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Comparative Mixed Risk Aversion: Definition and Application to Self-Protection and Willingness to Pay

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Abstract

We analyze the optimal choices of agents with utility functions whose derivatives alternate in sign, an important class that includes most of the functions commonly used in economics and finance (Mixed Risk Aversion, MRA, Caballé and Pomansky, 1996). We propose a comparative mixed risk aversion definition for this class of utility functions, namely, "More Risk Averse MRA", and provide a sufficient condition to compare individuals. We apply the model to optimal prevention and willingness to pay. More risk averse MRA agents spend less to reduce accident probabilities that are above ½. They spend more only when accident probabilities are below ½. Explanations in terms of risk premiums are provided. The results presented also allow for the presence of background risk.

Keywords: Mixed risk aversion, more risk averse MRA, self-protection, willingness to pay, background risk.

JEL classification: D80.

Introduction

The link between the structure of an agent's utility function and his action to reduce risk can be a subtle one. For example, following the contribution of Ehrlich and Becker (1972), who introduced the concepts of self-protection and self-insurance into the literature, Dionne and Eeckhoudt (1985) have shown that a more risk averse individual, in the sense of Arrow-Pratt, does not necessarily produce more self-protection activities[1].

The willingness-to-pay literature (Drèze, 1962; Jones-Lee, 1974; and Pratt and Zeckhauser, 1996) throws up another example. It is commonly acknowledged that a more risk averse decision-maker is not necessarily willing to pay more for a lower probability of accident (Eeckhoudt, Godfroid and Gollier, 1997). In a third example, McGuire, Pratt and Zeckhauser (1991) have shown that more risk averse individuals might choose more risky decisions (described as less insurance and more gamble) than less risk averse individuals. They obtained that this behavior depends upon a critical endogenous switching probability.

For many economic applications under risk and uncertainty, a simple concave transformation of a von Newmann-Morgenstern utility function (or an Arrow-Pratt increase in risk aversion) does not always yield intuitive changes in the decision variables affecting event probabilities or distribution functions. In the three previous examples, individual behavior affects the probability of events as well as the contingent outcomes. This behavior changes the outcomes distribution but does not necessarily increase risk in the sense of Rothschild-Stiglitz. It implies a first-order shift instead of a pure second-order one such as a mean preserving spread (Rothschild and

Stiglitz, 1970).[2] Consequently, to predict (risk averse) decision-makers' behavior, one needs restrictions on either utility or distribution functions that can take into account actions, such as self-protection, that may affect all distribution moments. In this paper, we look at restrictions made on utility functions. For an analysis of restrictions on distribution functions see Julien, Salanié and Salanié (1999), and for restrictions on loss functions see Lee (1998).

Section 1 presents the concept of mixed risk aversion (MRA) introduced in the literature by Caballé and Pomansky (1996) and offers the definition of "More Risk Averse MRA". Section 1 also proposes a transformation result that sets a sufficient condition to compare mixed risk averse individuals. We say that individual *v* is more risk averse MRA than individual *u* if he is more risk averse, more prudent, more temperate… or if the absolute ratio of the $n^{th}+2$ over the $n^{th}+1$ derivative of *v* is higher than the corresponding ratio of *u* for all positive integer *n*.

Section 2 shows how the concept of more risk averse MRA can be used to establish an exogenous threshold probability over which a more risk averse MRA agent invests less in self-protection activities and has a lower willingness to pay. We show that if agent *v* is more risk averse MRA than agent *u*, then *v* will select a higher level of self-protection and have a higher willingness to pay only if the accident probability is lower than ½[3]. This result is important since the threshold probability is no longer endogenous, and the great majority of risky situations that require self-protection (occupational safety, firearm safety, road safety, health care, environmental prevention, …) and public decisions on safety are characterized for events with a probability lower than ½.

We explain in detail why ½ is a critical value for obtaining the desired result. We also obtain that the switching probability of becoming a gambler is greater than ½ in the probability-improving environment of McGuire, Pratt and Zeckhauser[4]. Section 3 extends the above results to risky situations with background risk (Doherty and Schlesinger, 1983; Eeckhoudt and Kimball, 1992; Eeckhoudt, Gollier and Schlesinger, 1996). Concluding remarks are presented in Section 4.

1 Mixed risk aversion

Most of the utility functions commonly used in economics and finance such as the logarithmic and the power functions have derivatives with alternating signs showing positive odd derivatives and negative even derivatives. Caballé and Pomansky (1996) characterized the class of utility functions having this property which they called mixed risk aversion (MRA). These functions are properly characterized by the measure describing a mixture of exponential functions. Caballé and Pomansky have shown that the stochastic dominance and aggravation-of-risks concepts are more operative when applied to this class of utility functions. We build on this work by analyzing comparative mixed risk aversion.

1.1 Definition

Caballé and Pomansky defined mixed risk aversion as:

Definition 1: *(Caballé and Pomansky,* 1996*) A real-valued continuous utility function u defined on* (0,∞) *exhibits mixed risk aversion if and only if it has a completely monotone first derivative on* (0,∞) $\left(i.e.\ (-1)^n \ u^{(n+1)}\ (w) \geq 0,\ for\ \ n \geq 0 \right)$ *and u* (0) = 0.

Theorem 2.2 in Caballé and Pomansky (1996) shows that *u(w)* is a mixed risk aversion function if and only if it admits the following representation

$$u(w) = \int_0^\infty \frac{1 - e^{-wt}}{t}\, dF_u(t), \tag{1}$$

with

$$\int_1^\infty \frac{dF_u(t)}{t} < \infty .$$

Caballé and Pomansky (1996) also generalized the Arrow-Pratt index of absolute risk aversion to higher orders. They defined the n^{th} order index of absolute risk aversion as

$$A_n^u(w) = -\frac{u^{(n+2)}(w)}{u^{(n+1)}(w)}, \text{ for } n \geq 0.$$

A_0^u is the Arrow-Pratt index of absolute risk aversion, whereas A_1^u is the index of absolute prudence introduced by Kimball (1990) and A_2^u corresponds to the index of absolute temperance proposed by Eeckhoudt, Gollier and Schleisinger (1996).

The n^{th} order index of absolute risk aversion, $A_n^u(w)$, for *u* as given by (1) simplifies to*:*

$$A_n^u(w) = \frac{\int_0^\infty t^{n+1} e^{-wt}\, dF_u(t)}{\int_0^\infty t^n e^{-wt}\, dF_u(t)}.$$

This formulation of the n^{th} order index of absolute risk aversion provides equivalent characterizations for mixed risk aversion. In fact, the two following characterizations are equivalent:

i) $A_n^u(\cdot)$ is decreasing in *w* for all *w* and *n.*

or

ii) $A_n^u(w) \leq A_{n+1}^u(w)$ for all *w* and *n.*

To prove that $A_n^u(\cdot)$ is decreasing in *w* we apply the Cauchy-Schwartz inequality, i.e.,

$$\left(\int \varphi(t)\ \psi(t)\ dF_u(t)\right)^2 \leq \int \varphi^2(t)\ dF_u(t) \int \psi^2(t)\ dF_u(t),$$

to

$$\varphi(t) = t^{(n+2)/2}\, e^{-wt/2},\ \ \psi(t) = t^{n/2}\, e^{-wt/2}.$$

The equivalence between *i*) and *ii)* follows from $A_n^u(w) \geq 0,$ for all $n \geq 0,$ and the identity

$$\frac{d}{dw} A_n^u(w) = A_n^u(w)\left(A_n^u(w) - A_{n+1}^u(w)\right).$$

1.2 Mixed risk aversion and other concepts of attitude toward risk

Pratt and Zeckhauser (1987) introduced the concept of Proper Risk Aversion to predict lottery choices in the presence of an independent, undesirable lottery. A utility function is proper when an undesirable lottery can never be made desirable by the presence of another (independent) undesirable risk. Their concept is more general than mixed risk aversion, in the sense that (1) is sufficient for proper risk aversion but not necessary. Pratt and Zeckhauser have shown that mixtures of exponential utilities are proper and that properness implies decreasing absolute risk aversion (DARA). Brockett and Golden (1987) developed a parallel characterization of such functions and Hammond (1974) proposed a first application using a mixture (discrete) of exponential functions.

Mixed risk aversion implies standard risk aversion (Kimball, 1993) which implies properness (Pratt and Zeckhauser, 1987), which implies risk vulnerability (Gollier and Pratt, 1996)[5]. These three concepts were mainly developed to account for the presence of background risk; they are not directly useful for the purpose of comparing self-protection activities and willingness to pay choices' among risk averse decision makers. More restriction on the utility function is necessary and mixed risk aversion will be shown to yield interesting results.

1.3 Comparative mixed risk aversion

Consider two risk averse agents *u* and *v*. Following Pratt (1964), it has been established that comparative risk aversion amounts to applying a simple concave transformation *k* of a utility function: *v* is more risk averse than *u* if and only if $v = k(u)$ with $k'' < 0$. This type of comparison is not sufficient for comparing optimal decision variables for problems, such as self-protection or willingness to pay, that imply the variation of all moments of the distribution.

Definition 2: *Let u and v be two mixed risk averse utility functions. We say that v is more risk averse MRA than u if and only if* $A_n^u(w) \leq A_n^v(w)$*, for all n and w.*

The next proposition introduces a sufficient condition to compare mixed risk aversion.

Proposition 1: *Let u and v be two mixed risk averse utility functions described respectively by distribution functions* F_u *and* F_v*. If* $dF_v(\cdot)$ *dominates* $dF_u(\cdot)$ *in the sense of the Maximum Likelihood Ratio (i.e.* $\dfrac{dF_u(\cdot)}{dF_v(\cdot)}$ *is decreasing over* $(0,\infty)$*) then v is more risk averse MRA than u.*

Proof of Proposition 1: The proof uses characterization i) of mixed risk aversion presented in Section 1.1 and Theorem 4 in Jewitt (1987).

We discuss an example that will provide the intuition behind the proposition.

Consider

$$u(w) = -p_1 e^{-a_1 w} - p_2 e^{-a_2 w} - \ldots - p_m e^{-a_m w}$$
$$v(w) = -q_1 e^{-a_1 w} - q_2 e^{-a_2 w} - \ldots - q_m e^{-a_m w},$$

where p_i and q_i are probabilities, and a_i is a positive parameter for $i = 1, \ldots, m$ and $a_1 < a_2 < \ldots < a_m$. If $\frac{p_1}{q_1} \geq \ldots \geq \frac{p_m}{q_m}$, then from Proposition 1 we know that v is more risk averse, more prudent, more temperate,… than u, and, more generally, more risk averse MRA, that is

$$A_n^u(\cdot) \leq A_n^v(\cdot), \text{ for all } n \geq 0.$$

The intuition behind the proposition is quite simple. We know that u and v are mixtures of CARA utility functions. Consider, for the sake of illustration, the case where there are only two positive a_{i} (a_1 and a_2) in the example above. By transforming p_1 into q_1 lower than p_1, less weight is put upon the less risk averse CARA component of the u function (since $a_1 < a_2$). Of course lowering p_1 also implies that q_2 exceeds p_2 so that simultaneously more weight is placed upon the more risk averse component of u. So v is surely more risk averse than u and the proposition shows that this property automatically extends to all ratios of the successive derivatives of each utility function.

In Section 3, we show how the proposition can be extended to the presence of a background risk.

2 Applications

We now apply the comparative mixed risk aversion result to decisions by mixed risk averse agents to self-protection and to willingness to pay. Intuitively, we may expect a more risk averse MRA agent to exert more self-protection activities and to be willing to pay a higher monetary amount for a lower probability of accident. We will see that this result can be obtained only when the accident probability is sufficiently low.

2.1 Self-protection

The standard model for self-protection (Ehrlich and Becker, 1972) can be summarized as follows. Consider an individual with a von Neuman-Morgenstern utility function u and a non-random initial wealth w_0. The agent faces a risk of loss l and can invest a quantity x in self-protection activities, in order to reduce the probability of loss $(p(x))$, a decreasing function of x. The action cost c for one unit of x is fixed at $c \equiv 1$. With two states of the world, the optimal choice of self-protection is a solution of:

$$\max_x \; p(x)\, u\,(w_0 - l - x) + (1 - p(x))\, u\,(w_0 - x), \tag{2}$$

subject to the constraint $x \geq 0$. The first-order condition for an optimal choice x_u^* requires

$$p'(x)\left[u\,(w_0 - l - x) - u\,(w_0 - x)\right] - \left[p(x) u'(w_0 - l - x) + (1 - p(x))\, u'(w_0 - x)\right] = 0. \tag{3}$$

The second order necessary condition is

$$p''(x)\left[u(w_0-l-x)-u(w_0-x)\right]-2p'(x)\left[u'(w_0-l-x)-u'(w_0-x)\right]$$
$$+p(x)u''(w_0-l-x)+(1-p(x))u''(w_0-x)\leq 0.$$

Note that risk aversion is not sufficient to ensure the second-order condition (see Arnott, 1992, for details). In the remainder of this article, we assume that all conditions for allowing the solution of (3) to be a global maximum are met. Consequently all the derived results are restricted to these conditions as is usually done in this literature.

Self-protection activities do not necessarily reduce risk in the sense of Rothschild-Stiglitz, but do affect the probabilities of the various states as well as their contingent outcomes. The problem of analyzing the effect of risk aversion on optimal self-protection activities is different from that where probabilities are fixed, as in the context of market insurance or self-insurance. One consequence is that more risk averse agents in the sense of Arrow-Pratt will not necessarily choose a higher level of self-protection spending (see Dionne and Eeckhoudt, 1985). McGuire, Pratt and Zeckhauser (1991) have found an endogenous critical switching probability that depends on preferences and outcomes, and they interpret expenditures as gambling or less insurance. This endogenous switching probability is retrieved by Lee (1998). In this section, we show that the endogenous probability is lower than ½ for self-protection and willingness to pay and greater than ½ in the probability-improving environment of McGuire, Pratt and Zeckhauser (1991).

Julien, Salanié and Salanié (1999) have shown that if v is more risk averse than u in the sense of Arrow-Pratt, then there exists a threshold probability $\overline{p}$ such that self-protection is higher for v

than for *u* if and only if the probability of loss resulting from the optimal choice of *u* is less than $\overline{p}$, with

$$\overline{p} = \frac{\sum_{i=1}^{\infty} \frac{(-1)^i}{i!} l^i \left[v' u^{(i)} - u' v^{(i)}\right]}{\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} (-1)^i \, (-1)^j \, \frac{l^i l^j}{i!\, j!} \left[v^{(i)} u^{(j+1)} - u^{(i)} v^{(j+1)}\right]}. \tag{4}$$

This switching probability is endogenous since it depends on *u*, *v* and on outcomes. The critical value $\overline{p}$ is then different on a case-by-case basis. We now propose an exogenous probability. In fact, we can show the next proposition for the class of mixed risk averse utility functions.

Proposition 2: *Suppose the probability of loss p(x) is decreasing in x and let u and v be two mixed risk averse utility functions. Let* x_u^* *and* x_v^* *be the optimal level of effort selected respectively by agents u and v. Let v be more risk averse MRA than u, and suppose* x_v^* *is higher than* x_u^**, then* $p(x_u^*) < 1/2$.

Proof of Proposition 2: See Appendix A.

Proposition 2 can be stated equivalently as: Suppose agent *u* selects an effort x_u^*, such that his probability of accident $p(x_u^*)$ is higher than 1/2, then all more risk averse MRA agents *v* will select a level of effort x_v^* smaller than x_u^*: Prevention decreases with risk aversion if the probability of loss is sufficiently high.

To see why a more risk averse MRA individual may produce less self-protection, we first consider an agent with an exponential utility function. For an exponential utility function *v* with δ as the measure of absolute risk aversion, we draw the risk premium as a function of *p*, the probability of accident, as in Figure 1 (the π function). We see that the risk premium is increasing up to p^* and then decreasing.

Explicitly,

$$\pi = \frac{1}{\delta}\left[log\left[p\left(e^t - 1\right) + 1\right] - tp\right],$$

where $t = \delta l$.

Solving for p^* gives $p^* = (1/t) - 1/\left(e^t - 1\right)$. One can show that $p^* < 1/2$ for all δ[6]. Suppose now that a risk neutral agent minimizes the expected total loss ($x + p(x)l$) over *x* at x_u^* and that $p_u^* = p\left(x_u^*\right)$ is higher than ½. Then, for the risk averse individual *v*, spending more on self-protection than x_u^* (reducing *p*) will increase both the total expected loss and the risk premium (see Figure 1), which must be undesirable. For an exponential mixture, the same is true for every term in (1) and, hence, the exponential mixture (i.e. mixed risk averse) agent must spend less on self-protection than the risk neutral agent.[7]

(Figure 1 here)

Figure 1 also shows the difficulty of predicting self-protection decisions when $p_u^* < 1/2$. In this case, the effects on the risk premium may be positive for some p_u^* values and negative for other p_u^* values.

In Appendix B we evaluate the first-order condition (3) for an exponential utility function at the optimal level of a risk-neutral agent and give more insights as to why the sign of (3) is negative for $p_u^* \geq 1/2$, and undetermined otherwise. Figure 2 shows clearly that p_u^* must be lower than ½ to guarantee that the exponential risk averse (and so the mixed risk averse) agent will produce more prevention than the risk neutral decision-maker $((B1) > 0)$.

One corollary from Proposition 2 is that, at equilibrium, when the optimal choice of the risk-neutral decision-maker is higher than ½, all MRA agents will select a lower level of prevention and will have accident probabilities higher than ½.

By symmetry, it can be obtained that $\overline{p} \geq$ ½ when the winning probability $p(x)$ is increasing in x as in McGuire, Pratt and Zeckhauser (1991). The mathematical development is identical to that made in Proposition 2, with $1 - p(x)$ being the probability of loss for the modified problem. We then have the next result:

Corollary: *Under the same notation as in Proposition 2, with an increasing winning probability function of* x*,* $p(x)$*, if* $x_u^* \leq x_v^*$, then $p(x_u^*) \geq 1/2$.

2.2 Willingness to pay

Willingness to pay (*WTP*) is a guideline for public and private investment policies; and, according to *WTP*, public investment projects, such as health care, environmental prevention or road safety investments will be recommended only if the total benefits for the different agents benefiting from favorable probability changes exceed the capital cost of the project concerned. Alternative resource allocations are also compared on the basis of *WTP* (see Dionne and Lanoie, 2003, and Viscusi and Aldy, 2003, for recent surveys).

In some situations it is more appropriate to offer different bundles of risk to different individuals, if valuations of risk differ among agents. To establish such bundles, one then needs to know the *WTP* for the different risk averse categories. As we did for the self-protection model, we can use the concept of more risk aversion MRA to order *WTP* values. We can show the next result.

Proposition 3: *Let u and v be two mixed risk averse utility functions and WTP_u, WTP_v their corresponding amounts of willingness to pay. Let v be more risk averse MRA than u, and suppose that WTP_u* is smaller than WTP_v, *then the probability of a loss corresponding to the willingness to pay choice of u is lower than ½.*

Proof of Proposition 3: See Appendix A.

3 Background risk

Consider now the case where the individual u faces a background risk $(\tilde{\varepsilon})$ on wealth that is independent of the occurrence of an accident. Let's denote $\tilde{u}(w) = E_{\varepsilon}(u(w+\varepsilon)) = \int u(w+\varepsilon)\, dF(\varepsilon)$. We know that an individual with a utility function u and a background risk $\tilde{\varepsilon}$ behaves as an individual with utility function $\tilde{u}$ and no background risk. Kimball (1993) has shown that if u has a decreasing absolute risk aversion and a decreasing absolute prudence, then these properties hold for $\tilde{u}$. In other words, if u is standard risk averse then $\tilde{u}$ is also standard risk averse. We also know that mixed risk aversion implies standardness. Consequently, if u is mixed risk averse, then $\tilde{u}$ is also mixed risk averse and hence, for all $n \geq 0$, $-\frac{\int u^{(n+2)}(w+\varepsilon)dF(\varepsilon)}{\int u^{(n+1)}(w+\varepsilon)dF(\varepsilon)}$ is decreasing in w (following i in Section 1.1), which is an extension of Proposition 4 in Kimball (1993) to mixed risk aversion. Consequently, we can state the following proposition.

Proposition 4: *Let u and v be two mixed risk averse utility functions and suppose that v is more risk averse MRA than u, then both* $\tilde{u}$ *and* $\tilde{v}$ *are mixed risk averse utility functions and* $\tilde{v}$ *is more risk averse MRA than* $\tilde{u}$.

A detailed proof of Proposition 4 is in Dachraoui et al. (1999).

Proposition 4 allows us to extend the results of Section 2 directly to situations with a background risk. In particular, if the probability of loss resulting from the optimal self-protection choice of agent u is higher than ½, and if agent v is more risk averse MRA than u, then even in the presence of a background risk, it follows from Propositions 2 and 4 that agent v will select a smaller self-protection effort than agent u. It follows also from Propositions 3 and 4 that agent v will have a lower W*TP* than agent u when $p > ½$.

4 Conclusion

In this article we have characterized comparative mixed risk aversion and provided a sufficient condition to compare attitudes toward risk in the class of mixed risk averse utility functions. We have shown how this comparison of attitudes to risk can be useful in ordering optimal decision variables that affect all distribution moments among different mixed risk averse individuals. More risk averse MRA individuals select higher effort and have a higher willingness to pay, only if the probability of accident is lower than ½.

Many extensions of this article can be considered. First, it would be interesting to analyze how our measure of the comparative attitudes to risk generated by more risk aversion MRA can be useful in predicting the agent's action in a principal-agent framework when utility functions are not additively separable. How do different mixed risk averse agents choose optimal sharing contracts? A more difficult question would be to compare how different risk-sharing contracts are handled by different mixed risk averse agents.

Another extension is related to the willingness-to-pay literature. The study of willingness- to-pay aggregation is now possible in the class of MRA utility functions, since we have established an exogenous probability to compare optimal amounts.

Appendix A

Proof of Proposition 2

Proving Proposition 2 is equivalent to proving that $\overline{p} < 1/2$.

Let's denote

$$K_{ij} = (-1)^i \ (-1)^j \left[v^{(i)} \, u^{(j+1)} - u^{(i)} \, v^{(j+1)} \right].$$

One can show that:

$$K_{ij} \begin{cases} > 0 \ if \ i < j+1 \\ = 0 \ if \ i = j+1 \\ < 0 \ if \ i > j+1 \end{cases} . \tag{A1}$$

The denominator in (4) can be written as:

$$-l \sum_{i=1}^{\infty} \frac{(-1)^i}{i!} l^i \left[v' u^{(i+1)} - u' v^{(i+1)} \right] + \sum_{i=2}^{\infty} \sum_{j=1}^{\infty} \frac{l^{i+j}}{i! \, j!} K_{ij} \ ,$$

with $-l \sum_{i=1}^{\infty} \frac{(-1)^i}{i!} \ l^i \ \left[v' \ u^{(i+1)} - u' \ v^{(i+1)} \right] \geq 0$ from (A1).

Now we prove that

$$\sum_{i=2}^{\infty} \sum_{j=1}^{\infty} \frac{l^{i+j}}{i! \, j!} K_{ij} \ \geq 0 . \tag{A2}$$

First, note that

$$\begin{aligned} K_{ij} &= (-1)^i \ (-1)^j \left[v^{(i)} \ u^{(j+1)} - u^{(i)} \ v^{(j+1)} \right] \\ &= -(-1)^i \ (-1)^j \left[u^{(i)} \ v^{(j+1)} - v^{(i)} \ u^{(j+1)} \right] \\ &= -(-1)^{i-1} \ (-1)^{j+1} \left[u^{(i)} \ v^{(j+1)} - v^{(i)} \ u^{(j+1)} \right] \\ &= -K_{j+1,\, i-1}, \end{aligned} \tag{A3}$$

and hence

$$\frac{1}{i!\,j!}K_{ij}+\frac{1}{(j+1)\,!\,(i-1)\,!}K_{j+1,i-1}=(\frac{1}{i!\,j!}-\frac{1}{(j+1)\,!\,(i-1)\,!})K_{ij},$$

Moreover, since $i > j + 1$ if and only if $\frac{1}{i!\,j!}<\frac{1}{(j+1)!(i-1)!}$, from (A1) we have

$$\frac{1}{i!\,j!}K_{ij}+\frac{1}{(j+1)\,!\,(i-1)\,!}K_{j+1,i-1}\geq 0. \tag{A4}$$

Next we prove that (A4) is sufficient to get (A2).

We can write $\sum_{i=2}^{n}\sum_{j=1}^{n}\frac{l^{i+j}}{i!\,j!}K_{ij}$ as

$$\sum_{i=2}^{n}\sum_{j=1}^{n}\frac{l^{i+j}}{i!\,j!}K_{ij}=\sum_{k=3}^{2n}l^{k}(\sum_{\substack{i+j=k,\\ i\geq 2,\, j\geq 1}}\frac{1}{i!\,j!}K_{ij}). \tag{A5}$$

Depending on whether k is odd or even, the term inside the summation in (A5) can be written as:

- k is even $(k=2m,\ m\geq 2)$.

$$\begin{aligned}\sum_{\substack{i+j=2m,\\ i\geq 2,\, j\geq 1}}\frac{1}{i!\,j!}K_{ij}&=\sum_{\substack{i+j=2m,\\ i\leq p, i\geq 2,\, j\geq 1}}\frac{1}{i!\,j!}K_{ij}+\sum_{\substack{k+l=2m,\\ k\geq p+1, k\geq 2, l\geq 1}}\frac{1}{k!\,l!}K_{kl}\\ &=\sum_{\substack{i+j=2m,\\ i\leq p, i\geq 2,\, j\geq 1}}\frac{1}{i!\,j!}K_{ij}+\sum_{\substack{k+l=2m,\\ l\leq p-1, k\geq 2, l\geq 1}}\frac{1}{k!\,l!}K_{kl}.\end{aligned} \tag{A6}$$

Redefining indexes in the second term of the right hand side in the previous equation as $i=l+1$ and $j=k-1$, (A6) can be written as

$$\sum_{\substack{i+j=2m,\\ i\leq p, i\geq 2,\, j\geq 1}}\frac{1}{i!\,j!}K_{ij}+\sum_{\substack{i+j=2m,\\ i\leq p, i\geq 2, i\geq 1}}\frac{1}{(j+1)!(i-1)!}K_{j+1,i-1}=\sum_{\substack{i+j=2m,\\ i\leq p, i\geq 2, i\geq 1}}(\frac{1}{i!\,j!}K_{ij}+\frac{1}{(j+1)!(i-1)!}K_{j+1,i-1}),$$

which is positive from (A4).

- k is odd $(k=2m+1,\ m\geq 1)$.

Since $K_{m+1,m}=0,$ it follows that

$$\sum_{\substack{i+j=2m+1,\\ i\geq 2,\, j\geq 1}} \frac{1}{i!\,j!}K_{ij} = \sum_{\substack{i+j=2m+1,\\ i\leq p, i\geq 2,\, j\geq 1}} \frac{1}{i!\,j!}K_{ij} + \sum_{\substack{k+l=2m+1,\\ k\geq p+2, k\geq 2, l\geq 1}} \frac{1}{k!l!}K_{kl}$$

$$= \sum_{\substack{i+j=2m+1,\\ i\leq p, i\geq 2,\, j\geq 1}} \frac{1}{i!\,j!}K_{ij} + \sum_{\substack{k+l=2m+1,\\ l\leq p-1, k\geq 2, l\geq 1}} \frac{1}{k!l!}K_{kl}.$$

Once again redefining indexes in the second term in the right hand side of the previous equation as $i=l+1$ and $j=k-1,$ we get

$$\sum_{\substack{i+j=2m+1,\\ i\leq p, i\geq 2,\, j\geq 1}} \frac{1}{i!\,j!}K_{ij} + \sum_{\substack{k+l=2m+1,\\ l\leq p-1, k\geq 2, l\geq 1}} \frac{1}{k!l!}K_{kl} = \sum_{\substack{i+j=2m,\\ i\leq p, i\geq 2, i\geq 1}} \left(\frac{1}{i!\,j!}K_{ij} + \frac{1}{(j+1)!(i-1)!}K_{j+1,i-1}\right),$$

which is also positive from (A4).

As a result

$$\forall \;\; n\geq 2,\;\; \sum_{i=2}^{n}\sum_{j=1}^{n}\frac{l^{i+j}}{i!\,j!}\;\; K_{ij}\geq 0,$$

and at the limit we obtain

$$\sum_{i=2}^{\infty}\sum_{j=1}^{\infty}\frac{l^{i+j}}{i!\,j!}K_{ij}\;\;\geq 0\,.$$

The denominator in (4) is the sum of two positive terms. We can then write

$$\overline{p}\leq \frac{\displaystyle\sum_{i=1}^{\infty}\frac{(-1)^i}{i!}\,l^i\left[v'u^{(i)}-u'v^{(i)}\right]}{\displaystyle\sum_{i=1}^{\infty}\frac{(-1)^{i+1}}{i!}\,l^{i+1}\left[v'u^{(i+1)}-u'v^{(i+1)}\right]}$$

or

$$\overline{p}\leq \frac{\displaystyle\sum_{i=2}^{\infty}\frac{(-1)^i}{i!}\,l^i\left[v'u^{(i)}-u'v^{(i)}\right]}{\displaystyle\sum_{i=2}^{\infty}\frac{(-1)^i}{(i-1)!}\,l^i\left[v'u^{(i)}-u'v^{(i)}\right]}.$$

Since for $i \geq 2$, $i! > 2(i-1)!$, and we know that $(-1)^i\, l^i \left[v' u^{(i)} - u' v^{(i)}\right] > 0$, we then have

$$\frac{(-1)^i}{i!} l^i \left[v' u^{(i)} - u' v^{(i)}\right] < \frac{1}{2} \frac{(-1)^i}{(i-1)!} l^i \left[v' u^{(i)} - u' v^{(i)}\right].$$

Taking the summation over $i \geq 2$ gives $\bar{p} <$ 1/2.

Proof of Proposition 3

The expected utility for *u* is equal to:

$$U = pu(w_0 - l) + (1-p)u(w_0)$$

and that of individual *v* to:

$$V = pv(w_0 - l) + (1-p)v(w_0).$$

In order to obtain the willingness to pay for *u* (Drèze, 1962; Jones-Lee, 1974), we completely differentiate *U* with respect to *p* and w_0 to obtain:

$$WTP_u = \frac{dw_0}{dp} = \frac{u(w_0) - u(w_0 - l)}{pu'(w_0 - l) + (1-p)u'(w_0)} \tag{A7}$$

The same result holds for individual *v*

$$WTP_v = \frac{dw_0}{dp} = \frac{v(w_0) - v(w_0 - l)}{pv'(w_0 - l) + (1-p)v'(w_0)} \tag{A8}$$

The threshold probability $\bar{p}$ is solution of (A7) = (A8)

$$\bar{p} = \frac{v'(w_0)\Delta u - u'(w_0)\Delta v}{\Delta u' \Delta v - \Delta v' \Delta u} \tag{A9}$$

where v' and u' represent derivatives with respect to w_0.

With the Taylor expansion we have

$$\Delta u = \sum_{i=1}^{\infty} (-1)^i \frac{l^i}{i!} u^{(i)}(w_0 - x),\ \ \Delta v = \sum_{i=1}^{\infty} (-1)^i \frac{l^i}{i!} v^{(i)}(w_0 - x).$$

Similar values can be obtained for v' and u'.

We can then rewrite (A9) as:

$$\overline{p} = \frac{\sum_{i=1}^{\infty} \frac{(-1)^i}{i!} l^i \left[v' u^{(i)} - u' v^{(i)} \right]}{\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} (-1)^i (-1)^j \frac{l^i l^j}{i!\, j!} \left[v^{(i)} u^{(j+1)} - u^{(i)} v^{(j+1)} \right]}$$

The remainder of the proof to obtain that $\overline{p} < 1/2$ is the same as in Proposition 2.

Appendix B

The first order condition (3) of an exponential utility function evaluated at p_u^*

A Taylor expansion of the first order condition (3) around W-x- $p_u^* l$ gives

$$\sum_{n\geq 2}\frac{(-1)^n}{n!}\left[\left(1-p_u^*\right)^n-\left(-p_u^*\right)^n\right]l^{n-1}u^{(n)}\left(W-x-p_u^*l\right)-\sum_{n\geq 1}\frac{(-1)^n}{n!}\Big[p_u^*\left(1-p_u^*\right)^n + \left(1-p_u^*\right)\left(-p_u^*\right)\Big]l^n u^{(n+1)}\left(W-x-p_u^*l\right).$$

For an exponential utility function with absolute risk aversion δ, this expression becomes

$$le^{W-x-p_u^*l}\Bigg[\sum_{n\geq 2}\frac{1}{n!}\left(1-p_u^*\right)^n l^n\delta^n-\sum_{n\geq 2}\frac{1}{n!}\left(-p_u^*\right)^n l^n\delta^n-p_u^*\delta\sum_{n\geq 1}\frac{1}{n!}\left(1-p_u^*\right)^n l^n\delta^n - \left(1-p_u^*\right)\delta\sum_{n\geq 1}\frac{1}{n!}\left(-p_u^*\right)^n l^n\delta^n\Bigg].$$

The sign of this expression is that of the term inside the square brackets. The latter can be simplified to the next expression

$$\left(1-p_u^*t\right)exp\left(\left(1-p_u^*\right)t\right)-\left(1+\left(1-p_u^*\right)t\right)exp\left(-p_u^*t\right),\ \text{where } t=\delta l. \tag{B1}$$

For $p_u^* \geq 1/2$ the last expression is always negative no matter what the level of absolute risk aversion (δ) or the level of loss (l) are. If v is an exponential mixture, the same is true for every term. Hence the expression equivalent to (B1) is negative for v.

For $p_u^* < 1/2$ the sign of (B1) depends on δ and l (or $t = \delta l$) as illustrated in Figure 2. If v is an exponential mixture, the sign in expression (B1) is then undetermined.

(Figure 2 here)

Notes

[1] On this issue see also Briys and Schlesinger (1990), Julien, Salanié and Salanié (1999), Chiu (2000), Gollier and Eeckhoudt (2001), and Lee (1998). In fact, one cannot make any prediction on how a more risk averse agent will choose his optimal level of effort in a principal-agent relationship without introducing strong assumptions on the utility function (Arnott, 1992).

[2] For the self-protection example, the i^{th} moment of the gross expected loss is $p(x)l^i$, where p is the probability of accident, x is the level of self-protection and l is the amount of loss in case of accident. For the principal-agent problem where the outcomes distribution can be written as $F(l/x)$ it is also clear that the first derivative $F_x(l/x)$ affects more than just the mean of l.

[3] Julien, Salanié and Salanié (1999) derived independently a similar result. However, they did not study the existence of an exogenous boundary that will be effective for all risk averse agents and for all levels of loss.

[4] In their model, activity x increases the winning probability instead of decreasing the probability of loss as in the self-protection and willingness to pay applications.

[5] See Gollier and Pratt (1996) for a comparison of these three concepts (risk vulnerability, properness and standardness) proposed in the recent literature and related to the willingness to accept a risk when another independent background risk is added to random wealth. In this article, we consider only one random variable. See however Section 3.

[6] With $\lim_{\delta \to 0} p^* = 1/2$.

[7] The example is in the spirit of Eeckhoudt and Gollier (2001), who compared the optimal prevention of a risk averse (prudent) agent to the optimal prevention of a risk neutral decision maker.

References

Arnott, Richard. (1992). "Moral Hazard and Competitive Insurance Markets." In Georges Dionne (ed.), *Contributions to Insurance Economics*. Boston: Kluwer Academic Press.

Briys, Eric and Harris Schlesinger. (1990). "Risk-Aversion and the Propensities for Self-Insurance and Self-Protection." *Southern Economic Journal* 57, 458-467.

Brockett, Patrick L. and Linda L. Golden. (1987). "A Class of Utility Functions Containing All the Common Utility Functions." *Management Science* 33, 955-964.

Caballé, Jordi and Alexey Pomansky. (1996). "Mixed Risk Aversion." *Journal of Economic Theory* 71, 485-513.

Chiu, W. Henry. (2000). "On the Propensity to Self-Protect." *Journal of Risk and Insurance* 67, 555-577.

Dachraoui, Kaïs, Georges Dionne, Louis Eeckhoudt, and Philippe Godfroid. (1999). "Proper Risk Behavior," Working paper 99-01, Risk Management Chair, HEC Montréal (http://www.hec.ca/gestiondesrisques/99-01.pdf).

Dionne, Georges and Louis Eeckhoudt. (1985). "Self-Insurance, Self-Protection and Increased Risk-Aversion." *Economics Letters* 17, 39-42.

Dionne, Georges and Paul Lanoie. (2003). "Public Choice About the Value of a Statistical Life for Cost-Benefit Analyses: The Case of Road Safety." (http://www.aei-brookings.org/publications/abstract.php?pid=375), forthcoming in *Journal of Transport Economics and Policy*.

Doherty, Neil and Harris Schlesinger. (1983). "Optimal Insurance in Incomplete Markets." *Journal of Political Economy* 91, 1045-1054.

Drèze, Jacques. (1962). "L'utilité sociale d'une vie humaine." *Revue française de recherche opérationnelle* 6, 93-118.

Eeckhoudt, Louis, Philippe Godfroid, and Christian Gollier. (1996). "Willingness to Pay, Risk Premium and Risk Aversion." *Economics Letters* 55, 355-360.

Eeckhoudt, Louis and Christian Gollier. (2001). "The Impact of Prudence on Optimal Prevention." Mimeo. Université de Toulouse, France.

Eeckhoudt, Louis, Christian Gollier, and Harris Schlesinger. (1996). "Changes in Background Risk and Risk Taking Behavior." *Econometrica* 64, 683-689.

Eeckhoudt, Louis and Miles Kimball. (1992). "Background Risk, Prudence and the Demand for Insurance." In G. Dionne (ed.), *Contributions to Insurance Economics*. Boston: Kluwer Academic Press.

Ehrlich, Isaac and Gary Becker. (1972). "Market Insurance, Self-Insurance and Self-Protection." *Journal of Political Economy* 80, 623-648.

Gollier, Christian, and John W. Pratt. (1996). "Risk Vulnerability and the Tempering Effect of Background Risk." *Econometrica* 64, 1109-1123.

Hammond, John. S. (1974). "Simplifying the Choice Between Uncertain Prospects when Preference is Non-Linear." *Management Science* 20, 1047-1072.

Jewitt, Ian. (1987). "Risk Aversion and the Choice Between Risky Prospects: The Preservation of Comparative Statics Results." *The Review of Economic Studies* 54, 73-85.

Jones-Lee, Michael. (1974). "The Value of Changes in the Probability of Death or Injury." *Journal of Political Economy* 82, 835-849.

Julien, Bruno, Bernard Salanié, and François Salanié. (1999). "Should More Risk-Averse Agents Exert More Effort?" *Geneva Papers on Risk and Insurance Theory* 24, 19-28.

Kimball, Miles S. (1993). "Standard Risk Aversion." *Econometrica* 61, 589-611.

Lee, Kangoh. (1998). "Risk Aversion and Self-cum-Protection." *Journal of Risk and Uncertainty* 17, 139-150.

McGuire, Martin, John Pratt, and Richard Zeckhauser. (1991). "Paying to Improve Your Chances: Gambling or Insurance?" *Journal of Risk and Uncertainty* 4, 329-338.

Pratt, John W. and Richard Zeckhauser. (1987). "Proper Risk Aversion." *Econometrica* 55,143-154.

Pratt, John W. and Richard Zeckhauser. (1996). "Willingness to Pay and the Distribution of Risk and Wealth." *Journal of Political Economy* 104, 747-763.

Rothschild, Michael and Joseph Stiglitz. (1970). "Increasing Risk: A Definition." *Journal of Economic Theory* 2, 225-243.

Viscusi, W. Kip and Joseph E. Aldy. (2003). "The Value of a Statistical Life: A Critical Review of Market Estimates Throughout the World." *Journal of Risk and Uncertainty* 27, 5-76.

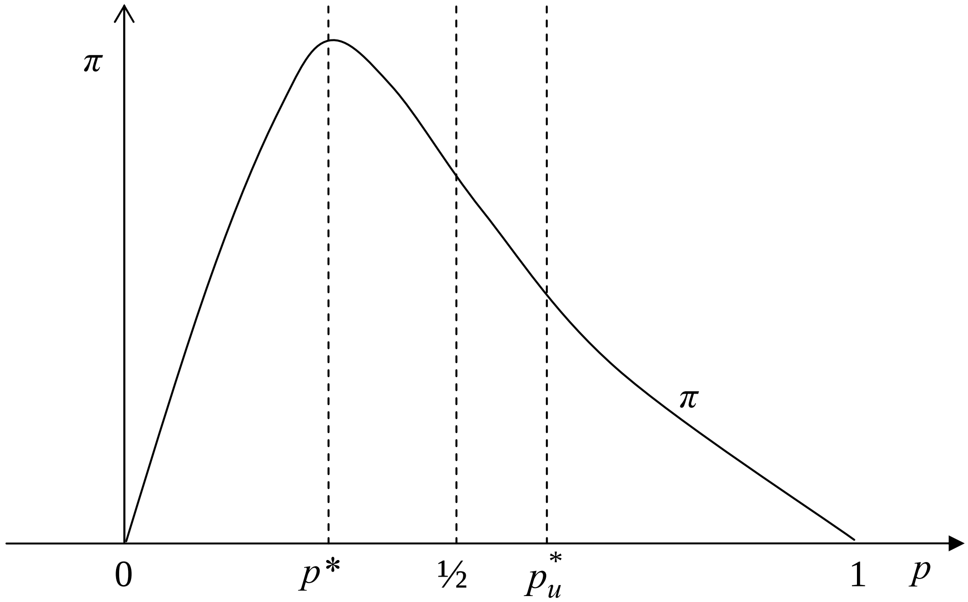

***Figure 1*: Risk Premium**

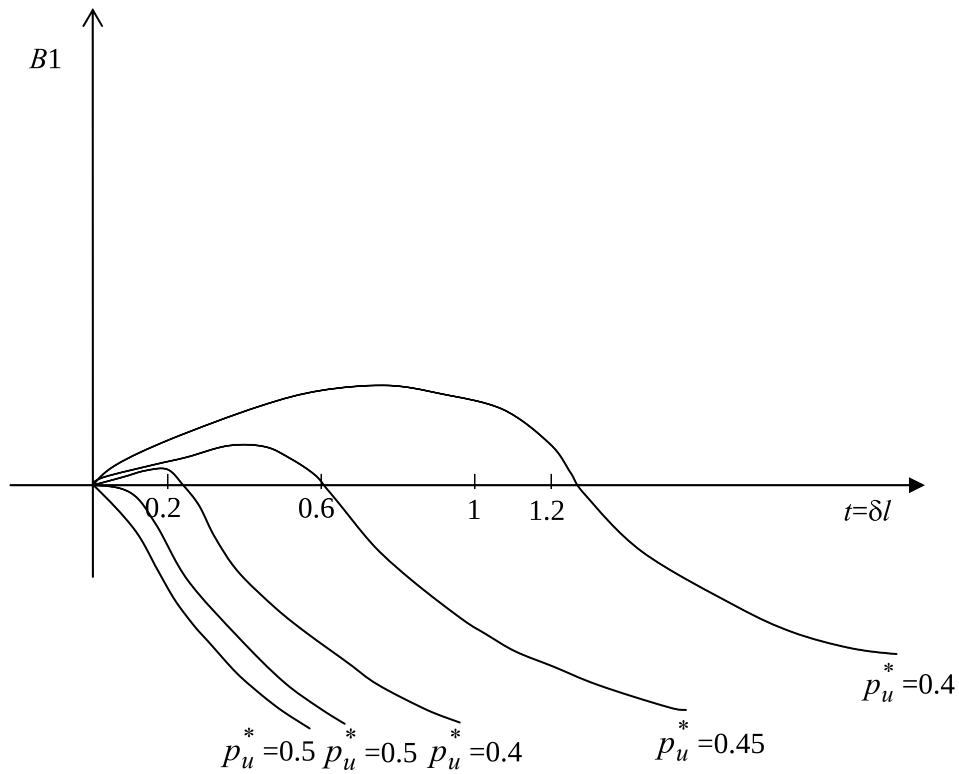

Figure 2*: First Order Condition for Different δ and *l