Autoregressive Conditional Duration (ACD) Models in Finance: A Survey of the Theoretical and Empirical Literature[†]

Maria Pacurar[*]

School of Business Administration, Dalhousie University

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Abstract

This paper provides an up-to-date survey of the main theoretical developments in ACD modeling and empirical studies using financial data. First, we discuss the properties of the standard ACD specification and its extensions, existing diagnostic tests, and joint models for the arrival times of events and some market characteristics. Then, we present the empirical applications of ACD models to different types of events, and identify possible directions for future research.

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[*]School of Business Administration, Dalhousie University, K.C. Rowe Management Building, 6100 University Ave., Halifax B3H 3J5, Canada. Phone: +1-(902) 494-3928. Fax: +1 (902) 494-1107. E-mail: maria.pacurar@dal.ca.

1 Introduction

Until two decades ago, most empirical studies in finance employed, as the finest frequency, daily data obtained by retaining either the first or the last observation of the day for the variable of interest (i.e., the closing price), thus neglecting all intraday events. However, due to the increased automatization of financial markets and the rapid developments in raising computer power, more and more exchanges have set up intraday databases that record every single transaction together with its characteristics (such as price, volume etc.). The availability of these low-cost intraday datasets fueled the development of a new area of financial research: high-frequency finance. Embracing finance, econometrics, and time series statistics, the analysis of high-frequency data (HFD) rapidly appeared as a promising avenue for research by facilitating a deeper understanding of market activity.[1] Interestingly, these developments have not been limited to academia, but have also affected the current trading environment. Over the last several years the speed of trading has been constantly increasing. Day-trading, once the exclusive territory of floor-traders, is now available to all investors. "High frequency finance hedge funds" have emerged as a new and successful category of hedge funds.

The intrinsic limit of high-frequency data is represented by the transaction or tick-by-tick data in which events are recorded one by one as they arise.[2] Consequently, the distinctive feature of this data is that observations are irregularly time-spaced. This feature challenges researchers as standard econometric techniques, as refined over the years, are no longer directly applicable.[3] Moreover, recent models from the market microstructure literature argue that time may convey information and should, therefore, be modeled as well. Motivated by these considerations, Engle and Russell (1998) developed the Autoregressive Conditional Duration (ACD) model whose explicit objective is the modeling

[1] Econometrics and finance journals (see, for instance, *Journal of Empirical Finance*, 1997; *Journal of Business and Economic Statistics*, 2000; *Empirical Economics*, 2006) have devoted special issues to the examination of high-frequency data; international conferences have focused on this field as well.

[2] Engle (2000) denotes them as "ultra-high frequency data".

[3] Other problems are associated with the use of transaction data, such as the bid-ask bounce, the discreteness of prices (see Gwilym and Sutcliffe, 2001, for a review).

of times between events. Since its introduction, the ACD model and its various extensions have become a leading tool in modeling the behavior of irregularly time-spaced financial data, opening the door to both theoretical and empirical developments. As illustrated by Engle and Russell (1998), Engle (2000), and Engle (2002), the ACD model shares many features with the GARCH model. A decade after its introduction, will it have the same success the GARCH model had in theoretical and empirical studies?

The objective of this paper is to review both the theoretical and empirical work that has been done on ACD models since their introduction a decade ago. ACD models have been partly covered in books such as Bauwens and Giot (2001), Engle and Russell (2002), Tsay (2002), and Hausch (2004), but none of them provides an exhaustive and up-to-date review of the published work on this subject. To our knowledge, this is the first survey-article on ACD models and it aims to offer a good understanding of the scope of current theoretical and empirical research, including recent findings.

The remainder of the paper is organized as follows. Section 2 is devoted to the theoretical developments on ACD models. We discuss the properties of the standard ACD specification and several of its extensions, existing diagnostic tests, and joint models for the arrival times of events and some market characteristics. Section 3 describes the applications of ACD methodology to different types of financial data, depending on the event of interest. Section 4 concludes the article.

2 The ACD model: theoretical developments

As we have already discussed, the main characteristic of HFD is the fact that they are irregularly time-spaced. Therefore, they are statistically viewed as point processes. A point process is "a special kind of stochastic process, one which generates a random collection of points on the time axis" (Bauwens and Giot, 2001, p.67)[4]. A high-frequency financial dataset contains a collection of financial events such as trades, quotes, etc. and,

[4] A popular point process is the Poisson process. See Hautsch (2004) for a useful review of the different possibilities to classify point processes models.

consequently, the times of these events represent the arrival times of the point process. When different characteristics are associated with an event (such as, for example, the price and the volume associated with a trade), they are called marks, and the double sequence of arrival times and marks is called a marked point process. Point processes are widely used in fields such as queueing theory and neuroscience but have attracted great interest in high-frequency finance over the last few years after Engle (2000) used them as a framework for the analysis of the trading process and of market behavior.

2.1 General setup

Let $\{t_0, t_1, ..., t_n, ...\}$ be a sequence of arrival times with $0 = t_0 \leq t_1 \leq ... \leq t_n \leq ...$. Thus, in this general setting simultaneous events are possible but, as discussed in Section 3, most of the papers analyzed exclude them. Let $N(t)$ be the number of events that have occurred by time $t \in [0, T]$ and $\{z_0, z_1, ...z_n, ..\}$ the sequence of marks associated with the arrival times $\{t_0, t_1, ..., t_n, ...\}$. Then, $t_{N(T)} = T$ is the last observed point of the sequence and $0 = t_0 \leq t_1 \leq ... \leq t_{N(T)} = T$ corresponds to the observed point process.

One common way of studying financial point processes (i.e., point processes that consist of arrival times of events linked to the trading process) is by modeling the process of durations between consecutive points.[5] Let $x_i = t_i - t_{i-1}$ denote the i^{th} duration between two events that occur at times t_{i-1} and t_i. The sequence $\left\{x_1, x_2, ..., x_{N(T)}\right\}$ has non-negative elements and this impacts the choice of the appropriate econometric models for durations.

In a very general setup and following Engle's framework (2000), we shall refer to the joint sequence of durations and marks:

$$\left\{(x_i, z_i), i = 1, ..., T\right\}.$$

[5] Other ways consist of modeling the intensity process or the counting process, but they are beyond the scope of our paper. For a thorough review of recent intensity models for financial point processes, we refer the interested reader to the book by Hautsch (2004). Winkelmann (1997) and Cameron and Trivedi (1998) provide comprehensive surveys of statistical and econometric techniques for the analysis of count data.

Denote the information set available at time t_{i-1} by $\mathcal{F}_{i-1}$. It includes past durations up to and including x_{i-1} but, as discussed in Section 3 it may also contain some pre-determined variables suggested by the microstructure literature. The i^{th} observation has joint density, conditional on $\mathcal{F}_{i-1}$, given by

$$(x_i, z_i)\,|\mathcal{F}_{i-1} \sim f\left(x_i, z_i|\breve{x}_{i-1}, \breve{z}_{i-1}; \theta_f\right), \tag{1}$$

where $\breve{x}_{i-1}$ and $\breve{z}_{i-1}$ denote the past of the variables X and Z, respectively, up to the $(i-1)^{th}$ transaction and $\theta_f \in \Theta$ is the set of parameters.

The joint density in (1) can be written as the product of the marginal density of the durations and the conditional density of the marks given the durations, all conditioned upon the past of durations and marks:

$$f\left(x_i, z_i|\breve{x}_{i-1}, \breve{z}_{i-1}; \theta_f\right) = g(x_i|\breve{x}_{i-1}, \breve{z}_{i-1}; \theta_x)q(z_i|x_i, \breve{x}_{i-1}, \breve{z}_{i-1}; \theta_z), \tag{2}$$

where $g(x_i|\breve{x}_{i-1}, \breve{z}_{i-1}; \theta_x)$ is the marginal density of the duration x_i with parameter θ_x, conditional on past durations and marks, and $q(z_i|x_i, \breve{x}_{i-1}, \breve{z}_{i-1}; \theta_z)$ is the conditional density of the mark z_i with parameter θ_z, and conditional on past durations and marks as well as the contemporaneous duration x_i. The log-likelihood is given by

$$\mathcal{L}(\theta_x, \theta_z) = \sum_{i=1}^{n} [\log g(x_i|\widetilde{x}_{i-1}, \widetilde{z}_{i-1}; \theta_x) + \log q(z_i|x_i, \widetilde{x}_{i-1}, \widetilde{z}_{i-1}; \theta_z)] \tag{3}$$

If the durations are considered weakly exogenous, cf. Engle, Hendry, and Richard (1983), with respect to the processes for the marks, then the two parts of the likelihood function could be maximized separately which simplifies the estimation (see, for instance, Engle, 2000).[6]

[6] Whereas no tests for weak exogeneity in this context exist, Dolado, Rodriguez-Poo, and Veredas (2004) recently derived a LM test-statistic useful to apply before separately estimating each density of the joint process.

2.2 Models for the durations

The ACD model introduced by Engle and Russell (1998) can be conceived as a marginal model of durations x_i . Let the conditional expected duration be

$$\psi_i \equiv E(x_i|\mathcal{F}_{i-1}) = \psi_i(\breve{x}_{i-1}, \breve{z}_{i-1}) \tag{4}$$

The main assumption of the ACD model is that the standardized durations

$$\varepsilon_i = \frac{x_i}{\psi_i} \tag{5}$$

are independent and identically distributed, that is iid with $E(\varepsilon_i) = 1$.[7]

This implies that $g(x_i|\breve{x}_{i-1}, \breve{z}_{i-1}; \theta_x) = g(x_i|\psi_i; \theta_x)$. Thus, all the temporal dependence of the duration process is captured by the conditional expected duration.

Let $p(\varepsilon, \theta_\varepsilon)$ be the density function for ε with parameters θ_ε. The multiplicative error structure of the model, together with the non-negativity of the duration sequence, requires that $p(\varepsilon, \theta_\varepsilon)$ has a non-negative support.[8] Then, $g(x_i|\mathcal{F}_{i-1}; \theta) = \psi_i^{-1} p(x_i/\psi_i; \theta_\varepsilon)$, where $\theta = (\theta_x, \theta_\varepsilon)$ is the vector of all the unknown parameters. The log-likelihood function is given by

$$L(\theta) = \sum_{i=1}^{N(T)} \log g(x_i|\mathcal{F}_{i-1}; \theta) = \sum_{i=1}^{N(T)} \left[\log p(\frac{x_i}{\psi_i}; \theta_\varepsilon) - \log \psi_i \right]. \tag{6}$$

Once a parametric distribution of ε has been specified, maximum likelihood estimates of θ can be obtained by using different numerical optimization algorithms.

The setup in equations (4)-(5) is very general and allows for a variety of models obtained by choosing different specifications for the expected duration, ψ_i and different distributions for ε. In the following subsections, we shall review the main types of ACD models that have been suggested in the financial econometrics literature as more flexible alternatives

[7] This assumption is without loss of generality. If $E(\varepsilon_i) \neq 1$, one can define $\varepsilon_i' = \varepsilon_i / E(\varepsilon_i)$ and $\psi_i' = \psi_i E(\varepsilon_i)$ so that $E(\varepsilon_i') = 1$ and $x_i = \psi_i' \varepsilon_i'$.

[8] Another way of dealing with the non-negativity of durations is to specify a (ARMA-type) model for log durations. However, this could pose some problems if some durations are exactly zero.

to the standard form originally proposed by Engle and Russell (1998).

2.2.1 The standard ACD model

The basic ACD model as proposed by Engle and Russell (1998) relies on a linear parameterization of (4) in which ψ_i depends on m past durations and q past expected durations:

$$\psi_i = \omega + \sum_{j=1}^{m} \alpha_j x_{i-j} + \sum_{j=1}^{q} \beta_j \psi_{i-j}. \tag{7}$$

This is referred to as the ACD(m, q) model. To ensure positive conditional durations for all possible realizations, sufficient but not necessary conditions are that $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$.

As it becomes apparent, the ACD model and the GARCH model of Bollerslev (1986) share several common features, the ACD model being commonly viewed as the counterpart of the GARCH model for duration data. Both models rely on a similar economic motivation following from the clustering of news and financial events in the markets. The autoregressive form of (7) allows for capturing the duration clustering observed in high-frequency data, i.e., small (large) durations being followed by other small (large) durations in a way similar to the GARCH model accounting for the volatility clustering. Just as a GARCH(1,1) is often found to suffice for removing the dependence in squared returns, a low order ACD model is often successful in removing the temporal dependence in durations.

The statistical properties of the ACD(1,1) model are well investigated: Engle and Russell (1998) derive its first two moments while Bauwens and Giot (2000) compute its autocorrelation function.

A very useful feature of the ACD model is that it can be formulated as an ARMA$(\max(m, q), q)$ model for durations x_i. Letting $\eta_i \equiv x_i - \psi_i$, which is a martingale difference by construction and rearranging terms, (7) becomes

$$x_i = \omega + \sum_{j=1}^{\max(m,q)} \left(\alpha_j + \beta_j\right) x_{i-j} - \sum_{j=1}^{q} \beta_j \eta_{i-j} + \eta_i. \tag{8}$$

It also follows from this ARMA representation that to ensure a well-defined process for durations, all the coefficients in the infinite-order AR representation implied by inverting the MA component must be non-negative; see Nelson and Cao (1992) for derivation of identical conditions to ensure non-negativity of GARCH models. From (8), in order for x_i to be covariance-stationary, sufficient conditions are that

$$\sum_{j=1}^{m} \alpha_j + \sum_{j=1}^{q} \beta_j < 1. \tag{9}$$

The stationarity and invertibility conditions require that the roots of $[1 - \alpha(L) - \beta(L)]$ and $[1 - \beta(L)]$, respectively, lie outside the unit circle where $\alpha(L)$ and $\beta(L)$ are the polynomials in terms of the lag operator L. Equation (8) can be used for computing forecasts of durations.

The conditional mean of x_i is by definition (4) equal to ψ_i. The unconditional mean of x_i is given by

$$E(x_i) = \frac{\omega}{\left(1 - \sum_{j=1}^{m} \alpha_j - \sum_{j=1}^{q} \beta_j\right)}. \tag{10}$$

The conditional variance of x_i based on (5) is

$$Var(x_i|\mathcal{F}_{i-1}) = \psi_i^2 Var(\varepsilon_i). \tag{11}$$

Thus, the model allows both for conditional overdispersion (when $Var(\varepsilon_i) > 1$) and underdispersion (when $Var(\varepsilon_i) < 1$).[9] Carrasco and Chen (2002) establish sufficient conditions to ensure β-mixing and finite higher-order moments for the ACD(m, q) model. Fernandes (2004) derives lower and upper bounds for the probability density function of stationary ACD(m, q) models.

[9]Dispersion is defined as the ratio of standard deviation to the mean.

2.2.2 Distributional assumptions and estimation

Any distribution defined on a positive support can be specified for $p(\varepsilon, \theta_\varepsilon)$; see Lancaster (1997) for several alternatives. A natural choice very convenient for estimation is the exponential distribution. Engle and Russell (1998) use the standard exponential distribution (that is, the shape parameter is equal to one) which leads to the so-called EACD model. A main advantage of this distribution is that it provides quasi-maximum likelihood (QML) estimators for the ACD parameters (Engle and Russell, 1998; Engle, 2002). Drost and Werker (2004) show that consistent estimates are obtained when the QML estimation is based on the standard gamma family (hence including the exponential).[10] The quasi-likelihood function takes the form:

$$L(\theta) = -\sum_{i=1}^{N(T)} \left[\frac{x_i}{\psi_i} + \log \psi_i \right]. \tag{12}$$

Following the similarity between the ACD and the GARCH model, the results of Lee and Hansen (1994) and Lumsdaine (1996) on the QMLE properties for the GARCH(1,1) model are formalized for the EACD(1,1) model by Engle and Russell (1998, Corollary, p.1135). Under the conditions of their theorem, consistent and asymptotically normal estimates of θ are obtained by maximizing the quasi-likelihood function given in (12), even if the distribution of ε, $p(\varepsilon, \theta_\varepsilon)$, is not exponential.[11] The standard errors need to be adjusted as in Bollerslev and Wooldridge (1992). The corollary also establishes that QML estimates of the ACD parameters can be obtained using standard GARCH software, in particular by considering the dependent variable equal to $\sqrt{x_i}$ and imposing a conditional mean equal to zero. However, a crucial assumption for obtaining QML consistent estimates of the ACD model is that the conditional expectation of durations, ψ_i is correctly specified. We shall discuss this moment restriction later when several types of specification tests for

[10] Gouriéroux et al. (1984) originally proved that if the conditional mean is correctly specified, even if the density is misspecified, consistent QML estimates can be obtained if and only if the assumed density belongs to the linear exponential family.

[11] The results are also valid for ACD models with unit roots, e.g., integrated models.

the ACD model are considered. Note also that the corollary is derived for the linear EACD(1,1) model and cannot necessarily be directly extended to more general ACD(m, q) models.

The QML estimation yields consistent estimates and the inference procedures in this case are straightforward to implement, but this comes at the cost of efficiency. In practice, fully efficient ML estimates might be preferred.[12] On the other hand, the choice of the distribution of the error term in (5) impacts the conditional intensity or hazard function of the ACD model.[13] The exponential specification implies a flat conditional hazard function which is quite restrictive and easily rejected in empirical financial applications (see Engle and Russell, 1998; Dufour and Engle, 2000a; Feng, Jiang, and Song, 2004; Lin and Tamvakis, 2004, among others). For greater flexibility, Engle and Russell (1998) use the standardized Weibull distribution with shape parameter equal to γ and scale parameter equal to one, the resulting model being called WACD. The Weibull distribution reduces to the exponential distribution if γ equals 1, but it allows for a increasing (decreasing) hazard function if $\gamma > 1$ ($\gamma < 1$). However, Engle and Russell (1998) find evidence of excess dispersion for the Weibull specification. Motivated by a descriptive analysis of empirical volume and price durations, Grammig and Maurer (2000) question the assumption of monotonicity of the hazard function in Engle and Russell's standard ACD models. They advocate the use of a Burr distribution[14] that contains the exponential, Weibull and log-logistic as special cases. The model is then called the Burr-ACD model. It is noteworthy, however, that not all the moments necessarily exist for the Burr distribution unless some restrictions are imposed on the parameters. This, in turn, may sometimes result in poor modeling of the higher (unconditional) moments of durations (see Bauwens, Galli, and Giot, 2003), thus jeopardizing, for example, its use in moment-matching based simulations.[15]

[12] This is similar to the ARCH literature where the normal distribution is often rejected in favor of some leptokurtic distribution for returns.

[13] Point processes are frequently formulated in terms of the intensity function. The hazard function is an alternative formulation of the same concept used for cross-sectional data (see Lancaster, 1997 for more information). In the ACD literature the two expressions are used interchangeably.

[14] The Burr distribution can be derived as a Gamma mixture of Weibull distributions; see Lancaster (1997).

[15] We refer the reader to Hautsch (2004) for more details on the properties of different distributions used

Lunde (1999) proposes the use of the generalized gamma distribution which leads to the GACD model. Both the Burr and the generalized gamma distributions allow for hump-shaped hazard functions (they both depend on two parameters) to describe situations where, for small durations, the hazard function is increasing and, for long durations, the hazard function is decreasing. The exact form of the distribution of ε has great importance in some applications of the ACD model, such as the ACD-GARCH class of models in which expected durations enter the conditional heteroskedasticity equation as explanatory variables. Monte Carlo evidence presented by Grammig and Maurer (2000) shows that imposing monotonic conditional hazard functions when the true data generating process requires non-monotonic hazard functions can have severe consequences for predicting expected durations because the estimators of the parameters of the autoregressive equation tend to be biased and inefficient. Hautsch (2002) specifies different ACD models based on the Generalized F distribution that includes as special cases the generalized gamma, Weibull and log-logistic distributions. Starting from a more general distribution allows one to test if the data support reductions to simpler distributions. It is common to estimate nonparametrically the unconditional distribution of durations x_i and use the shape of this distribution as an indication for choosing the density of ε.[16] More recently, De Luca and Gallo (2004) consider the use of a mixture of two distributions (in particular exponential) justified by the differences in information/behavior among different agents in the market.[17]

As an alternative to specifying a parametric distribution of ε, Engle (2000) and Engle and Russell (1998) use a semiparametric density estimation technique similar to the one used by Engle and Gonzalez-Rivera (1991) in the ARCH context. The conditional mean function of durations is parametrically specified and consistently estimated by QML, and then the baseline hazard is estimated nonparametrically using the standardized durations $\hat{\varepsilon}$ and a k-nearest neighbor estimator.

in the ACD framework.

[16] In practice, a Gamma kernel approach as proposed by Chen (2000), is frequently used to avoid the boundary bias of fixed kernels; see Grammig and Maurer (2000).

[17] The mixing parameter then has a financial interpretation as the proportion of informed (uninformed) agents but restrictively assumes that such a proportion is constant in a given time interval; see De Luca and Gallo (2004) for further discussion.

While crucial for the ACD model and its numerous extensions, the assumption of iid innovations ε_i in (5) may be too strong and inappropriate for describing the behavior of some financial durations. As discussed later, Zhang, Russell, and Tsay (2001) relax the independence hypothesis via a regime-switching model. Drost and Werker (2004) drop the iid assumption and consider that innovations ε_i may have dependencies of unknown conditional form, which brings in the question of efficiency in semiparametric estimation. Several semiparametric alternatives may be considered, such as Markov innovations and martingale innovations. The iid case of innovations with unknown density is also obtained under suitable restrictions. Using the concept of the efficient score function from the literature on semiparametric estimation,[18] the authors illustrate that even small dependencies in the innovations can trigger considerable efficiency gains in the efficient semiparametric procedures over the QML procedure.

The standard ACD model has been extended in several ways, directed mainly to improving the fitting of the stylized facts of financial durations. The strong similarity between the ACD and GARCH models nurtured the rapid expansion of alternative specifications of conditional durations. In the next subsection we review the most popular generalizations that have been proposed in the literature as well as some of the more recent and promising models.

2.2.3 Extensions of the standard ACD model

Persistence in durations. As we shall discuss in Section 3, empirical studies based on the linear model in (7) often reveal persistence in durations as the estimated coefficients on lagged variables add up nearly to one. Moreover, many financial duration series show a hyperbolic decay, i.e., significant autocorrelations up to long lags. This suggests that a better fit might be obtained by accounting for longer term dependence in durations. Indeed, the specification given by (8) shows that the standard ACD model imposes an exponential decay pattern on the autocorrelation function typical for stationary and

[18] See, for instance, Drost et al. (1997).

invertible ARMA processes. Or, this may be completely inappropriate in the presence of long memory processes. While the long-memory phenomenon has been extensively studied both in the theoretical and in the applied literature for time series and volatility[19], it has received considerably less attention in the literature on ACD models. Jasiak (1998) proposes the Fractionally Integrated ACD (FIACD) model, analogous to the FIGARCH model of Baillie, Bollerslev, and Mikkelsen (1996). The FIACD(m, d, q) model is defined as:

$$[1-\beta(L)]\,\psi_i = \omega + \left[1-\beta(L) - [1-\phi(L)]\,(1-L)^d\right] x_{i,} \tag{13}$$

where $\phi(L) = \alpha(L) + \beta(L)$ and the fractional differencing operator $(1-L)^d$ is given by $(1-L)^d = \sum_{k=0}^{\infty} \Gamma(k-d)\Gamma(k+1)^{-1}\Gamma(-d)^{-1}L^k$, Γ denoting the gamma function and $0 < d < 1$. When $d = 1$, the model is called Integrated ACD (IACD) by analogy with the IGARCH model of Engle and Bollerslev (1986). Building on the stationarity and ergodicity conditions derived by Bougerol and Picard (1992) for the IGARCH model, it can be shown that the FIACD model is strictly stationary and ergodic (see Jasiak, 1998 for further details).

As is well known from the literature on time series, long-memory may also occur because of the presence of structural breaks or regime-switching in the series. We shall later discuss some of the regime-switching models that have been proposed in the ACD literature.

Logarithmic-ACD models and asymmetric news impact curves. In the ACD(m, q) model (7) sufficient conditions are required for the parameters to ensure the positivity of durations. If one wants to add linearly in the autoregressive equation some variables taken from the microstructure literature and having expected negative coefficients, the durations might become negative. To avoid this situation, Bauwens and Giot (2000) introduce the more flexible logarithmic-ACD or Log-ACD(m, q) model in which the autoregressive equation is specified on the logarithm of the conditional duration ψ_i. The model is defined by (5) and one of the two proposed parameterizations of (4), referred

[19] See Baillie (1996) for a review, or Banerjee and Urga (2005) for a survey of more recent developments in the analysis of long-memory.

as Log-ACD$_1$ and Log-ACD$_2$, respectively[20]:

$$\ln\psi_i = \omega + \sum_{j=1}^{m}\alpha_j \ln x_{i-j} + \sum_{j=1}^{q}\beta_j \ln\psi_{i-j} = \omega + \sum_{j=1}^{m}\alpha_j \ln\varepsilon_{i-j} + \sum_{j=1}^{q}(\beta_j - \alpha_j)\ln\psi_{i-j}, \quad (14)$$

$$\ln\psi_i = \omega + \sum_{j=1}^{m}\alpha_j \varepsilon_{i-j} + \sum_{j=1}^{q}\beta_j \ln\psi_{i-j} = \omega + \sum_{j=1}^{m}\alpha_j (x_{i-j}/\psi_{i-j}) + \sum_{j=1}^{q}\beta_j \ln\psi_{i-j}. \quad (15)$$

Unlike the standard ACD(m, q) model, no non-negativity restrictions on the parameters of the autoregressive equation are needed to ensure the positivity of conditional durations. Covariance stationarity conditions are necessary ($|\alpha+\beta| < 1$ for the Log-ACD$_1$ model and $|\beta| < 1$ for the Log-ACD$_2$ model). In practice, the Log-ACD$_2$ model is often preferred as it seems to fit the data better. Bauwens, Galli, and Giot (2003) derive analytical expressions for the unconditional moments and the autocorrelation function of Log-ACD models. Unlike the ACD model whose autocorrelation function decreases geometrically at the rate $\alpha + \beta$ (see equation 8), the autocorrelation function of the Log-ACD model decreases at a rate less than β for small lags (see Bauwens, Galli, and Giot, 2003). The same probability distribution functions can be chosen for ε as in the ACD model, and the estimation can be analogously carried by maximum likelihood, considering the new definition of the conditional duration.

Tests for nonlinearity conducted by Engle and Russell (1998) on IBM trade duration data suggest that the standard ACD model given by (5) and (7) cannot fully capture nonlinear dependence between conditional duration and past information set. In particular, the authors report that conditional durations are overpredicted by the linear specification after very short or very long durations. This opened the door for new models looking for more flexible functional forms that allow for distinctive responses to small and large shocks. As can be easily seen in (14), the Log-ACD$_1$ model also allows for nonlinear

[20] Readers familiar with the GARCH literature will notice that the Log-ACD$_1$ model is analogous to the Log-GARCH model of Geweke (1986), while the Log-ACD$_2$ model resembles the EGARCH model proposed by Nelson (1991).

effects of short and long durations (i.e. when $\varepsilon_i < 1$ or $\varepsilon_i > 1$) without including any additional parameters. Dufour and Engle (2000a) argue that the Log-ACD model is likely to produce an over-adjustment of the conditional mean after very short durations because of the asymptotic convergence to minus infinity of log(0). Instead, they propose to model the news impact function with a piece-wise linear specification. The model is called the EXponential ACD or EXACD model due to its similarity to the EGARCH specification proposed by Nelson (1991), and it specifies the conditional duration as an asymmetric function of past durations:

$$\ln \psi_i = \omega + \sum_{j=1}^{m} \left[\alpha_j \varepsilon_{i-j} + \delta_j \left|\varepsilon_{i-j} - 1\right|\right] + \sum_{j=1}^{q} \beta_j \ln \psi_{i-j}. \tag{16}$$

Thus, the impact on the conditional duration is different, depending on the durations being shorter or longer than the conditional mean, i.e., $\varepsilon_i < 1$ involves a slope equal to $\alpha - \delta$ while $\varepsilon_i > 1$ determines a slope equal to $\alpha + \delta$.

Following the approach taken by Hentschel (1995) to developing a class of asymmetric GARCH models, Fernandes and Grammig (2006) introduce an interesting class of augmented ACD (AACD) models that encompass most of the specifications mentioned before as well as some models that have not been considered yet.[21] The AACD models are obtained by applying a Box-Cox transformation to the conditional duration process and a nonlinear function of ε_i that allows asymmetric responses to small and large shocks. The lowest-order parametrization is given by:

$$\psi_i^{\lambda} = \omega + \alpha \psi_{i-1}^{\lambda} \left[\left|\varepsilon_{i-1} - b\right| + c\left(\varepsilon_{i-1} - b\right)\right]^{v} + \beta \psi_{i-1}^{\lambda}. \tag{17}$$

In this specification, the asymmetric response to shocks and the shape of the piecewise function depend on parameters b and c, respectively, while parameter v induces concavity (convexity) of the shock impact curve for $v \leqslant 1$ ($v \geqslant 1$).

Building on the theoretical developments of Carrasco and Chen (2002), Fernandes

[21] Note, however, that they do not nest the TACD and STACD models presented later.

and Grammig (2006) derive sufficient conditions ensuring the existence of higher-order moments, strict stationarity, geometric ergodicity and β-mixing.[22] Under suitable restrictions several ACD models can be recovered: the standard ACD model, the Log-ACD models, the EXACD model, as well as other specifications inspired by the GARCH literature. Depending on the choice of the distribution of ε_i, the models can be estimated by maximum likelihood.

Regime-switching ACD models. Despite the evidence of nonlinearity reported by several studies (see Dufour and Engle, 2000a; Zhang, Russell, and Tsay, 2001; Taylor, 2004; Fernandes and Grammig, 2006; Meitz and Teräsvirta, 2006) the question of the type of nonlinear ACD model that would be the most appropriate needs further investigation. An alternative approach to dealing with the nonlinearity aspect-behavior evidenced by Engle and Russell (1998) involves considering the existence of different regimes with different dynamics corresponding to heavier or thinner trading periods. For instance, Zhang, Russell, and Tsay (2001) introduce a threshold ACD or TACD($m.q$) model in which the conditional duration depends nonlinearly on past information variables.[23] A K-regime TACD(m,q) model is given by

$$\begin{cases} x_i = \psi_i \varepsilon_i^{(k)} \\ \psi_i = \omega^{(k)} + \sum_{j=1}^{m} \alpha_j^{(k)} x_{i-j} + \sum_{j=1}^{q} \beta_j^{(k)} \psi_{i-j} \end{cases} \quad \text{if } x_{i-1} \in R_k, \tag{18}$$

where $R_k = [r_{k-1}, r_k]$, $k = 1, 2, ..., K$, with $K \in \mathbb{Z}^+$ being the number of regimes and $0 = r_0 < r_1 < ... < r_K = \infty$ are the threshold values. The regime-switching ACD parameters are denoted by $\omega^{(k)} > 0$, $\alpha_j^{(k)} \geqslant 0$ and $\beta_j^{(k)} \geqslant 0$. Moreover, for a fixed k, the error term $\varepsilon_i^{(k)}$ is an iid sequence with positive distribution that is regime-specific[24].

[22] Conditions for β-mixing and existence of moments for nonlinear ACD structures have been investigated by Meitz and Saikkonen (2004) for the first order case.

[23] The TACD model can be viewed as a generalization of the threshold-GARCH model of Rabemananjara and Zakoian (1993) and Zakoian (1994).

[24] The authors use the generalized gamma distribution. Note also that the choice of the threshold variable is not restricted to lag-1 duration.

Consequently, the different regimes of a TACD model have different duration persistence, conditional means and error distributions, which allows for greater flexibility compared to the ACD model. Conditions for geometric ergodicity and existence of moments are derived only for the TACD(1,1) model but they are difficult to generalize for higher order models. Moreover, the impact of the choice of the threshold variable still needs to be assessed. For a large number of regimes K, the estimation of the model may become computationally intensive[25] since it is performed by using a grid search across the threshold values r_1 and r_2 and by maximizing the likelihood function for each pair. An interesting finding of the Zhang, Russell, and Tsay (2001) study concerns the identification in the data analyzed of multiple structural breaks corresponding to some economic events. Following location of these break points, separate models are to be estimated for each subperiod.

The question of nonstationarity is addressed differently by Meitz and Teräsvirta (2006) who propose using ACD models with parameters changing smoothly over time. The logistic function is used as the transition function with the time as transition variable. Depending on the definition of time considered, time-varying ACD or TVACD models can be usefully applied either to test against parameter changes or to detect unsatisfactory removal of the intraday seasonality. The same authors also introduce an alternative to the TACD model. Building on the literature on smooth transition GARCH models,[26] they advocate the use of a smooth transition version of the ACD model, the STACD model, in which the transition between states is driven by a transition function. A logarithmic version may also be specified.

Latent factor-based models. Over recent years an area garnering substantial interest in the asset return literature has been that of stochastic volatility (SV) models. Modeling volatility as an unobserved latent variable has shown itself capable of capturing the dynamics of the financial series of returns in a better way than the competing

[25] Zhang, Russell, and Tsay (2001) estimate a 3-regime TACD(1,1) model while Bauwens, Giot, Grammig, and Veredas (2004) employ a logarithmic version of the same model.

[26] See, for example, Hagerud (1996), Gonzalez-Rivera (1998) or Lundbergh and Teräsvirta (2002).

GARCH models.[27] Given the similarity between ACD and GARCH models, it is not surprising that recent research on ACD models has taken this route. Bauwens and Veredas (2004) propose the stochastic conditional duration (SCD) model in which the conditional duration ψ_i is modeled as a latent variable (instead of being deterministic as in the ACD model). Economically, the latent variable may be thought of as capturing the unobservable information flow in the market. The SCD model as it was first proposed is given by

$$x_i = \psi_i \varepsilon_i, \tag{19}$$

$$\ln \psi_i = \omega + \beta \ln \psi_{i-1} + u_i, \qquad |\beta| < 1. \tag{20}$$

While equation (19) is identical to equation (5), the conditional duration in (20) is driven by a stationary first order autoregressive process AR(1).[28] Thus, the model has two sources of uncertainty, that is, ε_i for observed duration and u_i for conditional duration, which is expected to offer greater flexibility in describing the dynamics of the duration process. The following distributional assumptions are made:

$$\varepsilon_i \mid \mathcal{F}_{i-1} \sim iid\ p(\varepsilon_i) \quad \text{ and } \quad u_i \mid \mathcal{F}_{i-1} \sim iid\ \mathcal{N}(0, \sigma^2) \quad \text{with } u_i \text{ independent of } \varepsilon_i. \tag{21}$$

The same distribution may be chosen for ε_i as in the ACD framework and a different distribution for u_i can also be assumed. The original specification uses the Weibull and gamma distributions for ε_i but similar results are reported for both. Compared to the ACD model, the SCD model can generate a wider variety of hazard functions shapes (see Bauwens and Veredas, 2004 for further details). It follows from the specification in (19)-(21) that the SCD model is a mixture model. Therefore, its estimation is quite challenging

[27]See, for example, Danielsson (1994); Kim, Shephard, and Chib (1998); Ghysels, Harvey, and Renault (1996) on the advantages of using the SV framework relative to the GARCH framework.

[28]One may notice that no non-negativity constraints are imposed on the parameters to insure the positivity of durations. Also, the model mixes features of both the standard ACD model and the SV model introduced by Taylor (1982).

because the exact likelihood function cannot be derived in a closed form but involves a multidimensional integral whose evaluation requires extensive simulations, especially for large datasets (which is usually the case when working with high-frequency data).

Bauwens and Veredas (2004) propose use of the QML method with the Kalman filter after putting the model in a linear state space representation.[29] The procedure provides consistent and asymptotically normal estimators, but it is inefficient as it does not rely on the true likelihood of durations x_i. Strickland, Forbes, and Martin (2005) employ the Monte Carlo Markov Chain (MCMC) methodology which, like the QML is a complete method (i.e., permits estimation of both parameters and latent variables) but its implementation is rather time consuming.[30] An alternative method for the estimation of the SCD model is currently under investigation by Bauwens and Galli (2005) following the work of Liesenfeld and Richard (2003) on the application of the efficient importance sampling procedure to SV models. Ning (2004) investigates two other methods proposed in the SV literature: the empirical characteristic function and the GMM methods.

The leverage effect is well known in the volatility literature since Black (1976) first noted a negative correlation between current returns and future volatility. Feng, Jiang, and Song (2004) further develop the idea of asymmetric behavior of the expected duration in a SCD framework. Their model is characterized by the introduction of an intertemporal term ($ln\varepsilon_{i-1}$) in the latent function given in (20) to account for a leverage effect. The empirical study finds evidence of a positive relation between trade duration and conditional expected duration. The model is estimated using the Monte Carlo maximum-likelihood (MCML) method proposed by Durbin and Koopman (1997) in the general framework of non-gaussian state space models, and applied by Sandmann and Koopman (1998) to the estimation of SV models.

Given the rich potential of specifications and methods of estimation existing in the literature on SV models (see Broto and Ruiz, 2004 for a recent survey), we anticipate

[29] This procedure has been proposed independently by Nelson (1988) and Harvey, Ruiz, and Shephard (1994) for SV models.

[30] The SV literature shows the MCMC to have been more efficient than the QML and the generalized method of moments (GMM) techniques; see, for instance, Jacquier, Poison, and Rossi (1994).

further developments on SCD models in the near future. Formal comparisons of alternative estimation methods would be interesting.

As evidenced by (4) and (11), the ACD model does not allow for independent variation of the conditional mean and variance as higher order conditional moments are linked to specification of the conditional mean. Ghysels, Gouriéroux, and Jasiak (2004), referred to as GGJ hereafter, argue that this is a very restrictive assumption, especially when one is interested in analysis of market liquidity. Intertrade durations are an indicator of market liquidity because they measure the speed of the market but the variance of durations describes the risk on time associated to the liquidity risk. Therefore, GGJ introduce the Stochastic Volatility Duration (SVD) model in which the dynamics of the conditional mean and variance are untied by using two time varying factors instead of one. The SVD model extends the standard static exponential duration model with gamma heterogeneity from the literature on cross-sectional and panel data and given by $x_i = u_i/av_i$, where u_i and v_i are two independent variables with distributions standard exponential and gamma, respectively. This model can then be rewritten in terms of Gaussian factors as

$$x_i = \frac{H(1, F_{1i})}{aH(b, F_{2i})} \tag{22}$$

where a and b are positive parameters, F_{1i} and F_{2i} are iid standard normal variables, and $H(b, F) = G(b, \phi(F))$ where $G(b, .)$ is the quantile function of the gamma(b, b) distribution and $\phi()$ the c.d.f. of the standard normal distribution. GGJ propose to introduce dynamic patterns through the two underlying Gaussian factors by considering a VAR representation for the process $F_i = (F_{1i}, F_{2i})'$. The marginal distribution of F_i is constrained to be $N(0, I)-$ I being the identity matrix - to ensure that the marginal distribution of durations x_i belongs to the class of exponential distributions with gamma heterogeneity (i.e. Pareto distributions). Thus, the SVD model is given by (22) and

$$F_i = \sum_{j=1}^{p} \Lambda_j F_{i-j} + \varepsilon_i \tag{23}$$

where Λ_j is the matrix of autoregressive VAR parameters and ε_i is a Gaussian white noise with variance-covariance matrix $\sum(\Lambda)$ such that $Var(F_i) = I$.

While conceptually interesting, the use of the SVD model in applied research has been limited, due to its rather complicated estimation procedure. Indeed, the likelihood function is difficult to evaluate which is typical of the class of nonlinear dynamic factor models. GGJ suggest a two-step procedure in which parameters a and b are first estimated by QML, exploiting the fact that the marginal distribution of x_i is a Pareto distribution that depends only on the parameters a and b, and then the method of simulated moments[31] is used to get the autoregressive parameters Λ_j (see GGJ, 2004 for more details). It is noteworthy that the assumption of a Pareto distribution for the marginal distribution of x_i may be completely inappropriate for some duration processes, as evidenced by Bauwens, Giot, Grammig, and Veredas (2004) and this makes the first step of the estimation procedure unfeasible. Moreover, the same study finds a poorer predictive performance of the SVD model compared to ACD and Log-ACD models. More formal investigations of these alternative specifications for nonlinearity would be interesting.

2.2.4 Tests of the ACD model

A major issue when using ACD models is how to assess the adequacy of the estimated model. Even though, as discussed before, a variety of ACD specifications have been proposed in the literature, the question of evaluating a particular model has attracted far less interest. Most of the papers surveyed limit the testing to simple examinations of the standardized residuals. Recent papers, however, have proposed useful procedures for testing either the specification of the conditional mean function given in (4) or the specification of the distribution of the standardized durations $p(\varepsilon, \theta_\varepsilon)$ in (5). In the following, we briefly review these different approaches.

Basic residual examinations. The common way of evaluating ACD models consists of examining the dynamical and distributional properties of the estimated standardized

[31] See, for example, Gouriéroux and Monfort (1997).

residuals of an ACD model

$$\overset{\curlywedge}{\varepsilon}_i = \frac{x_i}{\overset{\curlywedge}{\psi}_i}, \quad i = 1, ..., T.$$

If the estimated model for the durations series is adequate, it follows that $\overset{\curlywedge}{\varepsilon}_i$ are iid. The approach used by Engle and Russell (1998) and largely adopted by subsequent authors consists of applying the Ljung-Box Q-statistic (see Ljung and Box, 1978) to the estimated residuals $\overset{\curlywedge}{\varepsilon}_i$ and to the squared estimated residuals $\overset{\curlywedge}{\varepsilon}_i^2$ to check for remaining serial dependence.[32] Practically all the papers surveyed employed this procedure. However, it is noteworthy that this approach is questionable. While the Ljung-Box test statistic is assumed to have an asymptotic χ^2 distribution under the null hypothesis of adequacy, no formal analysis exists that rigorously establishes this result in the context of ACD models. In fact, in a related context, Li and Mak (1994) show that this test statistic does not have the usual asymptotic χ^2 distribution under the null hypothesis when it is applied to standardized residuals of an estimated GARCH model. In the ACD framework, Li and Yu (2003) follow Li and Mak (1994) and propose a corrected statistic that results in a portmanteau test for the goodness-of-fit when ε_i follows the exponential distribution.[33] Like in the time series literature, additional examinations of residuals include visual check of the autocorrelation function of estimated residuals (see, for instance, Jasiak, 1998; Bauwens and Giot, 2000; Ghysels, Gouriéroux, and Jasiak, 2004; Bauwens, 2006). Furthermore, some papers (for example, Bauwens and Veredas, 2004; Ghysels, Gouriéroux, and Jasiak, 2004) compare the marginal density of durations derived from the model to the empirical marginal density directly obtained from the observed durations.

Beside testing for serial dependence in the estimated residuals, a few papers also test the moments conditions implied by the specified distribution of ε_i. According to the ACD model, the estimated residuals should have a mean of one. Engle and Russell (1998) propose a test for no excess dispersion of the estimated residuals when an exponential or

[32] When the Ljung-Box statistic is applied to the squares of the estimated residuals, it is known as the McLeod and Li (1983) test.

[33] According to Li and Yu (2003) the case of a Weibull distribution can be analyzed in the same way via the change of a variable.

Weibull distribution are assumed.[34] Bauwens and Veredas (2004) and De Luca and Gallo (2004) use QQ-plots to check the distribution assumptions while Prigent, Renault, and Scaillet (2001) employ Bartlett identity tests.

Testing the functional form of the conditional mean duration. As we have already mentioned, the validity of the conditional mean function is essential for the QML estimation of the ACD model. Sophisticated tests of no remaining ACD effects with rigorously proven asymptotic distributions have been recently developed in the literature. Following the work of Lundbergh and Teräsvirta (2002) on GARCH models who propose a Lagrange multiplier test of no residual ARCH,[35] Meitz and Teräsvirta (2006) propose a similar statistic of no residual ACD which is shown to be asymptotically equivalent to the Li and Yu (2003) test and also has a version robust to deviations from the assumed distribution. Alternatively, under the null hypothesis of adequacy of an ACD model, the fact that the estimated residuals $\hat{\varepsilon}_i$ are iid implies that their normalized spectral density equals the flat spectrum (that is, $1/2\pi$). Consequently, adequacy test statistics may be constructed by comparing an estimator of the spectral density of the estimated residuals and the flat spectrum. Building on Hong (1996, 1997), Duchesne and Pacurar (2005) make use of a kernel-based estimator of the normalized spectral density of the estimated residuals to construct such adequacy tests. Interestingly, a generalized version of the classic Box-Pierce/Ljung-Box test statistics is obtained as a special case (and therefore possesses a proven asymptotical distribution) but is shown to be less powerful. In a similar way, Duchesne and Hong (2001) use a wavelet-based estimator with a data-driven smoothing parameter.

The model presented in (7) assumes a linear dependence of the conditional duration on past information set. Engle and Russell (1998) propose a simple test for detecting potential nonlinear dependencies that is also applied by Zhang, Russell, and Tsay (2001) to motivate the introduction of their TACD model. The idea is to divide the durations

[34] In a Monte Carlo exercise, Fernandes and Grammig (2005) find poor performance of this overdispersion test compared to the nonparametric tests they propose.

[35] They also show that this test is asymptotically equivalent to the Li and Mak (1994) test.

into bins ranging from 0 to ∞ and then regress the estimated residuals on indicators of the size of the previous duration. Under the null hypothesis of the estimated residuals being iid, the coefficient of determination of this regression should be zero while under the alternative hypothesis, one can analyze the coefficients of indicators that are significant in order to identify sources of misspecification.

Meitz and Teräsvirta (2006) develop a useful and very general battery of tests of Lagrange multiplier type that allow extensive checks against different forms of misspecification of the functional form of the conditional mean duration: tests against higher-order ACD models, tests of linearity, and tests of parameters constancy. Finally, the omnibus procedure suggested by Hong and Lee (2003) for a large class of time series models and based on the generalized spectral density may also be used as a misspecification test of ACD models. As such, it is shown to be consistent against any type of pairwise serial dependence in the standardized durations.

Testing the distribution of the error term. Another possible source of misspecification for ACD models is the distribution of the error term. While the exponential distribution (or other member of the standard gamma family as shown by Drost and Werker, 2004) leads to consistent QML estimates, this procedure may be unsatisfactory in finite samples, as we have already mentioned. Only two of the theoretical papers surveyed paid attention to the development of elaborate tests against distributional misspecification. Fernandes and Grammig (2005) introduce two tests for distribution of the error term (the so-called D-test and H-test) based on comparison between parametric and nonparametric estimates of the density and of the hazard rate function of the estimated residuals, respectively. These tests inspect the whole distribution of the residuals, not only a limited number of moment restrictions, and are shown to be nuisance parameter free. It is noteworthy however, that the conditional mean function is assumed to be correctly specified and, therefore, a rejection could follow also from a misspecified conditional mean.

Another way to test ACD models consists of using the framework developed by Diebold, Gunther, and Tay (1998) for the evaluation of density forecasts. The main idea is that the sequence of probability integral transforms of the one-step-ahead density forecast has a distribution iid uniform $U(0,1)$ under the null hypothesis that the one-step-ahead prediction of the conditional density of durations is the correct density forecast for the data-generating process of durations. It follows that standard tests for uniformity and iid may then be used, but they do not generally indicate potential causes of the rejection of the null hypothesis. Therefore, the aforementioned authors recommend the use of graphical procedures such as examinations of histograms and autocorrelograms. This approach is adopted by Bauwens, Giot, Grammig, and Veredas (2004) for comparing the predictive abilities of the most popular ACD specifications. In the same context, Dufour and Engle (2000a) propose a new Lagrange Multiplier type of test for the uniformity and iid assumptions. Both approaches, however, assume the right conditional mean parameterization and, consequently, possible rejections may be due either to violation of distributional assumptions or violation of the conditional mean restriction.

Summing up, since existing misspecification tests of ACD models are directed toward detecting either distributional misspecification or functional misspecification, attention should be given in practice to the use of several complementary tests so that different sources of problems might be recognized. Unfortunately, none of these sophisticated procedures, to our knowledge has made its way to empirical studies so far.

2.3 Models for durations and marks

The models discussed above have been proposed for describing the dynamics of durations, x_i with respect to past information. In this respect, they are marginal models, following from the decomposition given by (2). But one may also be interested in the behavior of some marks (e.g., price, volume) given the durations and the past information, especially for testing some predictions from the market microstructure literature, as we shall discuss in the next section. In this case, the structure given by (2) provides a suitable framework for the joint modeling of durations between events of interest, x_i and some market characteristics, z_i. In most cases, only one mark is considered: the price.

The first joint model for durations and prices has been developed by Engle (2000).[36] He introduces an ACD-GARCH model based on the decomposition (2), the so-called Ultra-High-Frequency (UHF)-GARCH model. Durations between transactions are described by an ACD-type model conditional on the past while price changes are described by a GARCH model adapted to irregularly time-spaced data conditional on contemporaneous durations and the past. The adaptation consists of measuring the volatility per unit time. Under the assumption of weakly exogenous durations, the ACD model can be estimated first (cf. Engle, Hendry, and Richard; 1983), and then the contemporaneous duration and expected duration enter the GARCH model for volatility together with some other explanatory variables. As discussed later, the model can be used to investigate the relationship between current durations and volatility.

However, insights from the market microstructure literature suggest that it is possible that the volatility also impacts the duration process and ignoring this issue neglects part of the complex relationship between durations and volatility. Grammig and Wellner (2002) extend Engle (2000) approach by formulating a model for the interdependence of intraday volatility and duration between trades: the interdependent duration-volatility (IDV) model.

An alternative ACD-GARCH specification has been proposed by Ghysels and Jasiak (1998) based on the results of Drost and Nijman (1993) and Drost and Werker (1996) on the temporal aggregation of GARCH processes.[37] The model is formulated as a random coefficient GARCH where the parameters are driven by an ACD model for the durations between transactions. A GMM procedure is suggested for estimating the model but its application is rather computationally complex, which could explain the limited interest this model has received in empirical applications.

Whereas the models discussed above assume continuous distributions for price changes, other models have been put forward in the literature to specify the joint distribution of

[36] The working paper version dates from 1996.

[37] For a formal comparison of the UHF-GARCH and ACD-GARCH models, together with their advantages, we refer the interested reader to the work of Meddahi, Renault, and Werker (2006).

durations and price changes represented as a discrete variable. Bauwens and Giot (2003) and Russell and Engle (2005) propose competing risk models or transition models from the previous price change to the next one. The asymmetric Log-ACD model of Bauwens and Giot (2003) makes the durations dependent upon the direction of the previous price change, hence the asymmetric side. A Log-ACD model is used to describe the durations between two bid/ask quotes posted by a market maker but the model takes into account the direction of the mid bid/ask price change between the beginning and the end of a duration through a binary variable. Intraday transaction prices often take just a limited number of different values due to some institutional features with regard to price restrictions.[38] Therefore, Russell and Engle (2005) introduce the Autoregressive Conditional Multinomial (ACM) model to account for the discreteness of transaction prices. Building on (2) they propose using an ACD model for durations and a dynamic multinomial model for distribution of price changes conditional on past information and the contemporaneous duration. However, depending on the number of state changes considered, a large number of parameters might be needed, which complicates the estimation. A two-state only ACM model has been subsequently applied by Prigent, Renault, and Scaillet (2001) to option pricing.

Other approaches for studying causality relationships between durations and different marks have been developed based on the vector autoregressive (VAR) system used by Hasbrouck (1991). Hasbrouck (1991) analyzes the price impact of a trade on future prices but without assuming any influence of the timing of transactions on the distribution of marks, that is, $q(z_i|x_i, \breve{x}_{i-1}, \breve{z}_{i-1}; \theta_z) = q(z_i|\breve{z}_{i-1}; \theta_z)$ in (2). A simple bivariate model for changes in quotes and trade dynamics (e.g., trade sign) can be specified in transaction time. Dufour and Engle (2000b) extend Hasbrouck's model to allow durations to have an impact on price changes. An ACD model describes the dynamics of durations and the duration between transactions is then treated as a predetermined variable by allowing coefficients in both price changes and trade equations to be time-varying, depending on

[38] It has been also reported (see, for instance, Tsay, 2002; Bertram, 2004; Dionne, Duchesne, and Pacurar, 2005) that a large percentage of intraday transactions have no price change.

the duration. The model can easily be extended to include other variables, such as volume and volatility (Spierdijk, 2004; Manganelli, 2005).

An important assumption typically made in these joint models of durations and marks is that durations have some form of exogeneity (weak or strong, cf. Engle, Hendry, and Richard, 1983) which greatly simplifies the estimation and forecast procedures, respectively (see, for instance, Engle, 2000; Ghysels and Jasiak, 1998; Dufour and Engle, 2000b; Manganelli, 2005). The empirical consequences of removing the exogeneity assumption in VAR-type models are still open to question.[39]

3 Applications of ACD models in finance

As already mentioned in Section 2, ACD models can be used for modeling of arrival times of a variety of financial events. Most applications focus on the analysis of the trading process based on trade and price durations (as first initiated by Engle and Russell, 1997, 1998, and defined below) but other economic events, such as firm defaults or interventions of the Central Bank have also been considered. However, the importance of ACD models in finance so far stems from the relatively recent market microstructure literature that provides a strong economic motivation for use of these models besides the statistical justification already discussed (i.e., data being irregularly time-spaced). In the following, we present a review of the empirical applications of ACD methodology sorted according to the type of the event of interest. But first, we shall discuss an essential feature of all types of intraday durations.

3.1 Intraday seasonality

It is a well known fact that within a trading day financial markets are characterized by a strong seasonality. Initial intraday studies reporting seasonality used data resampled at regular time intervals (e.g., hourly, every half hour, every 5 minutes, etc.) and

[39] Moreover, Dufour and Engle (2000b, p. 2493) also state that "we do not have knowledge, to this date, of any theoretical model that shapes the reciprocal interactions of price, trade, and time...".

focused mainly on the behavior of the intraday volatility (see, for instance, Bollerslev and Domowitz, 1993; Andersen and Bollerslev, 1997, 1998; Beltratti and Morana, 1999). In the context of irregularly time-spaced intraday data, Engle and Russell (1998) report higher trading activity (hence, shorter durations on average) at the beginning and close of the trading day, and slower trading activity (longer durations) in the middle of the day. These patterns are linked to exchange features and traders' habits. Traders are very active at the opening as they engage in transactions to benefit from the overnight news. Similarly, at the closing, some traders want to close their positions before the end of the session. Lunchtime is naturally associated with less trading activity. Ignoring these intraday patterns would distort any estimation results. The procedure used most often in this literature for taking into account intraday seasonal effects is that originally employed by Engle and Russell (1998). It consists of decomposing intraday durations into a deterministic part based on the time of the day the duration arises, and a stochastic part modeling the durations' dynamics:

$$x_i = \widetilde{x}_i s\left(t_{i-1}\right), \tag{24}$$

where $\widetilde{x}_i$ denotes the "diurnally adjusted" durations and $s\left(t_i\right)$ denotes the seasonal factor at t_i. Equation (4) for the conditional duration becomes

$$\psi_i = E(\widetilde{x}_i|\mathcal{F}_{i-1}) s\left(t_{i-1}\right). \tag{25}$$

The two set of parameters of the conditional mean and of the seasonal factor, respectively, can be jointly estimated by maximum likelihood as in Engle and Russell (1998). However, due to numerical problems arising when trying to achieve convergence, it is more common to apply a two-step procedure in which the raw durations are first diurnally adjusted and then the ACD models are estimated on the deseasonalized durations, $\widetilde{x}_i$. Engle and Russell (1998) argue that both procedures give similar results due to the large size of intraday datasets.

With regard to the specification of the seasonal factor $s(t)$, two similar approaches are dominant in the empirical studies. The first is the one originated by Engle and Russell (1997, 1998) in which durations x_i are regressed on the time of the day using a piecewise linear spline or cubic spline specification and then diurnally adjusted durations $\widetilde{x}_i$ are obtained by taking the ratios of durations to fitted values. Second, Bauwens and Giot (2000) define the seasonal factor as the expectation of duration conditioned on time-of-day. This expectation is computed by averaging the raw durations over thirty-minute intervals for each day of the week (thus, the intraday seasonal factor is different for each day of the week). Cubic splines are then used to smooth the time-of-day function that displays the well-known inverted-U shape.

Some alternative procedures have also been applied in the literature. Tsay (2002) uses quadratic functions and indicator variables. Drost and Werker (2004) only include an indicator variable for lunchtime in the conditional mean duration because, for their data, trading intensity except for lunchtime seems almost constant. Dufour and Engle (2000a) include diurnal dummy variables in their vector autoregressive system but fail to identify any daily pattern, except for the first 30 minutes of the trading day that appear to have significantly different dynamics from the rest of the data.[40] Veredas, Rodriguez-Poo, and Espasa (2001) propose a joint estimation of the two components of durations, dynamics and seasonality with the dynamics specified by (Log)-ACD models and the seasonality left unspecified and therefore estimated nonparametrically.

Despite the need for some caution already expressed in the literature (see, for example, Bauwens et al., 2004; Meitz and Teräsvirta, 2006, among others), the impact of the deseasonalisation procedure on the results has been insufficiently investigated and should receive further consideration. It is noteworthy that eliminating intraday seasonality does not affect the main properties of the durations discussed below.

[40]Note also that it is a current practice in studies on irregularly time-spaced data to remove the transactions during the first minutes of the trading session (the opening trades).

3.2 Applications to trade durations

The most common type of event considered for defining financial durations is a trade. Thus, trade durations are simply the time intervals between consecutive transactions. Applications of ACD models to trade durations have been reported in numerous papers (see, among others, Engle and Russell, 1998; Jasiak, 1998; Engle, 2000; Zhang, Russell, and Tsay, 2001; Bauwens and Veredas, 2004; Manganelli, 2005; Bauwens, 2006). Generally, some authors were interested in the model specification necessary to fit the dynamics of different data while others focused on the testing of various market microstructure predictions.

3.2.1 Stylized facts of trade durations and model specification

Several stylized properties of trade durations that motivate the use of the ACD methodology have been identified in the empirical literature. First, all the papers report the phenomenon of clustering of trade durations, i.e., long (short) durations tend to be followed by long (short) durations that may be due to new information arising in clusters. This is also evidenced by the highly significant (positive) autocorrelations that generally start at a low value (around 0.10). The autocorrelation functions then decay slowly, indicating that persistence is an important issue when analyzing trade durations. Ljung-Box Q-statistics are often used to formally test the null hypothesis that the first (15 or 20) autocorrelations are 0. The statistics take very large values clearly indicating the presence of ACD effects, that is, duration clustering, at any reasonable level.[41] A slowly decaying autocorrelation function may be associated with a long-memory process, which motivated Jasiak (1998) to introduce the FIACD model. Evidence for long memory has been constantly reported for IBM trade durations, for instance (see, for example, Engle and Russell, 1998; Jasiak, 1998; Bauwens et al. 2004).

Second, the majority of papers report that trade durations are overdispersed, i.e., the

[41] An alternative test for ACD effects has been proposed by Duchesne and Pacurar (2005) using a frequency domain approach.

standard deviation is greater than the mean.[42] This can be tested formally using the dispersion test introduced by Engle and Russell (1998) or a Wald test for the equality of the first two sample moments, as in Dufour and Engle (2000a). Overdispersion is also apparent from the examination of the kernel densities of different trade durations that have a hump at very small durations and a long right tail (see, for instance, Bauwens et al., 2004; Bauwens and Giot, 2001; Engle and Russell, 1998).[43] This suggests that exponential distribution is not appropriate for unconditional distribution of trade durations (which does not mean however that conditional durations cannot be exponentially distributed).

Third, papers using transaction data reveal the existence of a large number of zero trade durations in the samples used (i.e., about two thirds of the observations for the IBM dataset originally used by Engle and Russell, 1998). Since the smallest time increment is a second, orders executed within a single second have the same time stamp. Following the original work of Engle and Russell (1998), the most common approach for dealing with zero durations consists of aggregating these simultaneous transactions. An average price weighted by volume is generally computed (when the variable price is also of interest), and all other transactions with the same time stamp are discarded. This procedure uses the microstructure argument that simultaneous observations correspond to split-transactions, that is, large orders broken into smaller orders to facilitate faster execution. When transactions do not have identical time stamps, identification of split-transactions may be more challenging. Grammig and Wellner (2002) consider as part of a large trade on the bid (ask) side of the order book only those trades with durations between sub-transactions of less than one second and whose prices are non-increasing (non-decreasing). Multiple transactions may, however, be informative as they reflect a rapid pace of the market. Zhang, Russell, and Tsay (2001) find that the exact number of multiple transactions

[42] Bauwens (2006) finds underdispersion for two out of four stocks considered from the Tokyo Stock Exchange. He eventually explains it by poor measurement of the very small durations of these stocks. Ghysels and Jasiak (1998) also report underdispersion for IBM trade durations computed with one month of data.

[43] One could argue that the hump close to the origin is an artifact of estimating the density of a positive variable using the kernel method. Therefore, most authors use the gamma kernel proposed by Chen (2000) and designed for this context.

does not carry information about future transaction rates, but the occurrence of multiple transactions does. Therefore, they incorporate this information into their TACD model through a lagged indicator variable for multiple transactions included as regressor in the conditional mean of durations. Bauwens (2006) artificially sets the duration between simultaneous trades at one second but we should mention the peculiar feature of his dataset from the Tokyo Stock Exchange in which two orders executed within two seconds have the same time stamp. An alternative explanation for zero durations is put forward by Veredas, Rodriguez-Poo, and Espasa (2001) based on the empirical observation of zero durations being clustered around round prices. Therefore, they argue that multiple transactions may occur because of many traders posting limit orders to be executed at round prices. Simply removing the simultaneous transactions certainly affects the dynamical properties of the trade durations. For instance, Veredas, Rodriguez-Poo, and Espasa (2001) and Bauwens (2006) report higher Q-statistics and residual autocorrelation, respectively, when zero durations are removed from the data. The parameters of the distribution of the innovation may also be altered. We believe that the literature still needs to explore alternative ways for dealing with zero durations as their impact is not sufficiently known.

It is noteworthy that a common practice when analyzing intraday durations consists of eliminating all interday (overnight) durations because they would distort the results. However, some alternative approaches have also been proposed. Manganelli (2005) treats the overnight period as if it was non-existent while Dufour and Engle (2000a) account for interday variations by including a dummy variable for the first observation of the trading day.

With regard to the model specification, most empirical applications of ACD models to trade durations have adopted linear ACD or Log-ACD specifications with low orders of lags, i.e., (1,1) or (2,2)-models, despite the findings discussed in Section 2 pointing out the need for nonlinear models. Further empirical evidence provided by Fernandes and Grammig (2006) shows that the problem of overpredicting short durations first identified by Engle and Russell (1998) can be palliated by allowing for a concave shocks impact curve.

Interestingly, several authors (Engle and Russell, 1998; Engle, 2000; Zhang, Russell, and Tsay, 2001; Fernandes and Grammig, 2006 among others) reveal substantial difficulties in completely removing dependence in the residual series, which suggests that the question of the most appropriate model for trade durations is far from being answered.[44] Bauwens et al. (2004) also report that none of the thirteen models considered (including ACD, log-ACD, TACD, SCD, and SVD models with various distributions for the innovation) provides a suitable specification for the conditional duration distribution. On the other hand, Dufour and Engle (2000a) find that the choice of the conditional distribution of durations apparently does not affect the out-of-sample predictions of the ACD model at short or longer horizons (but it becomes crucial when forecasting the whole density).

Many studies found evidence of high persistence of trade durations, the sum of the autoregressive coefficients (i.e., $\alpha+\beta$) being close to one while still in the stationary region (see, among others, Engle and Russell, 1998; Jasiak, 1998; Engle, 2000; Dufour and Engle, 2000a; Bauwens and Veredas, 2004).

However, few formal comparisons of existing ACD models have been made to date. Generally, studies limit to comparisons of a new proposed specification for the ACD model to the original Engle and Russell (1998)'s model.

Further investigation is imperatively needed as to the choice of the best model for different markets.

3.2.2 Tests of market microstructure hypotheses

As we have seen, the setup presented in Section 2 is very general and encompasses a variety of models. The autoregressive structure of the conditional duration function allows description of the duration clustering observed in intraday data, but this clustering phenomenon is not uniquely an empirical fact. The market microstructure literature offers some explanations for the autocorrelation of the duration process. However, the relationship between the literature on market microstructure and ACD models is

[44] A similar result is reported by Taylor (2004) for a sample of durations between non-zero price impact trades on the FTSE 100 index futures market.

reciprocally beneficial, the former providing theoretical explanations, the latter offering empirical support. An exhaustive review of market microstructure models is well beyond the scope of our paper and we, therefore, refer the interested reader to the excellent surveys of O'Hara (1995) or Madhavan (2000). Here, we shall limit our discussion to those theoretical models that provide insights on the use of the ACD framework and whose predictions have been empirically tested using ACD models.

Market microstructure is concerned with the study of the trading process and, as stated by O'Hara (1995, p.1), its research is "valuable for illuminating the behavior of prices and markets." Traditionally, price formation has been explained in the context of inventory models that focused on uncertainties in the order flow and the market maker's inventory position.[45] However, over the last several years, another branch of microstructure models has become more popular: information-based models that bring into play elements from asymmetric-information and adverse selection theory.[46] These models recognize the existence of different degrees of information in the market. Typically, two categories of traders are considered: informed and uninformed traders. Informed traders are assumed to possess private information and trade to take advantage of their superior information. Non-informed traders or liquidity traders trade for exogenous reasons, such as liquidity needs. The market maker or the specialist loses when trading with informed traders and has to compensate for these losses when trading with the uninformed traders, which explains the bid-ask spread. The critical aspect in these models is that uninformed traders may learn by observing the actions of informed traders, or put differently, informed traders disclose information through their trades. Thus, prices reflect all publicly available information but private information is also progressively revealed by observing the actions of the informed traders. Information may be conveyed through various trade characteristics, such as timing, price, and volume. Several information-based models try to explain the complex relationships between these microstructure variables.

Among the key variables considered, the timing of trades plays an important role.

[45] See, for instance, Garman (1976), Stoll (1978), Ho and Stoll (1981), among others.
[46] The basic ingredients of information-based models are attributed to Bagehot (1971).

Whereas initial microstructure models ignored the role of the time in the formation of prices, starting with the models of Diamond and Verrechia (1987) and Easley and O'Hara (1992), traders may learn from the timing of trades, hence the usefulness of ACD models for empirical investigations of trade durations.

Duration clustering is theoretically attributable to the presence of either informed traders or liquidity traders. According to the Easley and O'Hara (1992) model, informed traders only trade when new information enters the market while liquidity traders are assumed to trade with constant intensity. Thus, duration clustering occurs after information events because these increase the number of informed traders. Admati and Pfleiderer (1988) provide a different explanation of duration clustering. Their model distinguishes between two types of uninformed traders in addition to informed traders. Non-discretionary traders are similar to liquidity traders in the previous model whereas discretionary traders, while uninformed, can choose the timing of their trades. The authors show that the optimal behavior is a clumping behavior: discretionary traders select the same period for transacting in order to minimize the adverse selection costs and informed traders follow the pattern introduced by the discretionary traders.

In addition to providing theoretical explanations of the duration clustering phenomenon, market microstructure models make several predictions about the relationships between trade durations and other variables of the trading process.

Testable hypotheses linked to trade durations and empirical results From the papers we surveyed on trade durations, it appears that implications of the following main information-based models have been repeatedly tested using the ACD framework. These implications, sometimes contradictory, typically refer to the informational content of trade durations and the relationship between trading intensity and information-based trading.

Diamond and Verrechia (1987) use a rational expectation model with short-sale constraints. The main implication for empirical purposes is that when bad news enters the market, informed traders who do not own the stock cannot short-sale it because of the existing constraints. Hence, long durations are associated with bad news and should lead to declining prices.

In the Easley and O'Hara (1992) model, since informed traders trade only when there are information events (whether good or bad) that influence the asset price, short trade durations signify news arrival in the market and, hence, increased information-based trading. Consequently, the market maker needs to adjust his prices to reflect the increased uncertainty and risk of trading with informed traders, which translates into higher volatility and wider bid-ask spreads.[47]

Opposite relations between duration and volatility follow from the Admati-and-Pfleiderer model (1988) where frequent trading is associated with liquidity traders. Low trading means that liquidity (discretionary) traders are inactive, which leaves a higher proportion of informed traders on the market. This translates into higher adverse selection cost and higher volatility. Because of the lumping behavior of discretionary traders at equilibrium, we should also observe a clustering in trading volumes. Similarly, in the Foster and Viswanathan (1990) model the possibility of discretionary traders delaying trades creates patterns in trading behavior.

Empirical tests of these predictions have been reported by several authors. Engle (2000) applies several specifications of the UHF-GARCH model to IBM data. He finds a statistically significant negative relation between durations (expected durations) and volatility as expected from Easley and O'Hara (1992). Interestingly, the coefficient of the current duration in the mean equation is negative and, hence, consistent with the Diamond and Verrechia (1987) model, i.e., long durations will lead to declining prices. When a dummy variable for large lagged spreads is included in the variance equation it has a positive sign: as predicted by Easley and O'Hara (1992), wider spreads predict rising volatility.

Whereas in the Engle model (2000), volatility does not impact trading intensity, Grammig and Wellner (2002) study the interaction of volatility and trading intensity by

[47] Glosten and Milgrom (1985) first noted that when the probability of informed trading increases, the spreads become wider but time is not considered in their analysis.

specifically modeling the intraday interdependence between them. They apply the IDV model to secondary market trading after the Deutsche Telekom IPO in November 1996.[48] Their result is consistent with the Admati-and-Pfleiderer model (1988): lagged volatility which is conceived as an indicator of informed trading[49] has a significantly negative impact on transaction intensity.

When applying their TACD model to IBM data, Zhang, Russell, and Tsay (2001) find that the fast trading regime is characterized by wider spreads, larger volume, and higher volatility, all of which proxy for informed trading. Thus, the results are consistent with Easley and O'Hara (1992) model but also with Easley and O'Hara (1987) who suggest that the likelihood of informed trading is positively correlated with trading volume. Even if their analysis is limited to one stock, an interesting finding that deserves attention is the fact that fast and slow trading regimes have different dynamics, which may suggest that the results observed on frequently traded stocks are not necessarily valid for less frequently traded stocks.

To examine the relationship between trading intensity and intraday volatility, Feng, Jiang, and Song (2004) regress the realized volatility computed over 30-second time intervals against the forecast of trade duration based on the SCD models estimated with and without leverage (see Section 2). The models are applied to data for IBM, Boeing, and Coca Cola stocks. The results of the regression are consistent with the model of Easley and O'Hara (1992) as a significantly negative relation between trade durations and volatility is found for all three stocks.

Russell and Engle (2005) estimate a five-state ACM(3,3)-ACD(2,2) model on data for the Airgas stock traded on NYSE. They find that long durations are associated with falling prices, which is consistent with the predictions of Diamond and Verrechia (1987), and that the volatility per unit time is highest for short durations, as predicted by Easley and O'Hara (1992).

[48] Their interest in this event is motivated by the assertion made in the corporate finance literature that there is a large asymmetry of information in the market in the case of an IPO.

[49] See French and Roll (1986).

Another appealing approach for testing market microstructure hypotheses consists of using a VAR model that integrates several of the economic variables of interest. Dufour and Engle (2000b) analyze the price impact of trades in a large sample of 18 NYSE frequently traded stocks, using a bivariate 5-lags VAR-model for returns and trade sign in which the coefficients vary with the trading frequency as measured by trade durations. Their results show that shorter trade durations induce stronger positive autocorrelations of signed trades and larger quote revisions. For example, when a buy order is executed right after a previous order, its price impact is higher than that of a buy order arriving after a long time interval and also it becomes more likely to be followed by another buy order. When these results are associated with those of Hasbrouck (1991), it follows that short durations are associated with large spreads, large volumes, and high price impact of trades, consistent with Easley and O'Hara (1992) predictions.

More recently, other extensions of the Dufour and Engle VAR approach (2000b) have been proposed in the literature to include other variables such as the trade size and volatility. Spierdijk (2004) studies five frequently traded stocks from NYSE and reports similar results to Dufour and Engle (2000b). Moreover, she finds that large trades increase the speed of trading, while large returns decrease the trading intensity. Volatility is found to be higher when durations are short. Some of these results are also reported by Manganelli (2005) for a sample of 10 stocks from the NYSE. In particular, high volume is associated with increased volatility as predicted by Easley and O'Hara (1987): for example, larger trade sizes are more likely to be executed by informed traders and, hence, have a greater price impact. Also, durations have a negative impact on volatility that is consistent with Easley and O'Hara model (1992). An important element of Manganelli's study is the separate analysis of two groups of stocks classified according to their trading intensity. This is the first empirical study providing concrete evidence of a significant difference in dynamics between frequently traded and infrequently traded stocks. The usual relationships between duration, volume, and volatility are not empirically confirmed for his sample of infrequently traded stocks. It is also noteworthy that the empirically observed clustering in trading volumes (which is motivated theoretically by authors such as Foster and Viswanathan, 1990) is modeled by Manganelli (2005) by an Autoregressive Conditional Volume model similar to the ACD model.

Interestingly, all the papers mentioned above studied the equity market. Recently, Holder, Qi, and Sinha (2004) investigated the price formation process for the futures market. An approach similar to that of Dufour and Engle (2000b) is applied to transaction data for Treasury Note futures contracts traded at the Chicago Board of Trade. The analysis also includes the number of floor traders and the trading volume as explanatory variables. The major findings illustrate notable differences from the equity market. In particular, trade durations are significantly positively related to subsequent returns and the sign of trades.

Summing up, some remarks are noteworthy here. First, while all studies agree that trade durations have an informational content, the empirical results regarding the relationship between different trade variables are partially contradictory, in a way similar to theoretical microstructure models. Most of the papers seem to suggest that high trading periods are associated with high volumes and high volatilities and are due to the increased presence of informed traders in the market.

Second, the majority of papers reviewed use transaction data for very liquid blue chip stocks while a few studies suggest that the information dissemination for less frequently traded stocks might be quite different.

Finally, most of the results presented are obtained for the NYSE, a market that combines features of a price-driven market (presence of a market maker) with those of an order-driven market (existence of an order book). However, the learning process might be different in a pure order-driven market where no official market maker exists. Examinations of more markets/stocks would be valuable for a better understanding of the differences across market structures.

3.3 Applications to price durations

Instead of focusing the analysis on the arrival times of all transactions, examination of the arrival of particular events, such as a certain change in the price or the time necessary for trading a given amount of shares, may be needed. In the point process literature, retaining only the arrival times that are thought to carry some special information is called thinning the point process. Mostly used examples include price durations and volume durations.

First introduced in Engle and Russell (1997), price durations represent the times necessary for the price of a security to change by a given amount, C. The first point of the new thinned process usually corresponds to the first point of the original point process, $\tau_0 = t_0$. Then, if p_i is the price associated with the transaction time t_i, the series of price durations is generated by retaining all points i from the initial point process, $i > 1$, such that $|p_i - p_{i'}| \geq C$ where $i' < i$ is the index of the most recently selected point. To avoid the problem of the bid-ask bounce, Engle and Russell (1997, 1998) recommend defining price durations on the midprice of the bid-ask quote process.

The importance of price durations comes from their close relationship with the instantaneous volatility of price. As formally shown by Engle and Russell (1998), instantaneous intraday volatility is linked to the conditional hazard of price durations:

$$\sigma^2 \left(t|\mathcal{F}_{i-1}\right) = \left(\frac{C}{P(t)}\right)^2 h\left(x_i|\mathcal{F}_{i-1}\right) \tag{26}$$

where $\sigma^2 \left(t|\mathcal{F}_{i-1}\right)$ is the conditional instantaneous volatility , $P(t)$ is the the midquote price, and $h\left(x_i|\mathcal{F}_{i-1}\right)$ is the conditional hazard of price durations defined for the threshold C. Consequently, if an EACD model is applied on price durations $h\left(x_i|\mathcal{F}_{i-1}\right) = 1/\psi_i$.

3.3.1 Stylized facts of price durations and model specification

Studies investigating the empirical properties of price durations found the same characteristics that motivated the use of ACD models for trade durations: positive autocorrelations, overdispersion, a right-skewed shape, and strong intraday seasonality (see

Engle and Russell, 1998; Bauwens and Giot, 2000; Bauwens and Veredas, 2004; Bauwens et al., 2004, Fernandes and Grammig, 2006, among others). As price durations are inversely related to volatility, seasonal patters of price durations may also be interpreted as intraday patterns of volatility.

Interestingly, price durations appear to be easier to model with regard to fully eliminating the serial dependence of residuals through the use of an ACD model (Bauwens et al., 2004; Fernandes and Grammig, 2006).

With regard to model specifications, a limited number of comprehensive comparisons exist and their results are rather contradictory. Bauwens et al., (2004) report that less complex ACD models such as the ACD and the Log-ACD outperform more complex models like the TACD, SVD and SCD, as long as they are based on flexible innovation distributions, such as the generalized gamma and the Burr distribution. The use of non-monotonic hazard functions for price durations is also advocated by Grammig and Maurer (2000). On the other hand, Fernandes and Grammig (2006) found evidence against standard specifications and recommend the logarithmic class of their AACD family for increased flexibility. Again, more systematic investigations would be interesting.

3.3.2 Tests of market microstructure hypotheses and volatility modeling

The link of price durations to the volatility process makes the use of ACD models very appealing for testing market microstructure predictions in a very simple way. It is sufficient to add additional explanatory variables linked to different market characteristics into equation (4) for the conditional expected duration, and this also allows for a better model specification. In this respect, Log-ACD models have become very popular for they avoid the non-negativity constraints on parameters. Among the most relevant variables for investigating market microstructure effects, the lags of the following are typically used in analyses of stock markets (Engle and Russell, 1998; Bauwens and Giot, 2000; Bauwens and Giot, 2003; Bauwens and Veredas, 2004):

- The trading intensity: It is defined as the number of transactions during a price duration, divided by the value of this duration. According to Easley and O'Hara (1992), an increase in the trading intensity following an information event should be followed by shorter price durations as the market maker revises his quotes more frequently. A negative coefficient is found in empirical applications consistent with these predictions.

- The spread: In the Easley and O'Hara (1992) model, a high spread is associated with short durations, which is confirmed in empirical results by the negative coefficient of the lagged spread.

- The average volume per trade: Following implications of information-based models, a similar negative relationship between traded volume and price durations is typically found as a higher volume is indicator of informed trading and, hence, higher volatility.

Engle and Lange (2001) explore the depth of financial markets by defining an intraday statistic, referred to as VNET, as the number of shares purchased minus the number of shares sold over a price duration. Regressing VNET on several market variables, it is found that market depth varies with volume, transactions, and volatility.

Investigations of various relationships between market microstructure variables using ACD models for price durations have been also reported for derivatives markets (see Taylor, 2004 and Eom and Hahn, 2005 for the futures and option markets, respectively.)

An interesting application of ACD models for price durations has been proposed by Prigent, Renault, and Scaillet (2001). Building on a traditional binomial option pricing model, they relax some of its rigid assumptions by means of dynamic specifications. First, instead of considering fixed time intervals between price variations (jumps) of constant size, they employ a Log-ACD model for specifying the arrival times. Second, they model the probabilities of up and down moves by an ACM model with two states (that is, an Autoregressive Conditional Binomial model).

Starting from equation (26), ACD models for price durations can also be used as an alternative to standard GARCH models. This idea has been further explored by Giot (2000), Gerhard and Hautsch (2002), Kalimipalli and Warga (2002), and Giot (2002). Giot (2000) uses the estimated coefficients of an Log-ACD model applied to IBM durations for computing the intraday volatility directly from (26). Interesting insights come from the analysis of Gerhard and Hautsch (2002) on the LIFFE Bund future market. Their model, however, is a non-dynamic proportional intensity model for categorized durations including censoring effects due to market closure, and it does not belong to the ACD family. Kalimipalli and Warga (2002) use ACD-based volatility estimates as an explanatory variable in an ordered probit model for investigating the relationship between spreads, volatility, and volume for the ten most actively traded bonds on the Automated Bond System market maintained by NYSE. A statistically significant positive relationship between volatility and spreads is identified but, interestingly, a negative relationship between volume and spreads is found. According to Harris and Raviv (1993), this may be due to a lack of consensus among traders, therefore placing limit orders on both sides of the bid-ask spread. It also suggests a weak adverse-selection component of the spreads. Moreover, similar results are obtained when using GARCH-based volatilities, which confirms the robustness of the results.

Volatility is also an essential ingredient of risk management. Therefore, it is not surprising that its estimation based on price durations has finally been considered for constructing intraday Value at Risk (VaR) models. Giot (2002) quantifies the market risk based on intraday returns in a conditional parametric VaR framework. Three ARCH-type models are applied to the equidistantly time-spaced observations re-sampled from the irregularly time-spaced data, and a Log-ACD model serves to compute the volatility based on price durations. However, the results from the Log-ACD model are not completely satisfactory, the price durations based model failing most of the time for all the stocks considered[50]. As the author notes, possible explanations of this result may be found in the assumption of normality of intraday returns and the need of complicated time transformations to switch to the regularly time-spaced. It is noteworthy, however, that investigations of the benefits of using tick-by-tick data for risk management have only

[50] The performance of each model, including the Log-ACD based model, is assessed in a regularly time-spaced framework by computing its failure rate as the number of times returns are greater than the forecasted VaR.

just started. An alternative approach has been recently proposed by Dionne, Duchesne, and Pacurar (2005) based on an ACD-GARCH model within a Monte Carlo simulation framework.

3.4 Applications to volume durations and other economic events

Another way of thinning a point process generates volume durations defined as the times until a given aggregated volume is traded on a market. Volume durations were introduced by Gouriéroux, Jasiak, and Le Fol (1999) as reasonable measures of liquidity that account simultaneously for the time and volume dimensions of the trading process. According to the conventional definition of liquidity (Demsetz, 1968; Black, 1971; Glosten and Harris, 1988), an asset is liquid if it can be traded as fast as possible, in large quantities, and with no significant impact on the price. Consequently, while neglecting the price impact of volumes, volume durations may still be interpreted as the (time) cost of liquidity.

There are much fewer studies applying ACD models to volume durations than to trade and price durations. Their main objective typically consists of finding the appropriate ACD specifications (Bauwens et al., 2004; Bauwens and Veredas, 2004; Veredas, Rodriguez-Poo, and Espasa, 2005; Fernandes and Grammig, 2006). These papers report very different statistical properties of volume durations compared to trade and price durations. Initial autocorrelations are larger and the density of volume durations appears as clearly hump-shaped. While trade and price durations are overdispersed, volume durations exhibit underdispersion, that is, the standard deviation is smaller than the mean.

With regard to model specification, Bauwens et al. (2004) find that ACD and Log-ACD models based on the Burr or generalized gamma distribution are useful for modeling not only price durations but volume durations as well. As expected, the EACD and SVD models perform badly on volume durations as the exponential and Pareto distribution, respectively, cannot describe the hump shape of the unconditional distribution of volume durations.

In recent years, the ACD model has also been applied to other irregularly time-spaced financial data that are not linked to the intraday trading process. For example, Fischer and Zurlinden (2004) examine the time spacings between interventions by central banks on foreign exchange markets. Using daily data on spot transactions of the Federal Reserve, the Bundesbank, and the Swiss National Bank on the dollar market, the authors conclude that traditional variables of a central bank's reaction function for interventions (for example, the volume and direction of the intervention) do not improve the ACD specification in their sample.

An interesting application of the ACD methodology has been suggested by Christoffersen and Pelletier (2004) for backtesting a VaR model. The main idea is very simple: if one defines the event where the ex-post portfolio loss exceeds the ex-ante predicted VaR as a violation, the clustering of violations can be described by an ACD model. Under the null hypothesis that the VaR model is correctly specified for a confidence level $1-p$, the conditional expected duration until the next violation should be a constant equal to $1/p$ days which can be tested in several ways.

An attempt to apply ACD models to credit risk analysis can be found in Focardi and Fabozzi (2005) where defaults in a credit portfolio follow a point process. By using an ACD specification for the arrival times of defaults in a portfolio, one may estimate the aggregate loss directly without the need for modeling individual probabilities of defaults while accounting for the credit-risk contagion phenomenon through the clustering of the defaults. While conceptually interesting, this approach still has to be validated empirically, the authors investigating it only through simulations.

Given the wealth of possible specifications of the ACD models, we expect further applications of the ACD approach to other irregularly time-spaced events in the near future.

4 Conclusion

In this paper we have reviewed the theoretical and empirical literature on ACD models. Since its introduction by Engle and Russell (1998), several articles applying this class of models have already appeared. The motivation behind the use of ACD models is twofold. On the one hand, transaction data, increasingly available at a low cost over the last years, are irregularly time-spaced so that new econometric techniques are needed to deal with this feature. On the other hand, recent market microstructure models based on asymmetric-information theory argue for the role of time in the dissemination of information among different participants in the trading process.

As our survey shows, much progress has been realized in understanding ACD models. Initial research has been oriented towards proper modeling of the data at hand and has focused naturally on palliating some of the inconveniencies of the original model. Extensions have been undertaken simultaneously on two fronts: developing more flexible specifications of the conditional mean and looking for more flexible distributions of the error term. Importantly, the expansion of ACD models has been fueled by the strong similarity between the ACD and GARCH models which impacted the search for new specifications and estimation methods. Compared with the GARCH literature, however the current ACD literature can be considered rather young and not as rich. We believe that ACD modeling could nevertheless benefit by freeing itself from the GARCH influence.

Given the increasing variety of existing specifications, one would like to know which model is the most appropriate under specific circumstances. Very few studies compare different ACD models using the same data, and recent specifications are barely included. Typically, when a new type of ACD model has been proposed, its performance has been compared with that of the original ACD form of Engle and Russell (1998). In our opinion, extensive comparisons of several models using various evaluation criteria (both in-sample and out-of-sample) on different datasets would greatly benefit the applied econometrician. It is noteworthy that only the basic ACD and Log-ACD models are used in most of the empirical studies surveyed. Despite the evidence of nonlinearity reported by several authors, it is still not clear which type of nonlinear ACD model should be recommended for specific conditions. This same discrepancy between the theoretical and applied literature has been observed with regard to existing testing procedures for the ACD models. Even though several tests have been developed, their relative performance remains insufficiently investigated.

ACD models have been used for describing not only the durations between different market events, but also as a building block for jointly modeling duration and other market characteristics, such as duration and price. If initial interest focused on marginal models for the arrival times of events, joint models have quickly captured attention for the examination of several market microstructure theories. Multivariate models (for trades and quotes, for instance) may improve understanding of the complex relationships between several trade variables, such as price, volume, and volatility. As such, they may have implications for market designers and policy makers. As many studies use transaction data from NYSE, examination of data from other markets could help to evaluate the differences between trading systems regarding the price discovery process.

A common assumption when working with joint models for durations and marks is that of the exogeneity of durations, often considered without any formal testing. We expect more work on this issue in the near future as warnings for caution have already been made.

Examination of existing empirical studies on ACD models revealed other problems that should be addressed in future research. Intraday deseasonalization techniques seem to have a significant but not very well understood effect on any model used. The treatment of zero-durations arising from observations with the same time stamp does not reflect a consensus among researchers, even if the dominant trend consists of simply eliminating them. A few studies have also pointed out substantial differences between the dynamics of actively traded stocks used mostly in empirical applications and those of infrequently traded stocks.

The class of ACD-GARCH models, as well as the link between price durations and instantaneous volatility, could also serve in developing risk measures based on transaction data and that are useful for agents very active on the market. While some work has already been initiated, we think that this issue will receive further consideration.

A promising line of research seems to be the application of ACD models to irregularly time-spaced data other than intraday data on the trading process. Finally, we think that the use of ACD models by applied researchers could be encouraged by the integration of some of the existing models and testing procedures in popular softwares.

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