W_L}{I}$, and $r_H > R$. Although our contract seems less equitable than a standard debt contract, the entrepreneur ends up cheating with a smaller probability than if he was faced with a standard debt contract. This conclusion is straightforward when we look at equation (4). Taking the partial derivative with respect to r_L and r_H yields $\frac{@´}{@r_L} > 0$ and $\frac{@´}{@r_H} < 0$. Therefore, by increasing r_H and reducing r_L we reduce the probability that the entrepreneur will mis-report the true return on his risky project.

The second way that the …nancier sends an implicit signal about his willingness to audit is through the amount of money that he lends to the entrepreneur. By over-…nancing the entrepreneur's project the …nancier is implicitly saying that she has more to lose by not auditing. Therefore we should expect her to audit more often when she invests more money in the entrepreneur's project. Knowing that the …nancier will audit more often will induce the entrepreneur to cheat less often. This result is straightforward when we look at equation (4) that describes the probability with which the entrepreneur sends a false message. We see that the smaller ® is, the smaller will be the probability of committing fraud.

Our contract may explain why banks and other lenders accept a certain amount of perquisites consumption by the managers of corporation. By o¤ering the possibility to the entrepreneur to consume perquisites in the …rst period a …nancier presumes that the entrepreneur will not commit fraud as often since he knows that the …nancier has relatively more to lose by not auditing than if no perquisites are consumed. The ex-post moral hazard problem has, however, the major drawback that some projects which could be bene…cial to society are not undertaken. We therefore have some under-investment in the economy as a whole, even if individual projects are over-…nanced.

4 Conclusion

The goal of this paper was to derive the optimal contract between an entrepreneur who has private information about the realized return on a risky project and an uninformed …nancier. We concentrated on the problem of the entrepreneur having the possibility to mis-report the return in order to extract a rent from the …nancier. There was only one possible project, whose cost was …xed, and no e¤ort on the part of the entrepreneur was needed to make sure that the higher return is realized. In other words, we concentrated purely on an ex-post moral hazard problem à la Townsend (1979) and Gale and Hellwig (1985). The originality of our paper rests with the fact that we relaxed the perfect commitment assumption of the …nancier to an auditing strategy. This means that the optimality of the debt contract found in Townsend (1979), Gale and Hellwig (1985) and Dionne and Viala (1992) must be reconsidered.

We …nd that an entrepreneur is better o¤ in the low return state than in the high return state in the sense that his ex-post wealth is greater. We also …nd that an entrepreneur who wants to invest in a risky project will borrow more than he needs to …nance that project. Over-borrowing is also known in the literature as strategic borrowing. This term comes from the idea that players use debt as a strategic device when they are faced with information asymmetry. Here the over-borrowing is a way for the …nancier to signal to the entrepreneur that she has more to lose, and thus that the entrepreneur should commit fraud less often. The converse is true as well: Since the entrepreneur has more to gain, the …nancier should audit more often. Therefore debt is a way to solve part of the asymmetry problem by reducing one player's incentive to cheat, and by increasing the other player's incentive to audit.

A point that we have not address which would be interesting to see in the future is how borrowing constraint would affect the optimal contract. For example, if there are 10 identical projects whose initial investment cost is I, and there is only $10I$ available for investment, how will the optimal contract be affected? We cannot have over-investment in every project. Does this mean that we will eliminate a few project, or will we in fact just invest less in each?

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6 Appendix

Proof of theorem 1. Let's rewrite the equilibrium sextuplet as

$$PBNE = (\xi_L, \xi_H, \zeta_L, \zeta_H, \gamma_L, \gamma_H)$$

Looking at the left side of figure 2, it is obvious that $\gamma_H = 1$. Suppose Nature chooses the return to be low (W_L). Thus sending message W_L' always dominates sending message W_H' for the agent, whatever the principal does. By reporting a high return, the best the entrepreneur can do is get a payoff of $W_L - (1 + r_H)(1 - \alpha)\beta$. On the other hand by reporting a low return, the payoff to the entrepreneur is $W_L - (1 + r_L)(1 - \alpha)I$. Thus when the actual return is low, sending message W_L' dominates sending message W_H' if and only if $r_H > r_L$.

Suppose that r_H is indeed greater than r_L.[28] Then when the financier hears message W_H', she knows for sure that it is truthful, which means that γ_L is equal to zero. Thus the only meaningful strategy for the financier when a high return on investment message is sent is to never audit. This is straightforward since she gets $-c + (1 + r_H)(1 - \alpha)I$ if she audits, and $(1 + r_H)(1 - \alpha)I$ if she does not. We have now found three of the six elements of the sextuplet: ξ_H, ζ_L and γ_H. Lets now move to the right side of figure 2.

Let η_{ij}, $i, j \in \{L, H\}$ represent the probability that the entrepreneur tells the financier that the project's realization is j when it is i; in other words if $i \neq j$, then η_{ij} is the probability that the entrepreneur lies to the financier. This means that η_{HL} is the probability (in the mixed strategy sense) that the agent sends message W_L' when Nature chose the return to be high. In other words, η_{HL} is the probability that the agent lies about

[28] We will show later that it indeed is.

the investment yielding a high return. By Bayes' rule we can find the exact value of γ_L, the financier's posterior belief that the true return is low given that the entrepreneur sent message W_L^0. γ_L is equal to

$$\gamma_L = \frac{1 - \pi}{(1 - \pi) + \pi\eta_{HL}} \tag{19}$$

Only one strategy of the entrepreneur will induce the financier to be indifferent between auditing and not auditing a message. That strategy must be such that γ_L solves

$$(-c + (1 + r_H)(1 - \alpha)I)\gamma_L + (-c + (1 + r_L)(1 - \alpha)I)(1 - \gamma_L) = (1 + r_H)(1 - \alpha)I \tag{20}$$

and

$$\gamma_L = \frac{(r_H - r_L)(1 - \alpha)I - c}{(r_H - r_L)(1 - \alpha)I} \tag{21}$$

Substituting this value of γ_L in equation 1, we get that the agent's probability of saying the return on his investment is low given that it is in fact high is

$$\eta_{HL} = \left(\frac{c}{(r_H - r_L)(1 - \alpha)I - c}\right)\left(\frac{1 - \pi}{\pi}\right) \tag{22}$$

We now have five of the six elements of our sextuplet PBNE. All that is left to calculate is the auditing strategy of the financier when the agent reports a low return on investment. Her strategy must be such that the entrepreneur is indifferent between telling the truth and sending a false message, given that the return is high. Let ν_j, $j \in \{L; H\}$ represent the probability that the financier audits the entrepreneur's report of realization j. This means that ν_L is the probability (in a mixed strategy sense) of auditing a W_L^0 message. ν_L must then solve:

$$\begin{aligned} U(Y - (1 + r_H)(1 - \alpha)I + W_H) &= \nu U(Y - (1 + r_H)(1 - \alpha)I + W_H - k) \\ &\quad + (1 - \nu)U(Y - (1 + r_L)(1 - \alpha)I + W_H) \end{aligned} \tag{23}$$

which means that

$$\nu_L = \frac{U(Y - (1+r_L)(1-\alpha)I + W_H) - U(Y - (1+r_H)(1-\alpha)I + W_H)}{U(Y - (1+r_L)(1-\alpha)I + W_H) - U(Y - (1+r_H)(1-\alpha)I + W_H - k)} \tag{24}$$

Since all six elements of our PBNE have been found, the proof is done. $\square$

Proof of proposition 1. Taking the ratio of the necessary conditions yields

$$\frac{NC1}{NC2} = \frac{(1-\pi)U'(Y - (1+r_L)(1-\alpha)I + W_L)}{\pi U'(Y - (1+r_H)(1-\alpha)I + W_H)} = \frac{\frac{\alpha_L}{1-\alpha}}{\frac{\alpha_H}{1-\alpha}} \tag{25}$$

We want to show that the entrepreneur's wealth is greater in the high return state. If this is true, then the entrepreneur's marginal utility will be greater in the low return state

$$U'(Y - (1+r_L)(1-\alpha)I + W_L) < U'(Y - (1+r_H)(1-\alpha)I + W_H) \tag{26}$$

We know that

$$\frac{U'(Y - (1+r_L)(1-\alpha)I + W_L)}{U'(Y - (1+r_H)(1-\alpha)I + W_H)} = \left(\frac{\pi}{1-\pi}\right)\left(\frac{\alpha_L}{\alpha_H}\right) \tag{27}$$

Thus the entrepreneur's wealth is greater in the low return state if and only if $\pi\alpha_L < (1-\pi)\alpha_H$. Recall from equation (14) that

$$\alpha_H = (1-\alpha)\left(\frac{(1-\pi)(r_H - r_L)^2}{(\delta - r_H)(r_H - r_L)(\delta - (1-\pi)r_L - \pi r_H)}\right) - \alpha_L \tag{28}$$

We then get that $\pi\alpha_L < (1-\pi)\alpha_H$ if and only if

$$\pi\alpha_L < (1-\pi)\left[(1-\alpha)\left(\frac{(1-\pi)(r_H - r_L)^2}{(\delta - r_H)(r_H - r_L)[\delta - (1-\pi)r_L - \pi r_H]}\right) - \alpha_L\right] \tag{29}$$

and

$$\alpha_L < (1-\alpha)\left(\frac{(1-\pi)^2(r_H - r_L)^2}{(\delta - r_H)(r_H - r_L)[\delta - (1-\pi)r_L - \pi r_H]}\right) \tag{30}$$

Substituting for

$$\alpha_L = (1-\alpha)\left(\frac{(1-\pi)(r_H-r_L)^2 - (1-\pi)(\delta-r_L)^2 - \pi(\delta-r_H)^2}{(\delta-r_H)(r_H-r_L)\left[\delta-(1-\pi)r_L-\pi r_H\right]}\right) \tag{31}$$

given in equation (13) and simplifying, we get that $\pi\alpha_L < (1-\pi)\alpha_H$ if and only if

$$\pi(1-\pi)(r_H-r_L)^2 - (1-\pi)(\delta-r_L)^2 - \pi(\delta-r_H)^2 < 0 \tag{32}$$

Expanding the squares, we get

$$\pi(1-\pi)\left(r_H^2 - 2r_Hr_L + r_L^2\right) - (1-\pi)\left(\delta^2 - 2\delta r_L + r_L^2\right) - \pi\left(\delta^2 - 2\delta r_H + r_H^2\right) < 0 \tag{33}$$

Combining terms yields

$$\begin{aligned} 0 \;>\; & r_H^2\left[\pi(1-\pi) - \pi\right] + r_L^2\left[\pi(1-\pi) - (1-\pi)\right] + \delta^2\left[-(1-\pi) - \pi\right] \\ & - 2\pi(1-\pi)r_Hr_L + 2(1-\pi)\delta r_L + 2\pi\delta r_H \end{aligned} \tag{34}$$

Which simplifies to

$$\begin{aligned} 0 \;>\; & -\pi^2r_H^2 - (1-\pi)^2r_L^2 - \delta^2 \\ & - 2\pi(1-\pi)r_Hr_L + 2(1-\pi)\delta r_L + 2\pi\delta r_H \end{aligned} \tag{35}$$

and finally we get

$$-\left[\delta - \pi r_H - (1-\pi)r_L\right]^2 < 0 \tag{36}$$

which is obviously true. $\square$

Proof of proposition 2. We can rewrite NC1 and NC2 as

$$\frac{\theta(1-\pi)}{\frac{\alpha_L}{1-\alpha}}U'(Y-(1+r_L)(1-\alpha)I+W_L) = V' \tag{37}$$

$$\frac{\theta\pi}{\frac{\alpha_H}{1-\alpha}}U'(Y-(1+r_H)(1-\alpha)I+W_H) = V' \tag{38}$$

Expanding V' we get

$$\begin{aligned}\frac{\theta(1-\pi)}{\frac{\alpha_L}{1-\alpha}}U'(Y-(1+r_L)(1-\alpha)I+W_L) &= \theta(1-\pi)(1+r_L)U'(Y-(1+r_L)(1-\alpha)I+W_L)\\ &\quad+\theta\pi(1+r_H)U'(Y-(1+r_H)(1-\alpha)I+W_H)\\ &\quad-U'(Y-\alpha I)\end{aligned}\tag{39}$$

and

$$\begin{aligned}\frac{\theta\pi}{\frac{\alpha_H}{1-\alpha}}U'(Y-(1+r_H)(1-\alpha)I+W_H) &= \theta(1-\pi)(1+r_L)U'(Y-(1+r_L)(1-\alpha)I+W_L)\\ &\quad+\theta\pi(1+r_H)U'(Y-(1+r_H)(1-\alpha)I+W_H)\\ &\quad-U'(Y-\alpha I)\end{aligned}\tag{40}$$

Combining terms yields

$$\begin{aligned}\theta(1-\pi)U'(Y-(1+r_L)(1-\alpha)I+W_L) &= \left[\frac{\frac{\alpha_L}{1-\alpha}\theta\pi(1+r_H)}{1-\frac{\alpha_L}{1-\alpha}(1+r_L)}\right]U'(Y-(1+r_H)(1-\alpha)I+W_H)\\ &\quad-\left[\frac{\frac{\alpha_L}{1-\alpha}}{1-\frac{\alpha_L}{1-\alpha}(1+r_L)}\right]U'(Y-\alpha I)\end{aligned}\tag{41}$$

and

$$\begin{aligned}\theta\pi U'(Y-(1+r_H)(1-\alpha)I+W_H) &= \left[\frac{\frac{\alpha_H}{1-\alpha}\theta(1-\pi)(1+r_L)}{1-\frac{\alpha_H}{1-\alpha}(1+r_H)}\right]U'(Y-(1+r_L)(1-\alpha)I+W_L)\\ &\quad-\left[\frac{\frac{\alpha_H}{1-\alpha}}{1-\frac{\alpha_H}{1-\alpha}(1+r_H)}\right]U'(Y-\alpha I)\end{aligned}\tag{42}$$

Solving this system of equation yields

$$\frac{U'(Y-(1+r_L)(1-\alpha)I+W_L)}{U'(Y-\alpha I)} = -\left(\frac{1}{\theta(1-\pi)}\right)\frac{\frac{\alpha_L}{1-\alpha}}{1-\frac{\alpha_H}{1-\alpha}(1+r_H)-\frac{\alpha_L}{1-\alpha}(1+r_L)}\tag{43}$$

and

$$\frac{U'(Y-(1+r_H)(1-\alpha)I+W_H)}{U'(Y-\alpha I)} = -\left(\frac{1}{\theta\pi}\right)\frac{\frac{\alpha_H}{1-\alpha}}{1-\frac{\alpha_H}{1-\alpha}(1+r_H)-\frac{\alpha_L}{1-\alpha}(1+r_L)}\tag{44}$$

With many manipulations, it is possible to show that

$$1 - \frac{\alpha_H}{1-\alpha}(1+r_H) - \frac{\alpha_L}{1-\alpha}(1+r_L) = -\frac{(1-\pi)(r_H-r_L)(1+\delta)}{(\delta-\pi r_H-(1-\pi)r_L)(\delta-r_H)} < 0 \tag{45}$$

Substituting for the value of $\frac{\alpha_H}{(1-\alpha)}$ found in (14) and the value of $\frac{\alpha_L}{1-\alpha}$ found in (13) gives us

$$\frac{U'(Y-(1+r_L)(1-\alpha)I+W_L)}{U'(Y-\alpha I)} = \frac{(1-\pi)(r_H-r_L)^2-\pi(\delta-r_H)^2-(1-\pi)(\delta-r_L)^2}{\beta(1+\delta)(1-\pi)^2(r_H-r_L)^2} \tag{46}$$

and

$$\frac{U'(Y-(1+r_H)(1-\alpha)I+W_H)}{U'(Y-\alpha I)} = \frac{\pi(\delta-r_H)^2+(1-\pi)(\delta-r_L)^2}{\beta(1+\delta)\pi(1-\pi)(r_H-r_L)^2} \tag{47}$$

For the remainder of the proof, we will only need equation (46). It is easily shown that

$$\frac{\pi(\delta-r_H)^2+(1-\pi)(\delta-r_L)^2}{\pi(1-\pi)(r_H-r_L)^2} > 1 \tag{48}$$

since

$$\pi(\delta-r_H)^2+(1-\pi)(\delta-r_L)^2-\pi(1-\pi)(r_H-r_L)^2 = [\delta-\pi r_H-(1-\pi)r_L]^2 \tag{49}$$

which is obviously positive. In combination with the assumption that the financier does not discount the second period more than the entrepreneur (i.e.: $\beta \leq \frac{1}{(1+\delta)}, \frac{1}{\beta(1+\delta)} \geq 1$) it follows that

$$\left(\frac{\pi(\delta-r_H)^2+(1-\pi)(\delta-r_L)^2}{\pi(1-\pi)(r_H-r_L)^2}\right)\left(\frac{1}{\beta(1+\delta)}\right) > 1 \tag{50}$$

and thus

$$\frac{U'(Y-(1+r_H)(1-\alpha)I+W_H)}{U'(Y-\alpha I)} > 1 \tag{51}$$

Therefore the entrepreneur's wealth in the high return state must be smaller than his first period wealth. It is then possible to conclude that the proportion of the project that is self-financed must be smaller than

$$\alpha < \frac{(1 + r_H)I - W_H}{(2 + r_H)I} \tag{52}$$

On the other hand if the entrepreneur has a limited liability constraint, then we must have in the high ROI state that

$$(1 - \alpha)(1 + r_H)I \le W_H \tag{53}$$

which can also be written as

$$\alpha > \frac{(1 + r_H)I - W_H}{(1 + r_H)I} \tag{54}$$

Combining the inequalities in (51) and (53), we must have for the limited liability constraint and the necessary condition to hold that

$$\frac{(1 + r_H)I - W_H}{(2 + r_H)I} > \alpha > \frac{(1 + r_H)I - W_H}{(1 + r_H)I} \tag{55}$$

It is easily shown that equation (54) holds if and only if the numerators are negative: $(1 + r_H)I - W_H < 0$. Since

$$\alpha < \frac{(1 + r_H)I - W_H}{(2 + r_H)I} < 0 \tag{56}$$

we get that α must be negative. $\square$

Proof of corollary 1. We know from theorem 3 that the entrepreneur has lower marginal utility in the low ROI state than in the high ROI state. This means that his utility, and thus his ex-post wealth, is greater in the low ROI state. Thus,

$$W_L - (1 + r_L)(1 - \alpha)I > W_H - (1 + r_H)(1 - \alpha)I \geq 0 \tag{57}$$

Which simplifies to

$$(r_H - r_L)(1 - \alpha)I > W_H - W_L \tag{58}$$

We then have that the difference between the payments the entrepreneur makes in the high and low ROI states must be greater than the difference in the returns themselves. From our limited liability constraint in the low ROI state, we have that $W_L \geq (1 + r_L)(1 - \alpha)I$. Suppose that $W_L < I$, then either $r_L < 0$, or $\alpha > 0$. But from proposition 2 we know that the limited liability constraint in the high ROI state is a sufficient condition to get $\alpha < 0$. Thus it has to be that if $W_L < I$, then $r_L < 0$. $\square$

Proof of corollary 2. From the zero-profit condition we know that

$$
\begin{aligned}
(1+\delta)(1-\alpha)I = \; & (1-\pi)(1+r_L)(1-\alpha)I \\
& +\pi(1-\eta)(1+r_H)(1-\alpha)I \\
& +\pi\eta\nu(1+r_H)(1-\alpha)I \\
& +\pi\eta(1-\nu)(1+r_L)(1-\alpha)I \\
& -c\nu(1-\pi+\pi\eta)
\end{aligned}
\tag{59}
$$

Letting $I(1+\delta) = \pi W_H + (1-\pi)W_L - \varepsilon$ and rearranging the terms yield that

$$\pi[W_H - (1+r_H)I] + (1-\pi)[W_L - (1+r_L)I] - \varepsilon = -\pi\eta(r_H - r_L)I \tag{60}$$

Obviously the right hand side of this equation is negative. This means that the left hand side must also be negative, which happens if and only if

$$\varepsilon > \pi[W_H - (1+r_H)I] + (1-\pi)[W_L - (1+r_L)I] \tag{61}$$

As ε approaches zero, we then have that

$$\pi[W_H - (1+r_H)I] + (1-\pi)[W_L - (1+r_L)I] < 0 \tag{62}$$

which is not possible since the limited liability constrains us to have

$$
\begin{aligned}
W_H - (1+r_H)I &> 0 && (63)\\
W_L - (1+r_L)I &> 0 && (64)
\end{aligned}
$$

Therefore there are projects that have a positive net present value that will not be undertaken. $\square$

Proof of proposition 3: We will prove this proposition by contradiction. We will assume that there is an interior solution, and show that it is not possible. The first order conditions with respect to r_L and r_H in the case where the size of the investment is endogenous are given by

$$\begin{aligned}\frac{\partial EU}{\partial r_L} &= 0 = \theta(1-\pi)U'(Y-(1+r_L)(1-\alpha)I+\omega_L I)(1+r_L)\alpha_L I \quad (65)\\ &\quad -\theta(1-\pi)U'(Y-(1+r_L)(1-\alpha)I+\omega_L I)(1-\alpha)I\\ &\quad +\theta\pi U'(Y-(1+r_H)(1-\alpha)I+\omega_H I)(1+r_H)\alpha_L I\\ &\quad -U'(Y-\alpha I)\alpha_L I\end{aligned}$$

and

$$\begin{aligned}\frac{\partial EU}{\partial r_H} &= 0 = \theta\pi U'(Y-(1+r_H)(1-\alpha)I+\omega_H I)(1+r_H)\alpha_H I \quad (66)\\ &\quad -\theta\pi U'(Y-(1+r_H)(1-\alpha)I+\omega_H I)(1-\alpha)I\\ &\quad +\theta(1-\pi)U'(Y-(1+r_L)(1-\alpha)I+\omega_L I)(1+r_L)\alpha_H I\\ &\quad -U'(Y-\alpha I)\alpha_H I\end{aligned}$$

which are the same as the first order conditions when I is fixed. Combining these two first order conditions,[29] we get

$$\frac{U'(Y-(1+r_L)(1-\alpha)I+\omega_L I)}{U'(Y-\alpha I)} = \frac{(1-\pi)(r_H-r_L)^2-\pi(\delta-r_H)^2-(1-\pi)(\delta-r_L)^2}{\theta(1+\delta)(1-\pi)^2(r_H-r_L)^2} \quad (67)$$

and

$$\frac{U'(Y-(1+r_H)(1-\alpha)I+\omega_H I)}{U'(Y-\alpha I)} = \frac{\pi(\delta-r_H)^2+(1-\pi)(\delta-r_L)^2}{\theta(1+\delta)\pi(1-\pi)(r_H-r_L)^2} \quad (68)$$

[29] See equations (36) to (46). We can do this if there is an interior solution which means that $U'(.) \neq 0$

We can rewritte the first order condition with respect to the investment level as

$$\frac{1}{\theta} = \pi\omega_H \frac{U'(Y - (1+r_H)(1-\alpha)I + \omega_H I)}{U'(Y - \alpha I)} + (1-\pi)\omega_L \frac{U'(Y - (1+r_L)(1-\alpha)I + \omega_L I)}{U'(Y - \alpha I)} \tag{69}$$

Substituting (66) and (67) in (68) we get

$$\begin{aligned} \frac{1}{\theta} &= \pi\omega_H \frac{\pi(\delta - r_H)^2 + (1-\pi)(\delta - r_L)^2}{\theta(1+\delta)\pi(1-\pi)(r_H - r_L)^2} \\ &\quad + (1-\pi)\omega_L \frac{(1-\pi)(r_H - r_L)^2 - \pi(\delta - r_H)^2 - (1-\pi)(\delta - r_L)^2}{\theta(1+\delta)(1-\pi)^2(r_H - r_L)^2} \end{aligned} \tag{70}$$

Simplifying thoroughly yields

$$(1-\pi)(r_H - r_L)^2 \left(\frac{1+\delta-\omega_L}{\omega_H - \omega_L}\right) = \pi(\delta - r_H)^2 + (1-\pi)(\delta - r_L)^2 \tag{71}$$

Substracting $(1-\pi)(r_H - r_L)^2$ on each side of the equation, and using equation (48) gives us

$$(1-\pi)(r_H - r_L)^2 \left(\frac{1+\delta-\omega_H}{\omega_H - \omega_L}\right) = [\delta - \pi r_H - (1-\pi) r_L]^2 \tag{72}$$

We then see that if $\omega_H > 1+\delta$, then there will not exist a solution. We know from the limited liability condition that

$$\begin{aligned} \omega_H I &\geq (1+r_H)(1-\alpha)I \\ \omega_H &\geq (1+r_H)(1-\alpha) \end{aligned} \tag{73}$$

For there to be a solution, we need

$$1+\delta > \omega_H \geq (1+r_H)(1-\alpha) \tag{74}$$

which means that

$$\alpha > \frac{r_H - \delta}{1+r_H} > 0 \tag{75}$$

But we know from proposition 2 that ® < 0 when the limited liability condition is in e¤ect.[30] We thus get a contradiction. Therefore, there does not exist a solution to the entrepreneur' maximization problem when he can choose the level of his investment.²

[30] Recall that proposition 2 stated that the limited liability constraint was a su¢cient condition for ® < 0. We reached that conclusion by using the …rst order conditions of the entrepreneur's problem with I …xed with respect to r_H and r_L. Since we know that the …rst order conditions are basically the same in the two problems, proposition 2 holds even when the size of the investment is endogenous.

7 Tables and Figures

Table 1

Undiscounted monetary payo¤s to the entrepreneur and the …nancier contingent on the
and the state of the world.

State of the world	Action of Entrepreneur	Action of Financier	Payo¤ to Entrepreneur	
First Period	Borrow $(1 - \alpha)I$ Invest in project	Lend $(1 - \alpha)I$	$Y - \alpha I$	
Low Return	Report W_L	Audit	$Y - (1 + r_L)(1 - \alpha)I + W_L$	(1 +
Low Return	Report W_L	Don't audit	$Y - (1 + r_L)(1 - \alpha)I + W_L$	(1
Low Return	Report W_H	Audit	$Y - (1 + r_L)(1 - \alpha)I + W_L - k$	(1 +
Low Return	Report W_H	Don't audit	$Y - (1 + r_H)(1 - \alpha)I + W_L$	(1 -
High Return	Report W_L	Audit	$Y - (1 + r_H)(1 - \alpha)I + W_H - k$	(1 +
High Return	Report W_L	Don't audit	$Y - (1 + r_L)(1 - \alpha)I + W_H$	(1
High Return	Report W_H	Audit	$Y - (1 + r_H)(1 - \alpha)I + W_H$	(1 + r
High Return	Report W_H	Don't audit	$Y - (1 + r_H)(1 - \alpha)I + W_H$	(1

The contingent states in italics never occur in equilibrium: They represent actions that are o¤ th

Figure 1: Sequence of play.

Figure 2: Extensive form of the payment game.