Full Pooling in Multi-Period Contracting with Adverse Selection and Noncommitment

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Abstract

This paper analyses multi-period regulation or procurement policies under asymmetric information between the regulator and regulated firms. As well known in the literature, some degree of separation is always optimal under any form of commitment. In contrast, we show that full pooling is optimal under noncommitment when the discount factor is sufficiently high. We also discuss the meaning of full pooling under double randomization. Finally, we provide a graphical analysis of the second-best policy in terms of the regulator's commitment capacity.

JEL : D82, H57

Keywords : Incentives, multi-period contracts, regulation, procurement, renegotiation proofness, asymmetric information, full pooling.

Résumé

Cet article analyse les politiques de réglementation et d'achat public sur plusieurs périodes en présence d'asymétrie d'information entre le régulateur et les entreprises réglementées. Un résultat bien connu dans la littérature est qu'un certain degré de séparation est toujours optimal sous toute forme d'engagement des parties au contrat. En contraste, nous montrons que le plein mélange des types est optimal sans engagement des parties au contrat lorsque le facteur d'actualisation est suffisamment élevé. Nous discutons en détail de la définition de plein mélange des types. Finalement, nous proposons une analyse graphique des contrats optimaux en fonction des hypothèses d'engagement du régulateur.

JEL: D82, H57

Mots clés : Incitations, contrats sur plusieurs périodes, réglementation, achat public, à l'abri de la renégociation, information asymétrique, plein mélange.

1 Introduction

The study of incentives in procurement and regulation under asymmetric information is now a signi...cant subject of research. One of the main problems addressed in the recent literature is that of multi-period contracting. In this respect, La¤ont and Tirole (1993) have introduced a general framework that allows the consideration of dimerent assumptions about the parties' commitment capacity. They distinguish between three possibilities: full commitment, commitment and renegotiation, and noncommitment. Under full commitment, the regulator can fully commit to a long-term contract. Under noncommitment, the relationship between the regulator and the regulated ...rm is governed by a series of short-term contracts. In contrast to the two extreme forms of full commitment and noncommitment, commitment and renegotiation describes a situation where the parties can sign long-term contracts, but can alter the initial contract whenever this is mutually advantageous ex post. In this case, the relationship is essentially restricted to renegotiation-proof contracts and its optimal allocation results in a regulator's expected welfare intermediate between those of the full commitment and the noncommitment assumptions.

The di¤erent issues raised in designing the optimal incentive schemes under commitment and renegotiation and under noncommitment (the ratchet e¤ect, the take-the-money-and-run-strategy, the role of the discount rate, etc.) are somewhat intricate and some results are far from intuitive. One such result is how much pooling is optimal in the ...rst contracting period?

As well known in the literature, some degree of separation is always optimal under any form of commitment. In contrast, we are going to show that under noncommitment, full pooling is optimal when the discount factor is su¢ciently high. This means that separation becomes too costly in some circumstances.

The optimal scheme under full commitment is equivalent to a repetition of the optimal static scheme in each period. Such a repetition of the optimal static scheme is not feasible in long-term contracts without full commitment, because of a form of "ratcheting". This refers to the fact that any information obtained about the ...rm's type in the ...rst period will induce renegotiation in the second period so as to induce the ...rm to exert more e¤ort, whenever this can be mutually bene...cial ex post. If the discount factor is large, this implies that full separation may become too costly in terms of the extra rent that now has to be paid out to the e¢cient type. Some degree of pooling may then be preferable to full separation, because it reduces the impact of ratcheting by reducing the speed of information revelation. Speci...cally, partial pooling introduces some e¢ciency cost in the ...rst period (for both types the e¤ort level is distorted away from the ...rst best) but this is compensated by the lower overall rent paid to the e¢cient ...rm.

Under noncommitment the regulator is restricted in the timing of the transfers that can be made to the ...rm: because of his inability to commit, the regulator cannot promise to pay out rent in the second period (to be more precise, only self-enforceable promises are credible). As a consequence, compared to commitment and renegotiation, more rent must now be paid out in the ...rst period in order to obtain some degree of separation. This leads to the possibility of the "take-the-money-and-run" strategy, in that the ine⊄cient ...rm could pro...t by misrepresenting its type in the ...rst period (by choosing the larger transfer designed for the e⊄cient ...rm), but then quit the relationship in the second period. When this occurs, the two types' self-selection constraints are binding.

We ...rst show how the take-the-money-and-run strategy a¤ects the regulator's intertemporal welfare frontier. When the two self-selection constraints are binding, the new welfare frontier is always below that of commitment and renegotiation and its form depends on the discount factor. Because the takethe-money-and-run strategy is a consequence of the ratchet e¤ect (given the constraint on the timing of transfers) and because the impact of the ratchet e¤ect can be reduced by more pooling in the ...rst period, the optimal solution under noncommitment is generally characterized by more pooling than under commitment and renegotiation. In particular, the optimal solution may involve double randomization. We show how double randomization can improve the intertemporal welfare trade-o¤. We also discuss the meaning of full pooling under double randomization and show that full pooling is optimal under noncommitment when the discount factor is su¢ciently high.

Our result on full pooling contrasts with that of La¤ont and Tirole (1993) who obtained that full pooling can never be optimal even under no commitment. In fact, they limited their proof to a type of equilibrium (type I) which implies that the incentive constraint of the non-e¢cient ...rm is not binding and rules out the possibility of double-randomization. However, we show that double-randomization is optimal when the discount factor is su¢ciently large.

The paper develops as follows. Section 2 introduces the notation and the single-period model. Section 3 discusses the noncommitment model and

presents the main result of the paper. It also shows how the commitment and renegotiation case can be represented as a special case of the noncommitment model. Section 4 presents simulation results. The last section concludes.

2 The model

2.1 Full information benchmark

A project is to be undertaken in each period with monetary cost $C_{\dot{z}} = \bar{i} e_{\dot{z}}$, $\dot{z} = 1$; 2 where \bar{i} is an exogenous e¢ciency parameter and $e_{\dot{z}}$ is the producer's e¤ort in controlling costs¹. E¤ort entails a disutility in monetary equivalent of $\tilde{A}(e_{\dot{z}})$, a strictly increasing and strictly convex function with $\tilde{A}(0) = \tilde{A}^{0}(0) = 0$, and $\tilde{A}^{00}(e_{\dot{z}}) = 0$. The ...rm has the same type in each period.

The regulator does not observe $\overline{}$ nor e_i . $\overline{}$ can take two values $\underline{}$ and $\overline{}$ with $\overline{} > \overline{}$. We assume that the regulator has a prior about $\overline{}$ and we write $\circ = \Pr i = \underline{}$. To ...x ideas, consider ...rst the case where there is only one type in the economy.

For each period, the contract leads to a realized monetary cost and to a transfer to the ...rm. This can be represented by a pair $(C_{i}; t_{i})$ where t_{i} is the net transfer paid in addition to reimbursing C_{i} . The ...rm's utility level or surplus is $U_{i} = t_{i}$ i \tilde{A} (i C_{i}) when i > C_{i} . A cost target C_{i} > i can be realized with zero exort and we write $\tilde{A} = 0$ for such case.

The project has value S for consumers in each period. The net surplus of consumers is

$$S_{i} (1 + c_{i}) (C_{i} + t_{i})$$

where $C_{i} + t_{i}$ is the total monetary transfer to the ...rm and j is the marginal shadow cost of public funds². Total welfare in each period is the sum of the net consumers' surplus and of the producer's surplus:

$$W_{i} = S_{i} (1 + I) (C_{i} + t_{i}) + U_{i}:$$
(1)

Normalizing the individual rationality constraint U_{i} , 0, welfare is maximized by a contract such that $U_{i} = 0$ and $e_{i} = e^{\alpha}$ demed by

$$1 \stackrel{\sim}{_{\scriptstyle I}} \tilde{A}^{\emptyset}(e^{\alpha}) = 0:$$
 (2)

¹On procurement see also McAfee and McMillan (1987), Tirole (1986) and Riordan and Sappington (1988).

² For more details on the shadow cost of public funds, see Atkinson and Stiglitz (1980).

Under such a contract $C_{i} = \overline{i} e^{\alpha}$ and $t_{i} = \tilde{A}(e^{\alpha})$.

When there are two types and types are observable, dimerent contracts are omered to both types and expected welfare is

$$W_{i} = S_{i} \circ [(1 + s) (\underline{C}_{i} + \underline{t}_{i}) + \underline{U}_{i}] + (1 + s)^{\mathbf{f}} (1$$

where upper bar is refered to $\overline{}$.

Maximum expected welfare under full information (FI) is

$$W^{FI} = S_{i} (1 + s)^{E_{o_{-}}} + (1 + s)^{-} e^{\alpha} + \tilde{A} (e^{\alpha})^{\alpha}$$
(4)

Aggregate expected social welfare over the two periods is then equal to $(1 + \pm) W^{FI}$ where \pm is the discount factor.

2.2 One period asymmetrical information benchmark

When the e¢ciency parameter is unknown to the regulator³, the above ...rst best solutions are not implementable because of the incentive for the more e¢cient ...rm to misrepresent its type. To analyse this information problem, suppose for a moment there is only one period relationship. As is well known, the regulator can o¤er a menu of separating contracts, provided these satisfy the incentive compatibility constraints:

$$\underline{U} \quad \underline{t}_{i} \quad \tilde{A}(\underline{i}_{i} \quad \underline{C}) \quad \overline{t}_{i} \quad \tilde{A}(\underline{i}_{i} \quad \overline{C}); \quad \underline{IC}$$
(5)

$$\overline{U} \quad \widetilde{t}_i \quad \widetilde{A}(\overline{i}_i \quad \overline{C}) \quad \underline{t}_i \quad \widetilde{A}(\overline{i}_i \quad \underline{C}); \quad \overline{IC}$$
(6)

together with the individual rationality constraints

$$\underline{U}$$
 , 0; \underline{IR} (7)

$$\overline{U}$$
 , 0: \overline{IR} (8)

³ For models where the regulator does not observe costs, see Baron and Myerson (1982) and Baron and Besanko (1988). The asymmetric information considered here is of the adverse selection kind with no stochastic output but with an unobserved action. See Guesnerie and La¤ont (1984) and Picard (1987) for di¤erent adverse selection models and Dionne and Doherty (1994) and Fombaron (1997) for adverse selection models with stochastic output or result. For a recent model when moral hazard <u>and</u> adverse selection are present, see Lewis and Sappington (1997) and for issues related to repeated moral hazard and the role of memory, see the survey of Chiappori et al. (1994).

In the solution to this optimization program, the only binding constraints are IC and IR. Substituting from these two binding constraints and letting $c^- = \bar{i} \bar{i}$, we have

$$\underline{U} = \tilde{A}(\overline{i} \ \overline{C}) \ i \ \tilde{A}(\overline{i} \ \overline{C})$$
$$= \tilde{A}(\overline{e}) \ i \ \tilde{A}(\overline{e} \ i \ \overline{C})$$
(9)

where $\[ensuremath{\mathbb{C}}(\overline{e})\]$ is the e¢cient type's rent in terms of the ine¢cient type's e¤ort level. It can be veri...ed that $\[ensuremath{\mathbb{C}}^0(0) = 0, \[ensuremath{\mathbb{C}}^0(\overline{e}) > 0\]$ for $\overline{e} > 0\]$ and $\[ensuremath{\mathbb{C}}^{00}(\overline{e})\]$. Substituting in the expression for the expected welfare (equation 3), the regulator's problem reduces to ...nding \underline{e} and \overline{e} so as to minimize

$$\circ^{\mathbf{E}}(1+\mathbf{y})(\mathbf{\bar{i}} \mathbf{e} + \tilde{A}(\mathbf{e})) + \mathbf{g}^{\mathbf{c}}(\mathbf{e})^{\mathbf{m}} + (1\mathbf{i} \circ)(1+\mathbf{y})(\mathbf{\bar{i}} \mathbf{e} + \tilde{A}(\mathbf{e}))$$

In the solution, $\underline{e} = e^{\alpha}$ and \overline{e} satis...es:

$$1_{i} \tilde{A}^{\emptyset}(\bar{e}) = \frac{\sigma}{1_{i} \circ 1_{+s}} \bar{c}^{\emptyset}(\bar{e}) \quad \text{or} \quad \bar{e} = \bar{e}^{S}(\sigma) \quad (11)$$

where S stands for separation⁴.

It will be useful to keep in mind the comparative statics of the solution with respect to changes in °: First,

$$\frac{d\overline{e}^{S}(^{o})}{d^{o}} < 0 \tag{12}$$

with $0 \in \overline{e}^{S}(\circ)$ e^{*} as \circ varies between unity and zero. Second, writing the eccient types's rent as $U(\circ) = \mathbb{C}(\overline{e}^{S}(\circ))$, we have

$$\frac{dU(^{\circ})}{d^{\circ}} < 0 \tag{13}$$

with 0 $U(^{\circ})$ $^{\odot}(e^{\alpha})$.

Finally, the maximum expected welfare under asymmetric information (AI) as a function of $^{\circ}$ will be denoted:

$$W^{AI}(^{\circ}) = S_{i} \circ^{E} (1 +])(_{i} e^{a} + \tilde{A}(e^{a})) +]^{C}(\bar{e}^{S}(^{\circ}))^{a}$$

$$i (1 +])(1 +]^{E} i \bar{e}^{S}(^{\circ}) + \tilde{A}(\bar{e}^{S}(^{\circ}))^{a}$$
(14)

⁴We must emphasize that a separating solution is a general property in a principalagent framework (Stiglitz, 1977; La¤ont and Tirole, 1990). In other words, a pooling solution cannot be optimal.

which can be veri...ed to be strictly increasing in °. From now on, the ...rm's type is taken to be unobservable by the regulator⁵ in a two-period relationship.

2.3 Asymmetric information in a two-period relationship

The nature of the solution under asymmetric information in a multi-period relationship depends on the capacity for the principal to commit himself to a contract. Three types of commitment assumptions have been discussed in the literature: full commitment, commitment and renegotiation, and no commitment. Full commitment means that at the beginning of the ...rst period the regulator can o¤er an immutable menu of long-term contracts specifying net transfers and cost targets in each period. It is now well established in the literature that the optimal scheme under full commitment is a repetition of the static solution, that is $\underline{e}_1 = \underline{e}_2 = e^{\alpha}$ and $\overline{e}_1 = \overline{e}_2 = \overline{e}^S$ (°). In other words, the low-cost ...rm is induced to exert the ...rst-best level e^{α} at each date while the high-cost ...rm exerts the static second-best e^{α} ort \overline{e}^S (°) at each date. In general, however, such long term contracts are not renegotiation-proof (Dewatripont, 1989).

The full commitment is extreme because, at the beginning of the second period, the ...rm's type is known to the regulator. If the ...rm turns out to be the high-cost type, the planned exort level \overline{e}^{S} (°) can be improved upon ex-post: the second period welfare could be increased if the regulator oxered to renegotiate the initial contract so as to realize the ...rst-best cost $\overline{}_{i} e^{x}$. However this behavior can be anticipated and the lack of full commitment introduces some ine¢ciency because it is anticipated that the information revealed in period one cannot be disregarded in period two and lead to the ratcheting of the ine¢cient type's exort; this in turn increases the cost of separation at the beginning of the relationship. A less extreme commitment assumption is to suppose that parties can sign long term contracts but commit not to renegotiate. Such renegotiation-proof contracts were analysed by Laxont and Tirole (1990) as commitment and renegotiation⁶. We shall come

⁵ To simplify, we assume $S > (1 +)^{-}$. This ensures that the separating menu satisfying (11) is optimal for any ° 2 (0; 1), with $\overline{e}^{S}(\circ) ! 0$ as ° ! 1.

⁶On renegotiation with multiple screening variables, see Dewatripont and Maskin (1995).

back on this type of contract since, for some parameters values, it leads to an allocation that is identical to that obtained under no commitment.

Without any form of commitment whatsoever, there is an additional dif-...culty since now the regulator cannot defer to period two the payment of the rent needed to induce separation in the initial period: only short-term contracts are possible (or promises are credible only if they are self-enforceable). By revealing its type today, the e¢cient ...rm therefore jeopardizes its future rent⁷. As a consequence, if separation is to be obtained, the e¢cient ...rm must be paid its informational rent up-front at the beginning of the relationship. In other words, if there is separation, noncommitment adds a constraint on the timing of transfers.

This timing constraint leads to the possibility that the ine¢cient ...rm misrepresent its type in period one, pocket the larger incentive payment designed for the e¢cient type, and then quit the relationship in period two. Since the incentive payment that must now be paid in period one increases with the discount factor, the "take-the-money-and run" strategy will matter only when the discount factor is above some critical value. When this is the case, the incentive compatibility constraints of both the e¢cient and the ine¢cient type become binding in period one. We ...rst examine the e¤ect of a binding <u>IC</u> constraint and then consider the case where both constraints are binding. As we will see, when only the <u>IC</u> constraint is binding, only the e¢cient type randomnize. When both constraints are binding, there is the possibility that both types randomnize between the date one contracts (double randomization). Indeed, the fundamental result of this article is that, in the no commitment case, with a su¢ciently large \pm , the full pooling of types in period one is second-best optimal.

3 No commitment in a two-period relationship

We consider the possibility that, as in any adverse selection situation, it may be in the interest of the regulator to oxer a menu of contracts at date one so as to separate types. Let these contracts be denoted A^0 and A^1 . Since there is no commitment with respect to period two on either part, A^0 and

⁷This is the standard "ratchet exect" noted by various authors. See for instance Freixas et al. (1985).

A¹ are one-period contracts. That is, once the ...rm has picked one of these contracts, it must expect for date two the contract or menu of contracts that will then be ex post optimal from the regulator's point of view, given the information available to him at that date.

As it will become clear, in the solution, the e \mathbb{C} cient type or both types may randomize at date one between A⁰ and A¹. Assuming for the moment that only the e \mathbb{C} cient type randomizes, let A⁰ be the contract picked by the low-cost ...rm only (with probability x), while A¹ is picked by the high-cost ...rm and also (with probability 1 i x) by the low-cost ...rm. Let \underline{e}_1 denote the e \mathbb{C} cient type's e \mathbb{P} ort under contract A⁰ and \overline{e}_1 the ine \mathbb{C} cient type's e \mathbb{P} ort under contract A¹.

3.1 Randomization

Recalling that ° is the probability that the ...rm is the e \mathbb{C} cient type, contract A⁰ is chosen with probability x° and contract A¹ is chosen with probability (1 i x) ° + (1 i °). If the ...rm picks A⁰, it is known at date two that it is the e \mathbb{C} cient type. If it picks A¹, the date two posterior probability that the ...rm is the low-cost type is

$$^{\circ 1}(\mathbf{x}) = \frac{(1 \mathbf{i} \mathbf{x})^{\circ}}{(1 \mathbf{i} \mathbf{x})^{\circ} + (1 \mathbf{i}^{\circ})}$$

Note that x = 1 corresponds to full separation, i.e. the limiting case where contract A¹ is picked by the high-cost ...rm only, while x = 0 corresponds to full pooling, both types picking A¹ as if contract A⁰ were not o¤ered. In what follows, note that it is always optimal to design A¹so as to give zero rent to the ine¢cient type.

Following the choice of A¹, the period two scheme o¤ered to the ...rm is the optimal static menu for a probability ${}^{\circ 1}(x)$ that the ...rm is the e¢cient type. If it chooses A¹, the e¢cient ...rm can then expect in period two a rent equal to U(${}^{\circ 1}(x)$). Its total discounted rent in choosing A¹ is therefore ©(\overline{e}_1) + ±U(${}^{\circ 1}(x)$). To induce separation the same rent must be available with contract A⁰. With the latter contract no rent will be forthcoming in period two because the regulator is then fully informed of the ...rm's type. For separation to be possible at date one, the contract A⁰ must therefore pay an up-front a cash transfer equal to

$$\tilde{A}(\underline{e}_1) + {}^{\mathbb{C}}(\overline{e}_1) + \pm U({}^{o1}(x))^{a}$$

The ...rst term compensates for the ...rm's e^{p} ort \underline{e}_{1} , the second term is the rent element of the transfer.

Consider now the ineCcient ...rm's decision problem. If it picked A⁰, it would need to exert the exort $\underline{e}_1 + C^-$ to meet the imposed cost target under that contract, thus earning net utility is equal to

$$\tilde{A}(\underline{e}_1) + {}^{\otimes}(\overline{e}_1) + \pm U({}^{\circ 1}(x))^{a}_{i} \tilde{A}(\underline{e}_1 + {}^{\circ}):$$

Given that the ine¢cient ...rm earns zero rent under A¹, this ...rm will choose the latter contract only if the following self-selection constraint is satis...ed:

$$^{\odot}(\overline{e}_{1})_{i} ^{\odot}(\underline{e}_{1} + \mathbb{C}^{-}) + \pm U(^{\circ 1}(x)) \quad 0 \quad \overline{IC}$$

$$(15)$$

For a given randomization probability x, the period two welfare is

$$\Psi_{2}(\mathbf{x}) = {}^{\circ}\mathbf{x} {}^{\mathbf{f}}\mathbf{S}_{\mathbf{i}} (1 + \mathbf{y})(\underline{-}_{\mathbf{i}} e^{\mathbf{x}} + \tilde{A}(e^{\mathbf{x}}))^{\mathbf{x}} + ({}^{\circ}(1 \mathbf{i} \mathbf{x}) + 1 \mathbf{i} {}^{\circ}) W^{\mathbf{A}\mathbf{i}} {}^{\mathbf{f}}\mathbf{e}_{\mathbf{0}1}(\mathbf{x})^{\mathbf{x}} :$$
 (16)

For x given, the best date one contracts A^0 and A^1 must be designed so that \underline{e}_1 and \overline{e}_1 maximize the ...rst-period welfare

subject to the $\overline{\text{IC}}$ constraint (15). Let $\underline{e}_1(x)$ and $\overline{e}_1(x)$ denote the solution to this problem and let $\Re_1(x)$ be the corresponding optimal ...rst period welfare.

Whether or not the constraint \overline{IC} is binding in the latter optimization problem depends on the discount factor. When the constraint is not binding, the solution to the date one contract design problem is $\underline{e}_1(x) = e^x$, with $\overline{e}_1(x)$ strictly increasing in x and tending to the static second-best level $\overline{e}^{s}(^{\circ})$ as x tends to unity (full separation). At x = 0, we have the full pooling scheme and we denote the ineCcient type's exort by $\overline{e}_1(0) = \overline{e}^P$. When $\pm = 0$, the ...rst-period optimization problem is identical to the static one-period problem; since the ineCcient type's self-selection constraint is not binding in the latter problem, by continuity it is easily seen that the constraint is not binding in the present case for a small enough discount factor.⁸ In what follows we examine ...rst the case where the \overline{IC} constraint is not binding.

3.2 Non binding IC constraint

The randomization probability x determines how much information is revealed in the ...rst period. Changes in x generate a trade-o¤ between the ...rst and second-period welfare levels. In order to be able to make direct comparison with the full-commitment situation, as well as with the commitment and renegotiation case, we de...ne the "accounting" ...rst and second period welfare as

$$W_1(x) \quad (W_1(x) + {}^{\circ}x_{,\pm} U({}^{\circ 1}(x)))$$
 (18)

$$W_2(x) \quad \widehat{W}_2(x) = ^{\circ}x_{J} U(^{\circ 1}(x))$$
 (19)

Since $W_1 + \pm W_2 = \mathcal{W}_1 + \pm \mathcal{W}_2$, total discounted welfare is left unchanged by this transformation, so that working with the accounting levels of welfare rather than with the actual ones does not a ect the analysis.

(Figure 1, here)

Figure 1 compares the ...rst and second period "accounting" welfare levels for the two extreme cases of full separation (point S with x = 1) and full pooling (point P with x = 0). Point F in the ...gure depicts the per-period welfare levels that would be reached under full commitment; as discussed previously, this allows the second-best static welfare W^{AI} (°) in both periods. Under no commitment, the ...rst-period welfare with full separation is also the same as in the static solution; however, the second period is smaller because both types are required to supply the ...rst-best e^xort in period two, which implies that too much rent is then paid out. By contrast, under full pooling,

⁸ If the constraint is not binding at x = 1, it is easily veri...ed that it is not biding for smaller values of x.

the …rst-period welfare $W^P = W_1(0)$ is smaller than in the static solution; on the other hand, the date two welfare is then identical to the static solution. In …gure 1, the full separation scheme lies on a higher isowelfare line and would therefore be preferred to full pooling. Of course, the ranking would be reversed for a su¢ciently higher discount factor (i.e., a smaller slope for the isowelfare lines).

When the randomization probability x is allowed to vary between zero and one, $W_1(x)$ and $W_2(x)$ trace out an opportunity locus in the $(W_1; W_2)$ plane. Figure 2 is similar to the previous ...gure except that we have now drawn the welfare opportunity locus between S and P. Along the curve, the value of x decreases when we move from S to P and each point of the curve corresponds to a di¤erent semi-separating scheme. The slope of the locus is given by

$$\frac{dW_2}{dW_1} = \frac{W_2^0(x)}{W_1^0(x)}$$
(20)

Since increases in x imply a move towards the second-best static scheme in period one, the ...rst-period welfare satis...es $W_1^{0}(x) > 0$. On the other hand, increasing x brings the second period welfare further away from the static second-best static solution so that $W_2^{0}(x) < 0$ for $x \in 0$; at x = 0, the second period welfare is at a maximum and we have $W_2^{0}(x) = 0$. The slope of the SP locus is therefore negative for $x \in 0$ and is equal to zero at x = 0. The locus is concave in a neighborhood of full pooling but need not be concave everywhere.

The optimal scheme is obtained by a randomization probability x which maximizes the total discounted welfare $W_1(x) + \pm W_2(x)$. This is equivalent to choosing the point on the SP curve that lies on the highest isowelfare line. In the case represented in Figure 2 the optimal scheme is semi-separating. Clearly, when \pm is less than some critical value, the optimal scheme is the full separation solution x = 1. For \pm su¢ciently large, the solution is a semiseparating contract. The optimal x is non-increasing in \pm ; that is, the higher the discount rate the less information will the regulator want to extract

from the ...rm at date one.⁹ In particular, because the slope of SP is zero at x = 0, we have

$$\frac{d}{dx} (W_1(x) + \pm W_2(x))^{-1} > 0$$
(21)

which means that full pooling can never be a solution when the $\overline{\text{IC}}$ constraint is not binding. We write this as our ...rst proposition¹⁰.

Proposition 1 For a small enough discount rate, full pooling in period one is never optimal when the $\overline{\text{IC}}$ constraint is not binding.

3.3 Both self-selection constraints are binding

We now examine the consequences of a binding IC constraint on the intertemporal opportunity locus. When the self-selection constraints of both types are binding, the solution may involve randomization by both the low- and high-cost ...rm. For expository reasons, we examine ...rst the case where only the low-cost ...rm randomizes.

3.3.1 Simple randomization

First, even though both self-selection constraints are binding, full pooling is obviously always feasible so that point P of ...gure 2 must be part of the locus. Second, $W_2(x)$ is not a ected by a binding \overline{IC} constraint and it is therefore de...ned as in the previous section. Third, because \overline{IC} is binding, $W_1(x)$ is now smaller, at least for some x, so that the intertemporal locus is below the SP curve of Figure 2. Fourth, for \pm su¢ciently large, the constraint is binding for arbitrarily small deviations from the full pooling scheme at P. This follows from the fact that, for x close to zero, the \overline{IC} constraint is not binding only if

$$^{\odot}(e^{P})_{i} ^{\odot}(e^{\pi} + C^{-}) + \pm U(^{o}) = 0$$
 (22)

⁹Note that the optimal x is generally not a continuous function of \pm because the locus is typically not concave. In the simulations with a quadratic cost of e^xort, the locus becomes non concave for su¢ciently large values of °.

¹⁰Note that this case also corresponds to commitment and renegotiation.

where $d^{P} = d_{1}(0)$ is the ine¢cient type's exort level in a full pooling solution. Since d^{P} does not depend on \pm , it is clear that the preceding inequality cannot hold for \pm su¢ciently large. Finally, the opportunity locus may have a positive slope in a neighborhood of full pooling; that is, around full pooling, total discounted welfare may be decreasing in x. We write the latter statement as our next proposition.

Proposition 2 When only the low-cost ... rm randomizes, for \pm suCiently large -

$$\frac{d(W_1(x) + \pm W_2(x))}{dx} = \frac{1}{x=0} < 0$$
(23)

Proof. See the appendix.

Possible forms of the opportunity locus under simple randomization are represented in Figure 3. The SP curve is the locus when \pm is small and \overline{IC} is not binding. The locus shifts to the left when \pm increases su¢ciently and \overline{IC} becomes binding. The numerical simulations (see the next section) show that the locus may become convex everywhere. As a result, the only schemes worth considering on a curve such as S[®]P are full separation (point S[®]) or full pooling (point P). Note the contrast with the previous section: as earlier, the slope of the opportunity locus is zero at the full pooling point P, but this point may now be a strict local maximum (in fact, a corner solution).¹¹

3.3.2 Double randomization

We now allow both types to randomize. Let y > 0 denote the probability that the ine¢cient type chooses A⁰. Without loss of generality, we may take x _ y; that is, contract A⁰ is by convention the one that is more likely to be chosen by the e¢cient type. At date two, if contract A⁰ has been chosen, the posterior probability that the ...rm is the e¢cient type is given by

$$^{\circ 0}(x;y) = \frac{x^{\circ}}{x^{\circ} + y(1_{i}^{\circ})}$$
: (24)

¹¹That is, as before, $W_2^0(x) < 0$ for $x \notin 0$ and $W_2^0(0) = 0$, but we can now have $W_1^0(x) < 0$ in a neighborhood of x = 0.

If contract A^1 has been chosen, the posterior probability that the ...rm is the e¢cient type is

$${}^{\circ^{1}}(x;y) = \frac{(1 i x)^{\circ}}{(1 i x)^{\circ} + (1 i y)(1 i^{\circ})}:$$
 (25)

The equilibrium second-period contract is given by the optimal static scheme with respect to °° or °1, depending on what contract was chosen by the ...rm in the initial period. The expected welfare for period two is therefore

$$\mathfrak{B}_{2}(\mathbf{x};\mathbf{y}) = (\mathbf{x}^{\circ} + \mathbf{y}(1_{i}^{\circ})) W^{\mathsf{A}\mathsf{I}}[{}^{\circ\mathsf{o}}(\mathbf{x};\mathbf{y})] + ((1_{i}^{\circ}\mathbf{x})^{\circ} + (1_{i}^{\circ}\mathbf{y})(1_{i}^{\circ})) W^{\mathsf{A}\mathsf{I}}[{}^{\circ\mathsf{o}}(\mathbf{x};\mathbf{y})]$$
(26)

We now turn to the date one allocation. As before, \underline{e}_1 denotes the e¢cient ...rm's e¤ort under the ...rst period contract A⁰; the ine¢cient ...rm'e¤ort under the same contract is then $\underline{e}_1 + \overline{C}^-$. Similarly, under contract A¹, the ine¢cient ...rm's e¤ort is \overline{e}_1 and that of the e¢cient ...rm is then $\overline{e}_1 \downarrow \overline{C}^-$. In both contracts the ine¢cient ...rm earns zero rent, which takes care of the \overline{IC} constraint. Under A⁰ the e¢cient ...rms gets a ...rst period rent of ©($\underline{e}_1 + \overline{C}^-$) and can expect U [°⁰(x; y)] at date two. Under A¹ it gets a ...rst period rent of ©(\overline{e}_1) and can expect U [°¹(x; y)] at date two. The e¢cient ...rm is indi¤erent between both date one contracts if

$$^{\odot}(\overline{e}_{1}) + \pm U \stackrel{\mathbf{f}_{o1}}{\overset{\circ}{\overset{\circ}{}}}(x;y)^{\mathbf{m}} = ^{\odot}(\underline{e}_{1} + \mathbb{C}^{-}) + \pm U \stackrel{\mathbf{f}_{o0}}{\overset{\circ}{}}(x;y)^{\mathbf{m}}$$
(27)

For x and y given, the optimal ...rst period contracts are such that, subject to condition (27), \underline{e}_1 and \overline{e}_1 maximize

$$S_{i} \circ x^{\circ} (1 + \frac{1}{2})(\underline{-}_{i} \underline{e}_{1} + \tilde{A}(\underline{e}_{1})) + \frac{1}{2} \circ (\underline{e}_{1} + \underline{C}^{-})^{a}$$

$$i^{\circ} (1 + \frac{1}{2})(\underline{-}_{i} \underline{e}_{1} + \tilde{A}(\underline{e}_{1} + \underline{C}^{-})) + \frac{1}{2} \circ (\underline{e}_{1})^{a}$$

$$i^{\circ} (1 + \frac{1}{2})(\underline{-}_{i} \underline{e}_{1} + \tilde{A}(\underline{e}_{1} + \underline{C}^{-}))$$

$$i^{\circ} (1 + \frac{1}{2})(1 + \frac{1}{2})(\underline{-}_{i} \underline{e}_{1} + \tilde{A}(\underline{e}_{1}))$$

$$(28)$$

Let $\mathfrak{P}_1(x; y)$ denote the solution to this problem. Observe that x = y implies ${}^{\circ 0} = {}^{\circ 1} = {}^{\circ}$. When the randomization probabilities are the same for both types, the second period welfare is the second-best static welfare $W^{AI}({}^{\circ})$. Furthermore, the condition (27) is then ${}^{\odot}(\overline{e}_1) = {}^{\odot}(\underline{e}_1 + {}^{\circ})$ and

the solution to the period one problem is identical to the full pooling contract derived in the preceding section. That is, the …rst-period contracts A^0 and A^1 are identical and we have the full pooling level of welfare at date one¹²; the date two welfare is of course also the same as would be obtained following full pooling.

Lemma 1 For any x and y, x = y is equivalent to full pooling.

As in the preceding section, in order to draw the intertemporal opportunity locus, we introduce "accounting" levels of welfare now de...ned as

$$W_{1}(x; y) = \mathcal{W}_{1}(x; y) + {}^{\circ}x_{\pm} {}^{\circ}_{\mathbb{C}} U_{\mathbf{f}}^{\mathbf{f}_{01}}(x; y)_{\mathbf{a}}^{\mathbf{a}} i U_{\mathbf{f}}^{\mathbf{f}_{00}}(x; y)_{\mathbf{a}}^{\mathbf{a}}$$
(29)

$$W_{2}(x; y) = \Psi_{2}(x; y) | \circ x_{J} U^{L_{01}}(x; y)' | U^{L_{00}}(x; y)'$$
(30)

For a given y 2 (0; 1), we examine the opportunity locus generated by $W_1(x; y)$ and $W_2(x; y)$ when x varies in the interval [y; 1]. We know that the locus includes the full pooling point P when x = y. The slope of the locus is

$$\frac{dW_2}{dW_1} = \frac{@W_2(x; y) = @x}{@W_1(x; y) = @x}$$
(31)

It can be shown¹³ that the numerator of this expression is always negative (even at the full pooling x = y); the sign of the denominator is in general indeterminate, but it is positive at x = y. In particular, we have:

Lemma 2 For all y 2 (0; 1) and all \pm ,

$$\frac{@(W_1(x;y) + \pm W_2(x;y))}{@_X} = 0$$
(32)

¹²Let $\underline{e}_1(x; y)$ and $\overline{e}_1(x; y)$ denote the exort levels under the optimal ...rst period contracts. Then $\overline{e}_1(y; y) = \overline{e}^P$ and $\underline{e}_1(y; y) = \overline{e}^P$, and therefore $\mathfrak{P}_1(y; y) = W^P$.

¹³See the proof of the next lemma.

Proof. See the appendix.

This means that, under double randomization, a small deviation from full pooling to some degree of "asymmetric pooling" has only a second-order exect on total discounted welfare. Geometrically, the lemma implies that, under double randomization, the slope of the opportunity locus at the full pooling point P is

$$\frac{dW_2}{dW_1} = \frac{1}{x=y} = \frac{1}{1} \frac{1}{\pm}$$
 for any y 2 (0; 1) (33)

3.4 Optimal ...rst-period contracts

From the previous lemma, full pooling is a stationary value of the total discounted welfare with respect to small changes in x towards some degree of "asymmetric" pooling. Depending on the curvature of the total welfare function with respect to x, full pooling may therefore be a local minimum or a local maximum. We now show that, in a neighborhood of full pooling, the curvature of a locus does not depend on the randomization probability y, but only on the discount factor.

Lemma 3 There exists $\frac{b}{2}$ such that for all y 2 (0; 1)

$$\frac{{}^{@^{2}(W_{1}(x; y) + \pm W_{2}(x; y))}}{{}^{@}x^{2}} \int_{x=y}^{z} 0 \quad \text{if and only if} \quad \pm \quad \mathbf{\underline{b}} \quad (34)$$

Proof. See the appendix.

The last two lemmas show that, when the discount factor is greater than some critical value, full pooling is a strict local maximum within the class of all ...rst-period contracts designed so as to satisfy both <u>IC</u> and <u>IC</u>. An illustration is given in ...gure 4 which shows the opportunity locus as x varies in a neighborhood of the full-pooling point at x = y, for some given arbitrary y (the curvature in the ...gure assumes $\pm > \frac{1}{2}$).

Since we also know from proposition 2 that, for \pm suCiently large, a small deviation from full pooling to a semi-separating scheme under simple

randomixation decreases total discounted welfare, it follows that full pooling is a strict local maximum for su¢ciently large discount factors. The next proposition states that for su¢ciently large discount factors it is in fact a strict global maximum.

Proposition 3 For \pm su¢ciently large, the best ...rst-period scheme under no commitment is full pooling.

Proof. From proposition 9.11 in La¤ont and Tirole (1993), as ± gets arbitrarily large the optimal ...rst-period scheme is such that ${}^{o0}(x; y)_i {}^{o1}(x; y)$ gets arbitrarily small, which means an x arbitrarily close to y. But from the previous discussion, if ± is su¢ciently large, full pooling strictly dominates any scheme (x; y) with x close to y, for any y. Q.E.D.

Thus, when there is no commitment, for some su¢ciently large discount factors it is not in the interest of the principal to o¤er a menu of contracts in period one.

4 Simulations

The simulations are based on the quadratic utility of exort functions used by Laxont and Tirole in their Appendix 9.9.

Two types of equilibrium are of interest, depending on whether or not the incentive compatibility constraint of the ineCcient type is binding. Recall that, for given parameters, the take-the-money-and-run strategy will matter only when the discount factor is suCciently large. Table 1 reproduces the results from table 9.1 in La¤ont and Tirole. When $\pm = 0.01$ and $\pm = 0.1$, the IC constraint is not binding and the optimal scheme involves full separation in period one. For $\pm = 1$ and $\pm = 10$, the constraint is binding.

Figures 5 and 6 depict the intertemporal welfare loci for these values of the discount factor. When $\pm = 1$, the optimal scheme is also full separation. In ...gure 5a, the curve PS⁰ is the intertemporal welfare frontier with $\pm = 1$ generated by varying x, when only the eccient type is allowed to randomize. Point P corresponds to the full pooling scheme and S⁰ to full separation under the binding \overline{IC} constraint. Figure 5b is an enlargement of ...gure 5a in a neighborhood of the full pooling scheme; it depicts the trade-o¤s between the ...rst and second period welfare under double randomization, for di¤erent values of y. Note that each curve (for a given y > 0) is convex in a close neighborhood of P, emphasizing the fact that full pooling cannot be a solution. This corresponds to the case $\pm < \frac{b}{2}$ in lemma 3 where full pooling is a local minimum.

(Figure 5 here)

By contrast, when $\pm = 10$ the optimal scheme involves full pooling. Figure 6 is an enlargement in a neighborhood of full pooling for this value of the discount factor. Note that each intertemporal locus for y > 0 is concave in a close neighborhood of the full pooling point P, which corresponds to the fact that $\pm > \frac{1}{2}$. In this case, recalling that each locus for y > 0 is tangent to the isowelfare line through P, full pooling dominates any scheme in its neighborhood.

(Figure 6 here)

5 Conclusion

In this paper we have proposed a graphical analysis of multi-period procurement under asymmetric information. We have shown how the di¤erent commitment assumptions generate di¤erent allocation results over time. In particular, we have obtained that, when the discount factor is su¢ciently high, full pooling is optimal in the non-commitment model while it is never optimal under full commitment or under commitment and renegotiation.

Our analysis was based on the intertemporal welfare frontier, between the ...rst and second period welfare levels, generated by varying the degree of randomization that characterizes semi-separating incentive schemes. The shape of this frontier is determined by the prior with respect to the ...rm's type, by the e¢ciency parameters and by the commitment assumptions. The frontier emphasizes the trade-o¤ between e¢ciency and rent under commitment and renegotiation and shows how some degree of pooling (semi-pooling) permits more e¢ciency with less rent by reducing the speed of information revelation. Under noncommitment, when all self-selection constraints are binding, the frontier itself becomes a function of the discount factor. This demonstrates the di¢culty of ...nding simple solutions even in the two-period-two-type case.

Appendix

Proof of proposition 2: As discussed in the text, the $\overline{\text{IC}}$ constraint is binding for \pm su¢ciently large. To solve for the ...rst-period contracts, we therefore maximize (28) subject to

$$^{\odot}(\overline{e}_{1})_{i} ^{\odot}(\underline{e}_{1} + \mathbb{C}^{-}) + \pm U(^{\circ 1}(x)) = 0$$
(35)

Let ¹ denote the multiplier of (35). Then

$$\frac{dW_{1}(x)}{dx} = {}^{\circ}(1 +])^{\otimes} (\overline{}_{i} \overline{e}_{1} + \tilde{A}(\overline{e}_{1} i \ \mathbb{C}^{-}))_{i} (\underline{}_{i} \underline{e}_{1} + \tilde{A}(\underline{e}_{1}))^{a} (36)$$
$$i^{-1} \pm \frac{dU [{}^{\circ}(x)]}{dx}$$

where ¹, \underline{e}_1 and \overline{e}_1 are solution values as a function of x. At x = 0, it can be veri...ed that ¹ = 0, $\overline{e}_1 = \overline{e}^P$ and that \underline{e}_1 is de...ned by

$$^{\mathbb{C}}(\overline{e}^{P})_{i} \quad ^{\mathbb{C}}(\underline{e}_{1} + \mathbb{C}^{-}) + \pm U(^{\circ}) = 0$$
(37)

Thus,

$$\frac{dW_{1}(x)}{dx} = \circ (1 +) \circ (\overline{i} = \overline{e}^{P} + \widetilde{A}(\overline{e}^{P} + \overline{e}^{P})) = (\overline{i} = - (1 +) \circ (\overline{i} = \overline{e}^{P} + \widetilde{A}(\overline{e}^{P} + \overline{e}^{P})) = (\overline{e}^{P} + \widetilde{e}^{P} + \widetilde{e}^{P} + \widetilde{e}^{P}) = (\overline{e}^{P} + \overline{e}^{P} + \overline{e}^{P} + \overline{e}^{P}) = (\overline{e}^{P} + \overline{e}^{P} + \overline{e}^{P}$$

From (37), \underline{e}_1 is increasing in \pm which implies that $W_1^{\emptyset}(0)$ can be made negative for \pm su¢ciently large. The result then follows from the facts that $W_2^{\emptyset}(x) < 0$ for x $\mathbf{6}$ 0 and $W_2^{\emptyset}(0) = 0$. Q.E.D.

Proof of lemma 2: Using the envelope theorem,

$$\frac{@W_2(x;y)}{@x} = i {}^{\circ}x {}_{\circ}\frac{@}{@x} {}^{\otimes}U {}^{\mathbf{f}_{\circ 1}}(x;y) {}^{\mathbf{n}}i {}^{\mathsf{H}_{\circ 0}}(x;y) {}^{\mathbf{n}^{\mathbf{a}}} < 0$$
(39)

In particular, at $x = y_{i}$

$$\frac{@W_2(x;y)}{@x} = (1_i^{\circ})^{\circ 2} U^{0}(^{\circ}) + \frac{y}{1_i^{\circ}y} < 0$$
(40)

Regarding the ...rst period welfare, let ¹ denote the Lagrange multiplier associated with the constraint (27). Using the envelope theorem and substituting from (27),

$$\frac{@W_{1}(x; y)}{@x} = \circ(1 + 1)^{\circ} \left(\overline{a}_{i} = \overline{b}_{1} + \widetilde{A}(\overline{e}_{1} + \overline{a}_{i} + \overline{c}_{1})\right)_{i} \left(\overline{a}_{i} = \overline{b}_{1} + \widetilde{A}(\underline{e}_{1})\right)^{\circ} + \pm (1 + \circ x_{1})^{\circ} \frac{@}{@x} \left(\overline{b}_{0} + \overline{b}_{0}(x; y)\right)^{\circ} + U^{\circ} \left(\overline{b}_{0}(x; y)\right)^{\circ}$$
(41)

This expression may be either positive or negative. At x = y, it is easily seen that $^{1} = 0$ and we have

$$\frac{@W_{1}(x; y)}{@x} = \frac{1}{x = y} = \frac{1}{x} \pm (1 - \frac{1}{x})^{\circ 2} U^{0}(\circ) + \frac{1}{1 + \frac{y}{1 + y}} = 0$$
(42)

Combining (40) and (42), we get the statement in the lemma. Q.E.D.

Proof of lemma 3: Write $W(x; y) = W_1(x; y) + \pm W_2(x; y)$. Let ${}^1(x; y)$ be the multiplier of the constraint (27) in the ...rst-period optimization program. From the expression for ${}^{@}W_1 = {}^{@}x$ and ${}^{@}W_2 = {}^{@}x$ and using (27),

$$\frac{@W(x;y)}{@x} = (1 + \sqrt{2})^{\circ} \stackrel{\mathbf{f}}{(-i)} \overline{e_1} + \tilde{A}(\overline{e_1} + (\overline{e_1})) \stackrel{\mathbf{f}}{(-i)} (\frac{-i}{2} + (\overline{e_1}))^{\mathbf{a}} + \pm \frac{1}{@x} \stackrel{@}{@x} \stackrel{@}{U} \stackrel{\mathbf{f}}{(-i)} (x;y) \stackrel{\mathbf{a}}{i} \stackrel{U}{U} \stackrel{\mathbf{f}}{(-i)} (x;y) \stackrel{\mathbf{a}}{i} (x;y) \stackrel{\mathbf{f}}{(-i)} (x;y) \stackrel{\mathbf{a}}{(-i)} (x$$

where \underline{e}_1 , \overline{e}_1 and $\frac{1}{2}$ are short-hand for $\underline{e}_1(x; y)$, $\overline{e}_1(x; y)$ and $\frac{1}{2}(x; y)$. Noting that $\frac{1}{2}(y; y) = 0$ and using $\overline{e}_1(y; y) = \overline{e}^P$ and $\underline{e}_1(y; y) = \overline{e}^P$ i \mathbb{C}^- ,

$$\frac{{}^{@2}W(x;y)}{{}^{@}x^{2}} = (1 +)^{\circ}(1 + \tilde{A}^{0}(\bar{e}^{P} + \bar{e}^{-})) \quad \frac{{}^{@}\underline{e}_{1}}{{}^{@}x} + \pm C_{x} \frac{{}^{@}1}{{}^{@}x} \quad (44)$$

where all derivatives on the right-hand side are evaluated at x = y and where

$$= i^{\circ}(1i^{\circ}) U^{0}(^{\circ}) \frac{1}{1i^{\circ}} + \frac{1}{y^{\circ}} > 0$$
 (46)

Solving for the comparative statics of the ...rst-period program,

-

$$\frac{@\underline{e}_1}{@x} \Big[\frac{1}{x=y} = \frac{(1 i y) \pm C_x}{C^0(\overline{e}^P)} > 0$$
(47)

$$\frac{@\overline{e}_1}{@x}\Big]_{x=y} = \frac{y \pm c_x}{\overline{O}(\overline{e}^P)} < 0$$
(48)

$$\frac{e^{1}}{e_{x}} = \frac{B^{\odot^{0}}(\overline{e}^{P}) + y(1 + y(D \pm C_{x}))}{E^{\odot^{0}}(\overline{e}^{P})^{\frac{1}{2}}}$$
(49)

where

$$B (1_{i} \circ)(1 +)(1_{i} \tilde{A}^{0}(\overline{e}^{P})) < 0$$
(50)

$$D \quad \int \circ {}^{\mathbf{f}}(1+)\tilde{A}^{(0)}(\overline{e}^{P}_{i} \ \mathbb{C}^{-}) + \mathbb{I}^{(0)}(\overline{e}^{P})^{\mathbf{a}} + (1_{i} \circ)(1+)\tilde{A}^{(0)}(\overline{e}^{P}) > 0$$
(51)

Substituting in (44),

$$\frac{{}^{@}{}^{2}W(x;y)}{{}^{@}{x^{2}}} = E \quad \frac{{}^{\pm} {}^{C}{}_{x}}{{}^{\odot}{}^{0}(\overline{e}^{P})} \quad i \quad y(1 i \quad y)D \quad \frac{{}^{\pm} {}^{C}{}_{x}}{{}^{\odot}{}^{0}(\overline{e}^{P})} \quad (52)$$

where

$$\mathsf{E} \left(1 + \mathbf{J}\right)^{\mathbf{f}} \left(1_{\mathbf{i}} \tilde{\mathsf{A}}^{\emptyset}(\overline{\mathsf{e}}^{\mathsf{P}}_{\mathbf{i}} \mathbb{C}^{-})\right)_{\mathbf{i}} (1_{\mathbf{i}} \circ)(1_{\mathbf{i}} \tilde{\mathsf{A}}^{\emptyset}(\overline{\mathsf{e}}^{\mathsf{P}}))^{\mathbf{m}} > 0$$
(53)

where the sign follows from $\overline{e}^P_i \quad \mathbb{C}^- < e^{\alpha} < \overline{e}^P$. The sign of $@^2W = @x^2$ in (52) is the same as the sign of the expression

$$E_{i} y(1_{i} y)D_{\overline{\mathbb{O}^{0}(\overline{e}^{P})}^{\circ}} = E_{i} + \pm \frac{\circ(1_{i} \circ)D_{i}U^{0}(\circ)}{\circ}$$
(54)

Because E and D do not depend on y and given that E > 0, D > 0 and U⁰ < 0, there clearly exists a $\frac{1}{2}$ as stated in the lemma. Q.E.D.

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