Poll Subsidy and Excise Tax[*]

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Abstract

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JEL classification: H21, G22, D82

Keywords: Taxation, Insurance, Asymmetric Information, Ex Post Moral Hazard.

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Poll Subsidy and Excise Tax

ABSTRACT. Even if poll taxes are allegedly the most efficient form of taxation, governments are unwilling to use them because to do so corresponds to political suicide. Poll taxes are not always optimal, however, regardless of the political consequences. The goal of this paper is to present a case where an excise tax is preferred to a poll tax. We show that it is Pareto superior for the government to set the excise tax so high as to be able to give a lump-sum subsidy to all workers in the economy.

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1 Introduction

One reason why modern governments are unwilling to use poll taxes may be that to do so corresponds to political suicide. This is true even if poll taxes are generally perceived as the most efficient form of taxation. The goal of this paper is to present a simple case where a poll tax not only is not optimal, but where, in fact, a poll subsidy may be preferable. It is optimal to finance this poll subsidy through an excise tax on insurance benefits.

Although lump-sum taxes are efficient when there is perfect information, such may no longer be the case when information is not perfect. Eaton and Rosen (1980) and Peck (1998) demonstrate that it is possible to devise examples where lump-sum taxes are not optimal. The approach used by Eaton and Rosen (1980) is one where a worker must choose his labor supply without knowing what his future wage will be (this wage being uncorrelated from one agent to the next). They show that a tax on wages may yield a preferable outcome (smaller excess burden) than a poll tax. They explain this result by arguing that a wage tax provides some insurance in the sense that agents who have a lucky draw (higher wage) end up paying more in taxes. A poll tax, on the other hand, provides no such insurance.

Peck (1989) builds upon that result. He shows that, under certain circumstances, a tax on the profits of a corporation may yield a more preferable outcome than a lump-sum tax. His result stems from the use of an increasing return to scale technology at equilibrium. Still, it makes intuitive sense for corporations that face uncertain profits to prefer the use of a profit tax over a poll tax, since it reduces the risk of bankruptcy. We can see why that is if we look at a corporation that is making zero profits before taxes. With a profit tax, its total tax liability is zero, whereas with a poll tax, its tax liability is still on the books. Thus, a profit tax becomes a risk-sharing mechanism between the corporations and the government. This rationale for the use of a proportional tax is the same as that of Eaton and Rosen (1980).

The model we present examines the incidence of different types of taxes on the welfare of agents when markets are imperfect. The approach we shall take is one where a worker may suffer an injury that prevents him from working. This introduces some income uncertainty for the worker. Although the worker faces uncertainty regarding the future we allow an insurance market to exist to insure against that uncertainty. This differs from Eaton and Rosen (1980) and Peck (1989,1998), who do not allow an insurance market to exist.

The market imperfection we introduce is that the existence of the disability is not known for certain by the insurance company unless she conducts a costly audit of the disability insurance claim. This means that the worker may have the incentive to misreport the true state of the world to extract more money from the insurer. In other words, the worker possesses private information regarding the state of the world. The insurer can learn the state of the world if she conducts a costly audit. This approach, known as the costly state verification approach, was pioneered by Townsend (1979). Reinganum and Wilde (1985), Mookherjee and Png (1989), and Bond and Crocker (1997) also use this approach.

In such a setting, the traditional approach has been to say that the insurer can commit to an auditing strategy such that it is always in the worker's best interest to always tell the truth. Unfortunately, it may not be credible for the insurer to commit to such an auditing strategy, as argued by Graetz, Reinganum and Wilde (1986), Picard (1996), Khalil (1997) and Boyer (1998). Suppose the insurer announces an auditing strategy which guarantees that the worker has nothing to gain by reporting the wrong state of the world. If the worker believes in such an audit strategy, then he will always tell the truth. The insurer, upon hearing the worker's report, has no reason to audit since she knows that the worker has told the truth, and since audits are costly. By not auditing, the insurer saves the cost of auditing. We can thus see why it is not credible for the insurer to commit to such an auditing strategy. This means that the worker, anticipating the insurer's unwillingness to audit, will want to lie. The principal's impossibility to commit implies that the principal-agent problem between the insurer and the worker is not solved. Therefore, the optimal contract we derive is not incentive compatible.

Looking at what happens when the government budgetary needs are small (approaching zero), we demonstrate that when two types of taxes are allowed, workers are better off if the government taxes disability benefits in excess of its budgetary needs to offer a poll subsidy to all workers in the economy, or to subsidize insurance premiums. These results stem from the insurer's inability to commit to an auditing strategy because, in equilibrium, some workers claim for a disability that does not exist, and, more importantly, some workers are not caught claiming for a disability that does not exist. This means that some workers collect a benefit to which they are not entitled. With a benefits tax, proportionally more of the tax is borne by workers who lied to their insurance company. This is not the case with a poll or a premium tax, which are borne equally by every worker in the economy. Moreover, by taxing benefits, one may reduce insurance fraud in the economy. Fraud ir reduces even more if monies collected using the benefit tax are redistributed to workers in the form of premiums or poll subsidies.

The remainder of the paper is divided as follows. In the next section, we present the basic assumptions and the sequence of play between the government, the insurer and the worker. In section 3, we develop the model. We first present the game played by the worker and the insurer, given the taxes chosen by the government in the initial period. The optimal combination of taxes is presented in subsection 3.4. Finally, we conclude with a discussion of the results in section 4.

2 Basic Problem

2.1 Assumptions

We first present the assumptions of the model, and discuss some of the most important ones.

A.1. There are three kind of players in the economy: the workers, the insurers and the government. The workers have a vonNeumann-Morgenstern utility function over final wealth twice continuously differentiable, with $U'(.) > 0$ and $U''(.) < 0$. The insurers and the government are risk neutral. The number of workers (N) and of insurers (M) in the economy is large, with the number of workers being much larger than the number of insurers ($N >> M$).

A.2. There are only two possible states of the world: the worker is working or the worker is disabled. The worker is disabled with probability $\pi < \frac{1}{2}$. If disabled, the worker cannot work and suffers disutility D as a result of his disability.

A.3. The state of the world is an information known only to the worker. The insurer, however, can audit the worker at cost c to verify the state of the world.

A.4. The insurance market is competitive in the sense that the insurer makes zero expected profits. The disability insurance contract stipulates a premium (p) and a benefit (B) paid to the worker.

A.5. The action space for the worker is: report a disability (RD) and report no disability (ND). The action space for the insurer is: audit report (AR) and no audit (NR).

A.6. If caught reporting the wrong state of the world, the worker incurs penalty (disutility) $k < \infty$, which is a deadweight loss to society (i.e., neither the insurer nor the government collects it).

A.7. The worker has initial wealth Y, and may receive labor income W if he is working. This labor income cannot be observed by the insurer or by the government.

A.8. The insurer is not able to commit to an auditing strategy ex ante. This means that the principal-agent problem between the insurer and the worker is not solved.

A.9. The government needs to raise G dollars in the economy. This means that, on average, an amount $g = \frac{G}{N}$ (with N large) will be raised from each worker through taxes. Three types of taxes are permitted: a poll tax (T), an excise tax on the insurance premium (t_p), and an excise tax on the disability benefits received (t_b). The government is a perfect agent of the workers in the sense that it chooses the optimal tax scheme that maximizes the workers' expected utility. Since there is a large number of workers, the government knows with quasi-certainty that the amount G will be raised.

A.10. All taxes are collected after the worker has played, but before the insurer does.

The …rst six assumptions are typical of a costly state veri…cation problem as in Townsend (1979), Reinganum and Wilde (1985), and Mookherjee and Png (1989), although Reinganum and Wilde's agent is risk neutral. Using only two possible states greatly simpli…es the problem in two ways. First, it reduces the action space of the worker to only two actions: report a disability and do not report a disability. Second, it guarantees a unique equilibrium in mixed strategies for the game in which the worker and the insurer are involved.

Assumption A.6 needs some further explanation. The penalty in‡icted on workers when they are caught represents prison time. This penalty is paid by the worker, but is not collected by anyone in the economy. Therefore, it represents a deadweight loss to society. Assumption A.7 means that the labor income a worker earns is unobservable by any other player. Therefore, the insurer cannot decide to audit the worker's labor income instead of the worker's disability, and also that she cannot ask the government to audit the worker's labor income. This is important in the sense that, if the worker's labor income could be observed, the insurer would know for certain whether or not the worker is disabled, since a truly disabled worker cannot earn labor income. The impossibility to observe the worker's labor income also means that the government cannot tax labor income. This is why the taxation schemes are restricted to the three types of taxes presented in assumption A.9.

Assumption A.8 refers to the presumed impossibility for the insurer to commit to an auditing strategy. Khalil (1997) uses the case of the regulation of a monopolist with unknown cost to explain the logic behind the principal's inability to commit to an auditing strategy. He writes "since the optimal contract induces the agent to comply with the contract, from an ex post perspective the principal has no incentive to audit" (p.629). We know that it is possible to design a contract whereby it is optimal for the agent to always tell the truth. This revealing contract in a costly state veri…cation world relies on the assumption that the principal can commit to an auditing strategy ex ante. Ex post, however, there is no incentive for the principal to audit since she knows that the agent told the truth, as truth telling is always his best strategy. By not auditing, the principal saves the cost of auditing. Even if the contract stipulates an auditing strategy, it is in the ex post best interest of both players to renegotiate such a contract, say by splitting the saved cost of auditing between them.

Finally, A.9 and A.10 present the tax mechanism available to the government in this economy. The amount G may be viewed as the cost of administring the di¤erent taxes. The fact that government collects taxes before the insurer plays increase the penalty paid by workers who wrongfully …led a claim. This is true in the case of a bene…t tax. This tax-collection procedure does not penalize workers who truly suffered an accident, since they are indifferent as to when taxes are collected.

2.2 Timing

The sequence of play is shown in Figure 1. In the initial stage, the government sets the taxes to raise some amount of money $G \geq 0$ in the least painful way for the workers. This tax must be collected from the workers. The government can raise G using three types of taxes: a poll tax (T), an excise tax on the disability insurance premium (t_p), and an excise tax on the disability benefit received (t_b).

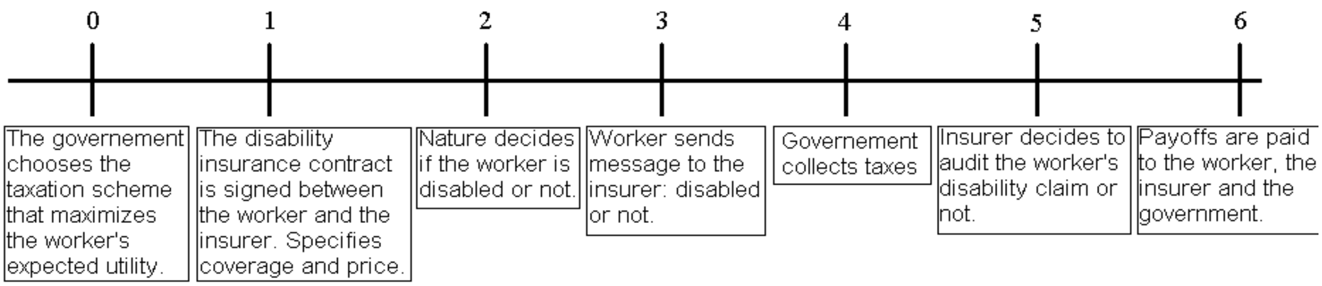

Figure 1: Sequence of play

In stage 1, the insurer offers a disability insurance contract to the worker. This contract specifies a coverage in case of a disability and a price that yields zero expected profits to the insurer. This contract does not specify an audit strategy, since such a strategy is not enforceable by the courts. The price of the insurance contract takes into account the worker's incentive to commit fraud with some probability.

In stages 2, 3, 5 and 6 the disability insurance claiming game between the insurer and the worker is played. This is a game of asymmetric information, where the worker knows the true state of the world (if he is disabled or not), and where the insurer does not unless she incurs an audit cost. Before the cost of auditing is incurred, the insurer does not know whether the message she received is truthful or not. This means that she cannot a priori differentiate a truthful message from an untruthful one. The insurer must assign some beliefs to each node in a given information set by updating her prior beliefs using Bayes' rule. In the last stage of the game, the payoffs to the players are paid.

The government, who collects taxes in stage 4, receives g per worker in expectation. The payoffs to the insurer and the worker depend on the actions taken by each player and Nature, and on the taxation system chosen by the government.

3 The Model

This game between the government, the insurer and the worker is solved using backward induction. We first derive the equilibrium to the disability insurance claiming game between the insurer and the worker.[1] Second, We derive the optimal disability insurance contract the insurer sells to the worker. Finally, the optimal taxation scheme shall be derived.

3.1 Claiming Game

The payoffs to the worker and the insurer are given in Table 1.

Table 1

Payoffs to the worker and the insurer contingent on their actions and the state of the world.

State of the world	Action of Worker	Action of Insurer	Payoff to Worker	Payoff to Insurer
No disability	Don't file	Audit	$U(Y - (1+t_p)p + W - T)$	$p - c$
No disability	Don't file	Don't audit	$U(Y - (1+t_p)p + W - T)$	p
No disability	File claim	Audit	$U(Y - (1+t_p)p + W - t_b B - T) - k$	$p - c$
No disability	File claim	Don't audit	$U(Y - (1+t_p)p + W + (1-t_b)B - T)$	$p - B$
Disability	File claim	Audit	$U(Y - (1+t_p)p + (1-t_b)B - T) - D$	$p - B - c$
Disability	File claim	Don't audit	$U(Y - (1+t_p)p + (1-t_b)B - T) - D$	$p - B$
Disability	Don't file	Audit	$U(Y - (1+t_p)p - T) - D$	$p - B - c$
Disability	Don't file	Don't audit	$U(Y - (1+t_p)p - T) - D$	p

The contingent states in italics never occur in equilibrium: they represent actions that are off the equilibrium path.

It is clear that the equilibrium of the game is Perfect Bayesian. A Perfect Bayesian Nash Equilibrium (PBNE) is such that no player has any incentive to deviate from his equilibrium strategy, and that the insurer has posterior beliefs in each of her information sets that were updated using Bayes' rule. The game's PBNE in mixed strategy is unique,[2] and is solved in the following proposition.

Proposition 1 Under A.2 ($\pi \le \frac{1}{2}$), the unique PBNE in mixed strategies without taxes[3] is:

1-The worker always files a claim if he is disabled;

[1] Since all insurers are the same and all workers are the same, we can limit our attention to one representative insurer and one representative worker.

[2] Gibbons (1992) and Myerson (1991) show that in a 2×2 game (two players with two possible actions each) there is at most one mixed strategy equilibrium.

[3] Subscript 0 refers to the case where there are no taxes.

2-The worker plays a mixed strategy between filing a claim (with probability η_i) and not filing if he is not disabled;

3-The insurer never audits a worker who does not file a claim;

4-The insurer plays a mixed strategy between auditing (with probability γ_i) and not auditing a worker who files a claim.

η_i and γ_i are given in equilibrium as

$$\eta_i = \left(\frac{c}{B_i - c}\right)\left(\frac{\pi}{1 - \pi}\right) \tag{1}$$

and

$$\gamma_i = \frac{U(Y - (1 + t_p)p_i + W + (1 - t_b)B_i - T) - U(Y - (1 + t_p)p_i + W - T)}{U(Y - (1 + t_p)p_i + W + (1 - t_b)B_i - T) - U(Y - (1 + t_p)p_i + W - T) + k} \tag{2}$$

where t_p is the premium tax rate, t_b is the benefit tax rate and T is the lump-sum tax, and where $i \in P(I)$ represents the different kind of taxes possible, $I = (T; p; b)$; i.e. i is the element of the power set of all possible combination of taxes $P(I)$.[4]

<ins>Proof</ins>: All the proofs are in the appendix.■

The intuition behind this equilibrium is straightforward. A worker who is truly disabled will never want to report that he is not disabled, and an insurer with whom no claim is filed has no reason to audit. The mixed strategy of a player is such that the other player is indifferent between his two possible actions. The worker sets his probability of filing a claim when he is not disabled such that the insurer is indifferent between auditing (with probability γ_i) and not auditing; while the insurer sets her probability of auditing when a disability is reported such that the worker who is not disabled is indifferent between reporting a disability (with probability η_i) and telling the truth. Note that for $\eta_i \in (0; 1)$, it has to be that $B_i > \frac{c}{(1-\pi)}$. We will show later that a sufficient condition for $B_i > \frac{c}{(1-\pi)}$ is that $\pi \leq \frac{1}{2}$.

We see that the shape of the equilibrium is independent of the taxation scheme used. The worker always files a claim if he is disabled, and the insurer never audits if no claim is filed. The only things that change are the equilibrium weights assigned to each action in each of the players' optimal strategy when a mixed strategy is used. This property of the equilibrium will simplify the analysis when the optimal contract between the worker and the insurer is considered.

[4] Recall that the number of elements of the power set of I is 2^n where n is the number of elements in set I. These elements are $\{T; b; p\}$, $\{T; p\}$, $\{T; b\}$, $\{b; p\}$, $\{p\}$, $\{T\}$, $\{b\}$, and $\emptyset$.

3.2 Insurance Contract

The problem for the insurer is to find a contract that maximizes the worker's expected utility given the taxation scheme chosen by the government in the initial period, given a zero-profit constraint for the insurer, and given that the worker and the insurer play the claiming game. The maximization problem for the insurer then is

$$\begin{aligned}\max_{p_T;B_T} EU_i = \; & \pi U(Y - (1+t_p)p_i + (1-t_b)B_i - T) - \pi D \\ & + (1-\pi)\eta_T(1-\nu_T)U(Y - (1+t_p)p_i + W + (1-t_b)B_i - T) \\ & + (1-\pi)\eta_T\nu_T U(Y - (1+t_p)p_i + W - T) - k \\ & + (1-\pi)(1-\eta_T)U(Y - (1+t_p)p_i + W - T)\end{aligned} \tag{3}$$

subject to four constraints

$$p_i = \pi B_i + (1-\pi)B_i\eta_i(1-\nu_i) + c\nu_i[\pi + (1-\pi)\eta_i] \tag{4}$$

$$\eta_i = \left(\frac{c}{B_i - c}\right)\left(\frac{\pi}{1-\pi}\right) \tag{5}$$

$$\nu_i = \frac{U(Y - (1+t_p)p_i + W + (1-t_b)B_i - T) - U(Y - (1+t_p)p_i + W - T)}{U(Y - (1+t_p)p_i + W + (1-t_b)B_i - T) - U(Y - (1+t_p)p_i + W - T) + k} \tag{6}$$

$$EU_i^* \geq \pi U(Y - T) - \pi D + (1-\pi)U(Y + W - T) \tag{7}$$

The first constraint is the zero expected profits constraint for the insurer. This constraint states that the premium the insurer collects must be equal to her expected payout. This expected payout includes the benefit paid, both to those who truly had a loss (πB_i) and to those who were not caught defrauding ($(1-\pi)B_i\eta_i(1-\nu_i)$), and the cost of the insurer's auditing strategy ($c\nu_i[\pi + (1-\pi)\eta_i]$). Constraints (6) and (5) represent the PBNE strategies of the players. When designing the contract, the principal must anticipate rationally what strategies will be played. The fourth constraint is the participation constraint. It states that the agent must be better off buying this contract then in autarchy.

This problem seems complicated to solve with two variables and four constraints. Fortunately, it is straightforward to simplify the maximization problem by substituting (6) and (5) and (4) into (3). We now have

$$\begin{aligned}\max_{B_i} EU_i = \; & \pi U\left(Y - (1+t_p)\pi\frac{B_i^2}{B_i - c} + (1-t_b)B_i - T\right) \\ & - \pi D + (1-\pi)U\left(Y - (1+t_p)\pi\frac{B_i^2}{B_i - c} + W - T\right)\end{aligned} \tag{8}$$

The participation constraint is redundant, since at $B_i = 0$, $p_i = 0$, which means that choosing $B_i = 0$ yields the participation constraint. The optimal disability insurance contract then solves

$$0 = -(1-\pi)\pi\frac{B_i(B_i-2c)}{(B_i-c)^2}(1+t_p)U'\left(Y-(1+t_p)\pi\frac{B_i^2}{B_i-c}+W-T\right) \tag{9}$$
$$+\pi\left[(1-t_b)-\frac{\pi B_i(B_i-2c)}{(B_i-c)^2}(1+t_p)\right]U'\left(Y-(1+t_p)\pi\frac{B_i^2}{B_i-c}+(1-t_b)B_i-T\right)$$

which we know yields a maximum since (8) is concave. This necessary first order condition may be rewritten as

$$\frac{U'\left(Y-(1+t_p)\pi\frac{B_i^2}{B_i-c}+(1-t_b)B_i-T\right)}{\Delta}=\frac{B_i(B_i-2c)}{(B_i-c)^2}\left(\frac{1+t_p}{1-t_b}\right) \tag{10}$$

where

$$\Delta=(1-\pi)U'\left(Y-(1+t_p)\pi\frac{B_i^2}{B_i-c}+W-T\right)+\pi U'\left(Y-(1+t_p)\pi\frac{B_i^2}{B_i-c}+(1-t_b)B_i-T\right) \tag{11}$$

It is clear here that $B_i > 2c$ is needed to achieve an optimum. To see why, note that the left hand side of (10) is always positive. We therefore need $B_i > 2c$ for the right side to be positive, since $\frac{1+t_p}{1-t_b} > 0$ and $B_i > 0$. Recall from proposition 1 that $\beta_i > \frac{c}{(1-\pi)}$ is a necessary condition for $\theta \in (0,1)$. We know from (10) that $\beta_i > 2c$. This means that $2c > \frac{c}{1-\pi}$ is a sufficient condition to have $\theta \in (0,1)$. Rearranging, we have that $\pi < \frac{1}{2}$ is a sufficient condition for θ to be a probability.

When the number of workers in the economy is large, the total tax raised from each worker using a poll tax, and excise tax on premium and an excise tax on benefits are, respectively, T, $t\pi\frac{B_i^2}{B_i-c}$ and $t_b\pi\frac{B_i^2}{B_i-c}$. The amount raised by the first two taxes is clear: each agent pays the poll tax T, and each agent pays $t_p p_p$ in taxes on the premium, since $p_p = \pi\frac{B_i^2}{B_i-c}$. The amount each workers pay in tax is known for certain for both the poll tax and the premium tax, and is the same for all workers. This is not the case with a benefit tax, since the tax paid is not the same for every worker. Workers pay tax rate t_b on all benefits received, if they receive a benefit. The amount workers can receive is B_i. They receive B_i with probability $\pi + (1-\pi)\theta$,[5] since taxes are paid before the decision to audit or not is made. The expected amount of taxes paid is thus $t_b B_i[\pi + (1-\pi)\theta]$. Substituting for the equilibrium value of θ yields the desired result.

3.3 Impact of Taxes

The probability that fraud is committed is given by θ_i. We see that, for any type of tax, θ_i decreases when B_i increases. To see why that is, consider what the insurer must pay if she does not audit. If

[5] With probability π a worker us disabled (in which case he receives the insurance benefit), and with probability $(1-\pi)\theta$ he is not disabled $(1-\pi)$, but he claims to be (θ).

the insurer has more to lose by paying a claim, then the worker will need to reduce his probability of committing fraud in order to keep the insurer indifferent between auditing and not auditing. The impact of the three taxes on fraud are stated in the following proposition.

Proposition 2 1-A premium tax always increases fraud;

2-A benefit tax reduces fraud if the coefficient of absolute risk aversion is high enough;

3-A poll tax increases fraud if the utility function displays decreasing absolute risk aversion.

In each case, the tax does not affect directly the probability of fraud. Rather, the taxes have an impact on benefits only. Since an increase in benefits decreases the probability of fraud (i.e., $\frac{\partial \theta}{\partial B}$), it follows that the tax's impact on fraud is the same as that on benefits.

It is interesting to observe, if the worker is sufficiently risk averse, that an increase in the benefit tax will induce him to choose a contract where the insurer pays a greater amount in benefits. Consider the two effects when taxes are levied on disability benefits. First, there is the value of insurance; second, there is the disutility of paying taxes. Ceteris paribus, a more risk-averse worker is willing to accept more disutility because he values more insurance, which is provided only through after-tax benefits. This means that, to receive a similar after-tax benefit, a more risk-averse worker needs to increase pre-tax benefits when taxes increase. He is willing therefore to pay more in taxes than a less risk-averse worker.

The consequence of a contract where the benefits paid by the insurer are greater is that the insurer has more to lose by not auditing. This means that the worker must alter his optimal reporting strategy to take into account this increased incentive for the insurer to audit. It is clear from looking at the worker's equilibrium reporting strategy that an increase in the benefit paid by the insurer reduces the worker's probability of filing a claim when he is not disabled. The benefit tax also has an impact on the payoff of workers, since they receive lower after-tax benefits than before. Therefore, workers have less to gain by filing a fraudulent claim, which means that they should be less willing to make them. The insurer is then able to reduce her equilibrium probability of auditing, which means that savings are made, since the amount of money devoted to auditing is reduced.

The impact of a premium tax is completely different from that of the benefit tax. Since the excise tax on the insurance premium is constructed similarly as a proportional loading factor on the premium, it is normal to expect the equilibrium benefit paid to be smaller. What this does is reduce the insurer's monetary incentive to audit, which means that the worker must increase his probability of fraud for the insurer to remain indifferent between auditing and not auditing.

Under the reasonable assumption that the worker's utility function does not display increasing absolute risk aversion, a poll tax will also increase fraud in the economy. The reason is that a poll tax uniformly reduces the wealth of workers, irrespective of their choice of insurance contract. Poorer workers are less willing to move away from the full insurance point if their utility function displays non-increasing absolute risk aversion. Since poorer workers choose a benefit closer to the actual loss they may suffer, and since the equilibrium benefit is greater than the loss, this means that the benefit is smaller when workers are poorer. Combined with the fact that there is more fraud when benefits are smaller, it has to be that fraud increases with the size of the poll tax.

It is therefore possible that workers will strictly prefer an excise tax on disability insurance benefits to a poll tax because it reduces the amount of fraud in the economy, and thus reduces waste. Also, a disability benefit tax has redistributive effects such that those who bear the greatest burden relatively are the workers who were successful in filing for disability benefits when they were not disabled. The trade-off between the efficiency of the poll tax and the redistributive effect of the excise tax on disability insurance benefits is more clear when we allow the government to use those two tax instruments in the economy.

3.4 Optimal Tax Scheme

Assume then that the government needs to raise some very small amount (i.e. $g \to 0$) from each worker. We are, in essence, examining a Pigouvian tax scheme where all (or almost all) the proceeds are redistributed to the workers. Can the workers Pareto rank the different taxes? The answer is yes. Looking at the interaction between two types of taxes, we can conclude that the premium tax is the least favored type of tax. In particular, we show that if the government is allowed to use two tax schemes, it should impose an excise tax on disability insurance benefits and redistribute the money either through a poll subsidy or a premium subsidy.[6]

As before, we assume that the government needs to raise some amount G through taxes. This amounts to an average of $g = \frac{G}{N}$ per worker if there are N workers in the economy. Since the government's goal is to maximize the worker's expected utility, it will choose the taxation scheme accordingly. The government knows what impact its choice of tax will have on the optimal contract, just as the insurer knows how the contract affects the claiming game. These taxes are chosen

[6] I will not examine the case where three taxes are possible since this would leave no degrees of freedom in the problem. There are only four possible combinations of actions. By allowing to choose three possible taxes, and to also choose the optimal benefit four decision variables are yielded. With four variables and four states the market is complete. This means that there are no degrees of freedom left.

optimally even if they lead to subsidies (negative taxes). The problem faced by the government is

$$\max_{t_b;t_p;T;B} EU_T = (1-\pi)U\left(Y-(1+t_p)\pi\frac{B_i^2}{B_i-c}+W-T\right) + \pi U\left(Y-(1+t_p)\pi\frac{B_i^2}{B_i-c}+(1-t_b)B_i-T\right)-\pi D \quad (12)$$

subject to the tax constraint

$$(t_b+t_p)\pi\frac{B_i^2}{B_i-c}+T-g=0 \quad (13)$$

and the first-order condition of the optimal choice of a disability insurance contract

$$0 = -(1-\pi)\pi\frac{B_i(B_i-2c)}{(B_i-c)^2}(1+t_p)U'\left(Y-(1+t_p)\pi\frac{B_i^2}{B_i-c}+W-T\right) + \pi\left[(1-t_b)-\frac{\pi B_i(B_i-2c)}{(B_i-c)^2}(1+t_p)\right]U'\left(Y-(1+t_p)\pi\frac{B_i^2}{B_i-c}+(1-t_b)B_i-T\right) \quad (14)$$

We let the government choose the combination of the two taxes that maximizes worker welfare ex ante.[7] There are three possible combinations of two taxes in this economy: benefit - poll, premium - benefit and poll - premium. Maximizing this program allows to state the following proposition concerning the benefit - poll case.

Proposition 3 If the amount of money needed (g) is small, then it is optimal for the government to levy an excise tax on disability benefits in excess of what is needed, and to redistribute the surplus to all workers in the form of a lump sum;in other words, to tax benefits and give a poll subsidy.

This result is very interesting. We have, in the presence of fraud and non-commitment on the part of insurers, that the government would maximize worker welfare by levying an excise tax on disability benefits instead of a poll tax.

What is most surprising about proposition 3 is that if workers were given the choice, they would vote for a government whose political platform on taxation is to over-tax disability benefits to subsidize all workers through lump-sum payments.[8] By taxing disability benefits, the government levies proportionally more money from workers who were not caught committing fraud. By allowing the government to redistribute these taxes through lump-sum subsidies to all workers in the economy, the government is capable of raising the excise tax on disability benefits to its optimal level. This means that even if the government raises no money in aggregate ($g = 0$), the workers are better

[7] The subscript on the benefit B is dropped for the rest of the paper; doing so will not confuse the reader.

[8] Since all workers are the same ex-ante, they all vote in the same manner.There is therefore no need to worry about the strategic voting behavior of agents in this model.

o¤ because those who commit fraud are implicitly penalized by paying taxes on the bene…ts they receive.

The two other possible combinations of taxes, poll - premium and premium - bene…t, are shown in this last proposition.

Proposition 4 *It is optimal for the government: 1- to tax bene…ts and subsidize premiums in the neighborhood of g ! 0; and 2- to levy a poll tax and not tax premiums.*

For the same reason as in the two other propositions, a bene…t tax is preferred to a premium tax because it imposes a greater burden on workers who are successful in …ling a fraudulent insurance disability claim. The other part of the proposition, that only a poll tax will be levied, comes from the fact that both the premium and the poll tax treat all workers in the same way, so that everyone in the economy ends up paying the same amount in tax. In other words, neither the premium nor the poll tax discriminate against workers who lie. It is clear then that if $g = 0$, then the government will not levy any tax.

If the government needs to pay for some expenditures, it is better for the workers that the government only levy a poll tax. This is due to the fact that premium taxes introduce ine¢ciencies that do not exist with a poll tax. It is interesting to note that levying a poll tax and no premium tax is in no way determined by the level of government expenditures. For any g, the workers are better o¤ if the government never taxes premiums.

4 Conclusion

In this paper, we presented a simple model where workers, who face uncertainty regarding their ability to work (they may become disabled through no fault of their own), may purchase insurance to mitigate this uncertainty. The workers, however, have proprietary information concerning their ex post ability to work. This means that they have an incentive to misreport their condition to collect disability bene…ts to which they are not entitled. If the insurer is not capable of committing credibly to an auditing strategy, then, in equilibrium, workers may commit fraud by announcing that they are disabled where in fact they are working.

In such an economy, we were able to observe the strategic complementation of taxes to increase welfare when the money collected using one tax scheme is redistributed to workers using a subsidy scheme. More to the point, we show that the optimal combination of the bene…t tax and the poll tax is such that the government should over-tax bene…ts in order to o¤er a lump-sum subsidy to all workers. These are the result of the workers' willingness to see their bene…ts taxed, since it lays a relatively greater burden of the tax on those workers who committed fraud and were not caught. We also show that government should tax bene…ts and subsidize premiums, for similar reasons.

These …ndings may be of interest to policymakers in an environment where workers may commit insurance fraud (whether it be disability insurance fraud, health care fraud or workers' compensation fraud). Because insurers may not be able to commit credibly to an auditing strategy, the government may want to tax insurance bene…ts and redistribute the money collected to all the participants in the economy through lump-sum subsidies, or premium subsidies.

There are two possible extensions at this point that may be worth pursuing in the future. The …rst relates to the inclusion of agents who never engage in fraud at all foe moral reasons. Picard (1996) shows that it is impossible to design a contract that separates opportunistic agents (those who play the game) from honest agents. By including honest agents who never bene…t from crime, it seems logical to expect that there is an even greater social good in taxing bene…ts and giving lump-sum subsidies. The reson is similar to those exposed in the paper: fraud is reduced, and a greater tax burden is laid on opportunistic agents.

The second possible extension would be to study the gain in welfare for agents who could not purchase insurance before, and who can now thanks to a lump-sum or premium subsidy. In the model we presented, there were no exogenous premium loading. If there existed a loading, then it is quite possible that some agents would have chosen not to purchase insurance. By subsidizing premiums by taxing bene…ts, the government could increase welfare by allowing agents to have access to more a¤ordable insurance.

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6 Appendix

Proof of proposition 1. The proof done using backward induction is similar to Gibbons (1992). The solution to this game is a sextuple. Looking at the left-hand side of Figure 2, it is clear that $\alpha_{DF} = 0$. Suppose the worker is disabled. Then filing a claim (FC) dominates not filing (DF), whatever the insurer does. By not filing, the best the worker can do is get a payoff of $U(Y - (1 + t_p)p_i - T) - D$. On the other hand, by filing a claim, the payoff to the agent is $U(Y - (1 + t_p)p_i + (1 - t_b)B_i - T) - D$.

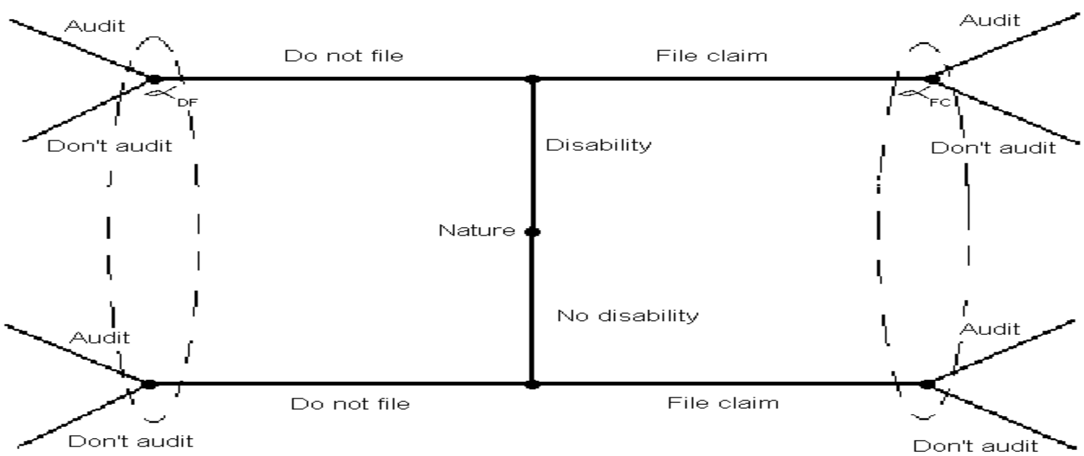

Figure 2: Extensive form of game.

When the insurer sees the agent play DF, she knows for sure that he is not disabled. Therefore, the insurer knows with probability one that she is at the lower node of the left-hand side information set. Consequently, the only meaningful strategy for the insurer when DF is played is to never audit. This is straightforward, since the insurer gets $p - c$ if she audits, and p if she does not. We have now found three of the six elements of the sextuple. Let's now move to the right side of the figure, where things are much more interesting.

Let γ_i be the probability (in a mixed strategy sense) of auditing a filed claim. The strategy of the insurer on the right-hand side of the information set must be such that the worker is indifferent between filing a claim and not filing, given that he is not disabled. To do so, γ_i must solve

$$\begin{aligned} U(Y - (1 + t_p)p_i + W - T) &= \gamma\left[U(Y - (1 + t_p)p_i + W - t_bB_i - T) - k\right] \\ &\quad + (1 - \gamma)U(Y - (1 + t_p)p_i + W + (1 - t_b)B_i - T) \end{aligned} \qquad (15)$$

which means that

$$\gamma_i = \frac{U\left(Y - (1+t_p)\,p_i + W + (1-t_b)\,B_i - T\right) - U\left(Y - (1+t_p)\,p_i + W - T\right)}{U\left(Y - (1+t_p)\,p_i + W + (1-t_b)\,B_i - T\right) - U\left(Y - (1+t_p)\,p_i + W - t_b B_i - T\right) + k} \tag{16}$$

All that is left to calculate is the belief of the insurer on right-hand side of the information set and the strategy of the worker given that he is not disabled. Let η be the probability (in the mixed strategy sense) that the worker files a claim when he is not disabled. Using Bayes' rule, we can find the exact value of α_{FC}, the insurer's posterior belief that the worker is indeed disabled. α_{FC} is equal to

$$\alpha_{FC} = \frac{\pi}{\pi + (1-\pi)\,\eta_i} \tag{17}$$

Only one strategy of the worker will induce the insurer to be indifferent between auditing and not auditing. That strategy is such that α_{FC} solves

$$(-c - B_i)\,\alpha_{FC} + (-c)(1 - \alpha_{FC}) = -B_i \tag{18}$$

which means that

$$\alpha_{FC} = \frac{B_i - c}{B_i} \tag{19}$$

Substituting for α_{FC} in (17), yields that the agent's probability of committing fraud is[9]

$$\eta_i = \left(\frac{c}{B_i - c}\right)\left(\frac{\pi}{1-\pi}\right) \tag{20}$$

Since all six elements have been found, the proof is done. $\square$

<u>Proof of proposition 2</u>. In all three cases, it is sufficient to show the impact of the taxes on the equilibrium benefits, since taxes have no direct impact on the probability of fraud. Let

$$\begin{aligned} \Omega = {} & -(1-\pi)\,\pi\frac{B_i\,(B_i - 2c)}{(B_i - c)^2}\,(1+t_p)\,U'\left(Y - (1+t_p)\,\pi\frac{B_i^2}{B_i - c} + W - T\right) \\ & + \pi\left[(1-t_b) - \frac{\pi B_i\,(B_i - 2c)}{(B_i - c)^2}\,(1+t_p)\right] U'\left(Y - (1+t_p)\,\pi\frac{B_i^2}{B_i - c} + (1-t_b)\,B_i - T\right) \end{aligned} \tag{21}$$

be the first-order condition of the problem.

In the first case, we want to show that $\frac{dB_p}{dt_p} = -\frac{\partial\Omega/\partial t_p}{\partial\Omega/\partial B_p} < 0$.[10] Taking the partial of Ω_p with respect to t_p and B_p and letting $t_b = T = 0$ yields

$$\frac{\partial\Omega}{\partial t_p} = -(1-\pi)\,\pi\frac{B_p\,(B_p - 2c)}{(B_p - c)^2}U'\left(Y - (1+t_p)\,\pi\frac{B_p^2}{B_p - c} + W\right) \tag{22}$$

---

[9] Note that we need to assume that $\pi < \frac{B_0 - c}{B_0}$ for the reporting probability to be in the zero-one interval. If not, then the agent will always commit fraud when he has a low loss. A sufficient condition is to assume that $\pi < \frac{1}{2}$ since, as we can see in the first-order condition $B_0 > 2c$ (see next footnote).

[10] We will denote by subscript p the case of a premium tax, b, the case of a benefit tax and T, the case of a lump-sum tax.

$$+ (1-\pi)\,\pi^2 (1+t_p)\,\frac{B_p^3 (B_p - 2c)}{(B_p - c)^3} U''\left(Y - (1+t_p)\,\pi\frac{B_p^2}{B_p - c} + W\right)$$

and

$$\begin{aligned}\frac{\partial \Omega}{\partial B_p} = & -2(1-\pi)\,\pi(1+t_p)\,\frac{c^2}{(B_p-c)^3} U'\left(Y - (1+t_p)\,\pi\frac{B_p^2}{B_p-c} + W\right) \\ & + (1-\pi)\left[\pi(1+t_p)\,\frac{B_p(B_p-2c)}{(B_p-c)^2}\right]^2 U''\left(Y - (1+t_p)\,\pi\frac{B_p^2}{B_p-c} + W\right) \\ & - 2\pi^2(1+t_p)\,\frac{c^2}{(B_p-c)^3} U'\left(Y - (1+t_p)\,\pi\frac{B_p^2}{B_p-c} + B_p\right) \\ & + \pi\left[1 - (1+t_p)\,\frac{\pi B_p(B_p-2c)}{(B_p-c)^2}\right]^2 U''\left(Y - (1+t_p)\,\pi\frac{B_p^2}{B_p-c} + B_p\right)\end{aligned} \tag{23}$$

Obviously, both $\frac{\partial \Omega}{\partial t_p}$ and $\frac{\partial \Omega}{\partial B_p}$ are negative at $t_p = T = 0$. This means that $\frac{dB_p}{dt_p} = -\frac{\partial \Omega_p/\partial t_p}{\partial \Omega_p/\partial B_p} < 0$, and that $\frac{\partial \eta_p}{\partial t_p} > 0$. This completes the first part of the proof.

Looking at the impact of t_b on B_b, we want to show that $\frac{dB_b}{dt_b} = -\frac{\partial \Omega_b/\partial t_b}{\partial \Omega_b/\partial B_b} > 0$. Taking the partial derivative of Ω with respect to t_b and $B_b$and letting $t_p = T = 0$ yields

$$\begin{aligned}\frac{\partial \Omega}{\partial t_b} = & -U'\left(Y - \pi\frac{B_b^2}{B_b-c} + (1-t_b)B_b\right) \\ & - B_b\left[(1-t_b) - \frac{\pi B_b(B_b-2c)}{(B_b-c)^2}\right] U''\left(Y - \pi\frac{B_b^2}{B_b-c} + (1-t_b)B_b\right)\end{aligned} \tag{24}$$

and

$$\begin{aligned}\frac{\partial \Omega}{\partial B_b} = & -2(1-\pi)\left[\frac{c^2}{(B_b-c)^3}\right] U'\left(Y - \pi\frac{B_b^2}{B_b-c} + W\right) \\ & + \pi(1-\pi)\left[\frac{B_b(B_b-2c)}{(B_b-c)^2}\right]^2 U''\left(Y - \pi\frac{B_b^2}{B_b-c} + W\right) \\ & - 2\pi\left[\frac{c^2}{(B_b-c)^3}\right] U'\left(Y - \pi\frac{B_b^2}{B_b-c} + (1-t_b)B_b\right) \\ & + \left[(1-t_b) - \frac{\pi B_b(B_b-2c)}{(B_b-c)^2}\right]^2 U''\left(Y - \pi\frac{B_b^2}{B_b-c} + (1-t_b)B_b\right)\end{aligned} \tag{25}$$

It is clear that $\frac{\partial \Omega}{\partial B_b} < 0$ since $U'(\cdot) > 0$, $U''(\cdot) < 0$. This means that $\text{sign}\left(\frac{dB_b}{dt_b}\right) = \text{sign}\left(\frac{\partial \Omega}{\partial t_b}\right)$. Rearranging gives us that $\frac{dB_b}{dt_b} > 0$ if and only if

$$R_A = -\frac{U''\left(Y - \pi\frac{B_b^2}{B_b-c} + (1-t_b)B_b\right)}{U'\left(Y - \pi\frac{B_b^2}{B_b-c} + (1-t_b)B_b\right)} > \frac{1}{B_b\left[(1-t_b) - \frac{\pi B_b(B_b-2c)}{(B_b-c)^2}\right]} \tag{26}$$

This equation means that if the worker's coefficient of absolute risk aversion (R_A) is large enough, then an increase in the disability insurance tax rate will induce an increase in the pre-tax benefit paid by the insurer. The second part of the corollary is done.

Finally, the impact of a poll tax on the benefit is given by $\frac{dB_T}{dT} = -\frac{\partial \Omega_T / \partial T}{\partial \Omega_T / \partial B_T}$. Taking the partial of Ω_T with respect to T and B_T and letting $t_p = t_b = 0$ yields

$$\begin{aligned} \frac{\partial \Omega}{\partial T} = \; & (1-\pi)\pi \frac{B_T(B_T-2c)}{(B_T-c)^2} U''\left(Y - \pi \frac{B_T^2}{B_T-c} + W - T\right) \\ & - \pi \left[1 - \frac{\pi B_T(B_T-2c)}{(B_T-c)^2}\right] U''\left(Y - \pi \frac{B_T^2}{B_T-c} + B_T - T\right) \end{aligned} \tag{27}$$

and

$$\begin{aligned} \frac{\partial \Omega}{\partial B_T} = \; & -2(1-\pi)\pi \frac{c^2}{(B_p-c)^3} U'\left(Y - \pi \frac{B_T^2}{B_T-c} + W - T\right) \\ & + (1-\pi)\left[\pi \frac{B_T(B_T-2c)}{(B_T-c)^2}\right]^2 U''\left(Y - \pi \frac{B_T^2}{B_T-c} + W - T\right) \\ & - 2\pi^2 \frac{c^2}{(B_p-c)^3} U'\left(Y - \pi \frac{B_T^2}{B_T-c} + B_T - T\right) \\ & + \pi \left[1 - \frac{\pi B_T(B_T-2c)}{(B_T-c)^2}\right]^2 U''\left(Y - \pi \frac{B_T^2}{B_T-c} + B_T - T\right) \end{aligned} \tag{28}$$

We want to show that $\frac{dB_T}{dT} = -\frac{\partial \Omega / \partial T}{\partial \Omega / \partial B_T} \leq 0$. It is clear that $\frac{\partial \Omega}{\partial B_T} < 0$. Therefore, $\frac{dB_T}{dT} \leq 0$ if and only if $\frac{\partial \Omega}{\partial T} \leq 0$. This occurs if and only if

$$\begin{aligned} 0 \geq \; & (1-\pi) \frac{B_T(B_T-2c)}{(B_T-c)^2} U''\left(Y - \pi \frac{B_T^2}{B_T-c} + W - T\right) \\ & - \left[1 - \frac{\pi B_T(B_T-2c)}{(B_T-c)^2}\right] U''\left(Y - \pi \frac{B_T^2}{B_T-c} + B_T - T\right) \end{aligned} \tag{29}$$

and

$$\frac{1 - \frac{\pi B_T(B_T-2c)}{(B_T-c)^2}}{(1-\pi)\frac{B_T(B_T-2c)}{(B_T-c)^2}} \leq \frac{U''\left(Y - \pi \frac{B_T^2}{B_T-c} + W - T\right)}{U''\left(Y - \pi \frac{B_T^2}{B_T-c} + B_T - T\right)} \tag{30}$$

We know from the first order condition that

$$\frac{1 - \frac{\pi B_T(B_T-2c)}{(B_T-c)^2}}{(1-\pi)\frac{B_T(B_T-2c)}{(B_T-c)^2}} = \frac{U'\left(Y - \pi \frac{B_T^2}{B_T-c} + W - T\right)}{U'\left(Y - \pi \frac{B_T^2}{B_T-c} + B_T - T\right)} \tag{31}$$

Substituting in the previous equation and rearranging gives us that $\frac{\partial \Omega}{\partial T} \leq 0$ if and only if

$$-\frac{U''\left(Y-\pi\frac{B_T^2}{B_T-c}+B_T-T\right)}{U'\left(Y-\pi\frac{B_T^2}{B_T-c}+B_T-T\right)} \leq -\frac{U''\left(Y-\pi\frac{B_T^2}{B_T-c}+W-T\right)}{U'\left(Y-\pi\frac{B_T^2}{B_T-c}+W-T\right)} \tag{32}$$

which is true if the utility function displays decreasing absolute risk aversion and if $B_T \geq W$. To show that $B_T \geq W$, let $B_T = W$ in the first order condition (and $t_p = t_b = 0$). It is then clear that the first-order condition is positive. This then means that the worker is capable of increasing his utility by choosing a $B_T \geq W$. The first-order condition as $B_T = W$ is positive if and only if

$$\begin{aligned} 0 < \; & -(1-\pi)\pi\frac{W(W-2c)}{(W-c)^2}U'\left(Y-\pi\frac{W^2}{W-c}+W-T\right) \\ & +\pi\left[1-\pi\frac{W(W-2c)}{(W-c)^2}\right]U'\left(Y-\pi\frac{W^2}{W-c}+W-T\right) \end{aligned} \tag{33}$$

Dividing everywhere by $\pi U'(:) > 0$, we obtain

$$0 < 1-\pi\frac{W(W-2c)}{(W-c)^2}-(1-\pi)\frac{W(W-2c)}{(W-c)^2} \tag{34}$$

Simplifying yields

$$0 < (W-c)^2 - W(W-2c) \tag{35}$$

which is always true since $c^2 > 0$. Thus, as the poll tax increases, the equilibrium benefit decreases and fraud increases. This completes the proof. $\square$

Proof of proposition 3. The Lagrangian problem of the government is[11]

$$\begin{aligned} \max_{t_b;T;B;\mu_1;\mu_2} W = \; & \pi U\left(Y-\pi\frac{B^2}{B-c}+(1-t_b)B-T\right) \\ & +(1-\pi)U\left(Y-\pi\frac{B^2}{B-c}+W-T\right) \\ & +\mu_1\left[t_b\pi\frac{B^2}{B-c}+T-g\right] \\ & +\mu_2\left[\begin{array}{c}\pi\left[(1-t_b)-\pi\frac{B(B-2c)}{(B-c)^2}\right]U'\left(Y-\pi\frac{B^2}{B-c}+(1-t_b)B_b-T\right) \\ -(1-\pi)\pi\frac{B(B-2c)}{(B-c)^2}U'\left(Y-\pi\frac{B^2}{B-c}+W-T\right)\end{array}\right] \end{aligned} \tag{36}$$

The five first-order conditions are

$$\begin{aligned} \frac{\partial L}{\partial t_b} = 0 = \; & -B\pi U'\left(Y-\pi\frac{B^2}{B-c}+(1-t_b)B-T\right)+\mu_1\pi\frac{B^2}{B-c} \\ & -\mu_2\left[\begin{array}{c}B\left[(1-t_b)-\pi\frac{B(B-2c)}{(B-c)^2}\right]U''\left(Y-\pi\frac{B^2}{B-c}+(1-t_b)B_b-T\right) \\ +\pi U'\left(Y-\pi\frac{B^2}{B-c}+(1-t_b)B_b-T\right)\end{array}\right] \end{aligned} \tag{37}$$

[11] We will drop the subscript on premiums and benefits in the remainder of the paper to lighten the presentation.

$$
\begin{aligned}
\frac{\partial L}{\partial T} = {} & 0 = -\pi U'\left(Y - \pi\frac{B^2}{B-c} + (1-t_b)B - T\right) \\
& - (1-\pi)U'\left(Y - \pi\frac{B^2}{B-c} + W - T\right) + \mu_1 \\
& - \mu_2 \left[\begin{array}{c} \pi\left[(1-t_b) - \pi\frac{B(B-2c)}{(B-c)^2}\right]U''\left(Y - \pi\frac{B^2}{B-c} + (1-t_b)B_b - T\right) \\ + (1-\pi)\pi\frac{B(B-2c)}{(B-c)^2}U''\left(Y - \pi\frac{B^2}{B-c} + W - T\right) \end{array}\right]
\end{aligned} \tag{38}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial B} = {} & 0 = \pi\left[(1-t_b) - \pi\frac{B(B-2c)}{(B-c)^2}\right]U'\left(Y - \pi\frac{B^2}{B-c} + (1-t_b)B - T\right) \\
& - (1-\pi)\pi\frac{B(B-2c)}{(B-c)^2}U'\left(Y - \pi\frac{B^2}{B-c} + W - T\right) + \mu_1 t_b \pi\frac{B^2}{B-c} \\
& + \mu_2 \left[\begin{array}{c} -\pi^2\frac{c^2}{(B-c)^3}U'\left(Y - \pi\frac{B^2}{B-c} + (1-t_b)B_b - T\right) \\ -(1-\pi)\pi\frac{c^2}{(B-c)^3}U'\left(Y - \pi\frac{B^2}{B-c} + W - T\right) \\ +\pi\left[(1-t_b) - \pi\frac{B(B-2c)}{(B-c)^2}\right]^2 U''\left(Y - \pi\frac{B^2}{B-c} + (1-t_b)B_b - T\right) \\ +(1-\pi)\left[\pi\frac{B(B-2c)}{(B-c)^2}\right]^2 U''\left(Y - \pi\frac{B^2}{B-c} + W - T\right) \end{array}\right]
\end{aligned} \tag{39}
$$

$$
\frac{\partial L}{\partial \mu_1} = 0 = t_b \pi \frac{B^2}{B-c} + T - g \tag{40}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial \mu_2} = {} & 0 = \pi\left[(1-t_b) - \pi\frac{B(B-2c)}{(B-c)^2}\right]U'\left(Y - \pi\frac{B^2}{B-c} + (1-t_b)B_b - T\right) \\
& - (1-\pi)\pi\frac{B(B-2c)}{(B-c)^2}U'\left(Y - \pi\frac{B^2}{B-c} + W - T\right)
\end{aligned} \tag{41}
$$

It is clear that we can rewrite (39), as

$$
\begin{aligned}
\frac{\partial L}{\partial B} = {} & 0 = \frac{\partial L}{\partial \mu_2} + \mu_1 t_b \pi\frac{B^2}{B-c} \\
& + \mu_2 \left[\begin{array}{c} -\pi^2\frac{c^2}{(B-c)^3}U'\left(Y - \pi\frac{B^2}{B-c} + (1-t_b)B_b - T\right) \\ -(1-\pi)\pi\frac{c^2}{(B-c)^3}U'\left(Y - \pi\frac{B^2}{B-c} + W - T\right) \\ +\pi\left[(1-t_b) - \pi\frac{B(B-2c)}{(B-c)^2}\right]^2 U''\left(Y - \pi\frac{B^2}{B-c} + (1-t_b)B_b - T\right) \\ +(1-\pi)\left[\pi\frac{B(B-2c)}{(B-c)^2}\right]^2 U''\left(Y - \pi\frac{B^2}{B-c} + W - T\right) \end{array}\right]
\end{aligned} \tag{42}
$$

We know that $\frac{\partial L}{\partial \mu_2} = 0$ from (41). Thus, it has to be that $\mu_1 t_b \pi \frac{B^2}{B-c} \geq 0$, since μ_2 is non-negative and the term multiplying μ_2 in (42) is negative. Since $\mu_1 \geq 0$, we end up with $t_b \geq 0$. Therefore the government will never want to subsidize disability insurance benefits. We can rewrite (40) as $T = g - t_b \pi \frac{B^2}{B-c}$. This means that as the amount of money needed approaches zero ($g \to 0$), the poll tax becomes non-positive, since $T = -t_b \pi B \leq 0$. Therefore, it is optimal for the government to give lump-sum subsidies to the workers by using an excise tax on disability benefits.

It remains to be proven that t_b is not equal to zero. Suppose $t_b = 0$. Then, from (42), $\mu_2 = 0$. From (38), we then have that

$$\mu_1 = \pi U'\left(Y - \pi\frac{B^2}{B-c} + (1-t_b)B - T\right) + (1-\pi)U'\left(Y - \pi\frac{B^2}{B-c} + W - T\right) \tag{43}$$

Substituting this value of μ_1 into (37) and combining terms yields

$$(1-\pi)B\pi U'\left(Y - \pi\frac{B^2}{B-c} + (1-t_b)B - T\right) = \pi B(1-\pi)U'\left(Y - \pi\frac{B^2}{B-c} + W - T\right) \tag{44}$$

Simplifying, we obtain

$$U'\left(Y - \pi\frac{B^2}{B-c} + (1-t_b)B - T\right) = U'\left(Y - \pi\frac{B^2}{B-c} + W - T\right) \tag{45}$$

With $t_b = 0$, T must also equal zero as $g \to 0$. Thus

$$U'\left(Y - \pi\frac{B^2}{B-c} + B\right) = U'\left(Y - \pi\frac{B^2}{B-c} + W\right) \tag{46}$$

which means that $B = W$. All that is left to prove is that $B \neq W$. From (41), letting $T = t_b = 0$, and $B = W$ yields

$$0 = \pi\left[1 - \pi\frac{W(W-2c)}{(W-c)^2}\right]U'\left(Y - \pi\frac{W^2}{W-c} + W\right) - (1-\pi)\pi\frac{W(W-2c)}{(W-c)^2}U'\left(Y - \pi\frac{W^2}{W-c} + W\right) \tag{47}$$

Simplifying, we obtain

$$1 - \frac{W(W-2c)}{(W-c)^2} = 0 \tag{48}$$

which is true if and only if

$$W(W-2c) = (W-c)^2 \tag{49}$$

This occurs if and only if $c = 0$. By assumption $c > 0$. This means that $B \neq W$, which means that $t_b \neq 0$. This completes the proof. ■

<u>Proof of proposition 4</u>. When benefits and premiums can be taxed, the Lagrangian problem of the government is

$$\max_{t_b, t_p, B, \mu_1, \mu_2} W = \pi U\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + (1-t_b)B\right) \tag{50}$$

$$
\begin{aligned}
&+(1-\pi)U\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W\right)\\
&+\mu_1\left[(t_b+t_p)\pi\frac{B^2}{B-c}-g\right]\\
&+\mu_2\left[\begin{array}{c}\pi\left[(1-t_b)-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right)\\
-(1-\pi)(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W\right)\end{array}\right]
\end{aligned}
$$

The five first-order conditions are

$$
\begin{aligned}
\frac{\partial L}{\partial t_b} = 0 = &-B\pi U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right)+\mu_1\pi\frac{B^2}{B-c}\\
&-\mu_2\left[\begin{array}{c}B\left[(1-t_b)-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right)\\
+\pi U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right)\end{array}\right]
\end{aligned}
\tag{51}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial t_p} = 0 = &-\pi^2\frac{B^2}{B-c}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right)\\
&-(1-\pi)\pi\frac{B^2}{B-c}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W\right)+\mu_1\pi\frac{B^2}{B-c}\\
&-\mu_2\left[\begin{array}{c}-\pi\left[(1-t_b)-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]\pi\frac{B^2}{B-c}U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right)\\
-\pi\frac{B(B-2c)}{(B-c)^2}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right)\\
+(1-\pi)\pi(1+t_p)\frac{B(B-2c)}{(B-c)^2}\pi\frac{B^2}{B-c}U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W\right)\\
-(1-\pi)\pi\frac{B(B-2c)}{(B-c)^2}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W\right)\end{array}\right]
\end{aligned}
\tag{52}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial B} = 0 = &\,\pi\left[(1-t_b)-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right)\\
&-(1-\pi)(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W\right)\\
&+\mu_1\left[(t_b+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]\\
&+\mu_2\left[\begin{array}{c}-\pi^2(1+t_p)\frac{c^2}{(B-c)^3}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right)\\
-(1-\pi)(1+t_p)\pi\frac{c^2}{(B-c)^3}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W\right)\\
+\pi\left[(1-t_b)-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]^2U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right)\\
+(1-\pi)\left[(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]^2U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W\right)\end{array}\right]
\end{aligned}
\tag{53}
$$

$$
\frac{\partial L}{\partial \mu_1} = 0 = (t_b+t_p)\pi\frac{B^2}{B-c}-g \tag{54}
$$

$$\frac{\partial L}{\partial \mu_2} = 0 = \pi\left[(1-t_b) - (1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right) \tag{55}$$
$$-(1-\pi)(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W\right)$$

We can rewrite $\frac{\partial L}{\partial B}=0$, as

$$\frac{\partial L}{\partial B} = 0 = \frac{\partial L}{\partial \mu_2} + \mu_1\left[(t_b+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right] \tag{56}$$
$$+\mu_2\begin{bmatrix} -\pi^2(1+t_p)\frac{c^2}{(B-c)^3}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right) \\ -(1-\pi)(1+t_p)\pi\frac{c^2}{(B-c)^3}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W\right) \\ +\pi\left[(1-t_b)-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]^2U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right) \\ +(1-\pi)\left[(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]^2U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W\right)\end{bmatrix}$$

We know that $\frac{\partial L}{\partial \mu_2}=0$. Since the term multiplying μ_2 is negative, and since $\mu_2 \geq 0$, is has to be that $(t_b+t_p)\frac{B(B-2c)}{(B-c)^2} \geq 0$. We know from $\frac{\partial L}{\partial \mu_1}=0$ that $t_b = \frac{g}{\pi B^2}(B-c) - t_p$. This means that

$$\frac{g}{\pi B^2}(B-c) - t_p + t_p\frac{B(B-2c)}{(B-c)^2} \geq 0 \tag{57}$$

This is true if

$$t_p \leq \frac{g}{\pi B^2 c^2}(B-c)^3 \tag{58}$$

Letting $g \to 0$, I get that $t_p \leq 0$. This means that $t_b \geq 0$ as $g \to 0$. The remaining step is to show that $t_b \neq 0$, which is straightforward. At $t_b = 0$ and $t_p = 0$, $\mu_2 = 0$ from (56). Using (51) and (52), and letting $\mu_2 = 0$ imply that

$$\mu_1\left(\frac{B^2}{B-c}\right) = B\pi U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right) \tag{59}$$

and

$$\mu_1\left(\frac{B^2}{B-c}\right) = \pi^2\frac{B^2}{B-c}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right) \tag{60}$$
$$+(1-\pi)\pi\frac{B^2}{B-c}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W\right)$$

This means that, as we let $t_b = 0$ and $t_p = 0$

$$\frac{U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right)}{\pi U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+(1-t_b)B\right)+(1-\pi)U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W\right)} = \frac{B}{B-c} \tag{61}$$

We know from (10) what the left side of (61) is (with $T = 0$). Making the substitution yields

$$\frac{B(B-2c)}{(B-c)^2}\left(\frac{1+t_p}{1-t_b}\right) = \frac{B}{B-c} \tag{62}$$

Simplifying, we obtain

$$\frac{(B-2c)}{(B-c)}\left(\frac{1+t_p}{1-t_b}\right) = 1 \tag{63}$$

which cannot happen if $t_b = t_p = 0$. Thus, $t_b > 0$ and $t_p < 0$.

The second part of the proof is similar. When a poll and a premium tax can be levied, the Lagrangian problem of the government is

$$\begin{aligned}
\max_{T;t_p;B;\mu_1;\mu_2} W \quad = \quad & \pi U\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + B - T\right) \qquad (64)\\
& + (1-\pi)U\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + W - T\right)\\
& + \mu_1\left[T + t_p\pi\frac{B^2}{B-c} - g\right]\\
& + \mu_2\left[\begin{array}{l}\pi\left[1-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right)\\ -(1-\pi)(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right)\end{array}\right]
\end{aligned}$$

The five first-order conditions are

$$\begin{aligned}
\frac{\partial L}{\partial T} \quad = \quad & 0 = -\pi U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right) \qquad (65)\\
& -(1-\pi)U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right)\\
& +\mu_1 - \mu_2\left[\begin{array}{l}\pi\left[1-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right)\\ -(1-\pi)(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right)\end{array}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial L}{\partial t_p} \quad = \quad & 0 = -\pi^2\frac{B^2}{B-c}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right) \qquad (66)\\
& -(1-\pi)\pi\frac{B^2}{B-c}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right) + \mu_1\pi\frac{B^2}{B-c}\\
& -\mu_2\left[\begin{array}{l}-\pi\left[1-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]\pi\frac{B^2}{B-c}U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right)\\ -\pi\pi\frac{B(B-2c)}{(B-c)^2}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right)\\ +(1-\pi)(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\pi\frac{B^2}{B-c}U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right)\\ -(1-\pi)\pi\frac{B(B-2c)}{(B-c)^2}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right)\end{array}\right]
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial L}{\partial B} = {} & 0 = \pi\left[1-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right) \\
& -(1-\pi)(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right) \\
& +\mu_1\left[t_p\pi\frac{B(B-2c)}{(B-c)^2}\right] \\
& +\mu_2\begin{bmatrix}
-\pi^2(1+t_p)\frac{c^2}{(B-c)^3}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right) \\
-(1-\pi)(1+t_p)\pi\frac{c^2}{(B-c)^3}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right) \\
+\pi\left[1-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]^2U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right) \\
+(1-\pi)\left[(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]^2U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right)
\end{bmatrix}
\end{aligned}
\tag{67}
$$

$$
\frac{\partial L}{\partial \mu_1} = 0 = T + t_p\pi\frac{B^2}{B-c} - g \tag{68}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial \mu_2} = {} & 0 = \pi\left[1-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right) \\
& -(1-\pi)(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right)
\end{aligned}
\tag{69}
$$

Since $\frac{\partial L}{\partial \mu_2} = 0$, we can rewrite $\frac{\partial L}{\partial B} = 0$ as

$$
\begin{aligned}
\frac{\partial L}{\partial B} = {} & 0 = \frac{\partial L}{\partial \mu_2} + \mu_1 t_p\pi\frac{B(B-2c)}{(B-c)^2} \\
& +\mu_2\begin{bmatrix}
-\pi^2(1+t_p)\frac{c^2}{(B-c)^3}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right) \\
-(1-\pi)(1+t_p)\pi\frac{c^2}{(B-c)^3}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right) \\
+\pi\left[1-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]^2U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right) \\
+(1-\pi)\left[(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]^2U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right)
\end{bmatrix}
\end{aligned}
\tag{70}
$$

Since $\frac{\partial L}{\partial \mu_2} = 0$, we have

$$
t_p = -\left(\frac{(B-c)^2}{\pi B(B-2c)}\right)\left(\frac{\mu_2}{\mu_1}\right)\begin{bmatrix}
-\pi^2(1+t_p)\frac{c^2}{(B-c)^3}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right) \\
-(1-\pi)(1+t_p)\pi\frac{c^2}{(B-c)^3}U'\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right) \\
+\pi\left[1-(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]^2U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+B-T\right) \\
+(1-\pi)\left[(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]^2U''\left(Y-(1+t_p)\pi\frac{B^2}{B-c}+W-T\right)
\end{bmatrix}
\tag{71}
$$

The remainder of the proof will show that $\mu_2 = 0$, and that $\mu_1 \neq 0$. From $\frac{\partial L}{\partial T} = 0$ and $\frac{\partial L}{\partial t_p} = 0$, we obtain

$$0 = \mu_1 \left[\begin{array}{c} \pi U'\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + B - T\right) \\ + (1-\pi)\, U'\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + W - T\right) \end{array} \right] - \mu_2 \left[\begin{array}{c} \pi\left[1 - (1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right] U''\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + B - T\right) \\ - (1-\pi)(1+t_p)\pi\frac{B(B-2c)}{(B-c)^2} U''\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + W - T\right) \end{array} \right] \tag{72}$$

$$0 = \mu_1 \left[\begin{array}{c} \pi U'\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + B - T\right) \\ + (1-\pi)\, U'\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + W - T\right) \end{array} \right] - \frac{\mu_2}{\pi\frac{B^2}{B-c}} \left[\begin{array}{c} -\pi\left[1 - (1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right]\pi\frac{B^2}{B-c} U''\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + B - T\right) \\ -\pi\pi\frac{B(B-2c)}{(B-c)^2} U'\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + B - T\right) \\ + (1-\pi)\pi(1+t_p)\frac{B(B-2c)}{(B-c)^2}\pi\frac{B^2}{B-c} U''\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + W - T\right) \\ - (1-\pi)\pi\frac{B(B-2c)}{(B-c)^2} U'\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + W - T\right) \end{array} \right] \tag{73}$$

If μ_2 is not equal to zero, these two equalities hold if and only if

$$\frac{\left[\begin{array}{c} \left[1 - (1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right] U''\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + B - T\right) \\ - (1-\pi)(1+t_p)\frac{B(B-2c)}{(B-c)^2} U''\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + W - T\right) \end{array} \right]}{\left[\begin{array}{c} \pi U'\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + B - T\right) \\ + (1-\pi)\, U'\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + W - T\right) \end{array} \right]} = -\frac{(B-2c)}{2\pi B(B-c)} \tag{74}$$

Assume $U'''(:) > 0$.[12] If

$$0 = \left[1 - (1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right] U'\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + B - T\right) - (1-\pi)(1+t_p)\frac{B(B-2c)}{(B-c)^2} U'\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + W - T\right) \tag{75}$$

from $\frac{\partial L}{\partial \mu_2} = 0$, then

$$0 \leq \left[1 - (1+t_p)\pi\frac{B(B-2c)}{(B-c)^2}\right] U''\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + B - T\right) - (1-\pi)(1+t_p)\frac{B(B-2c)}{(B-c)^2} U''\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + W - T\right) \tag{76}$$

[12] A positive third derivative of the utility function is implied by a DARA utility function.

Thus, the left-hand side of (74) is positive and its right-hand side is negative, which is impossible. This means that $\mu_2 = 0$ and thus

$$\mu_1 = \pi U'\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + B - T\right) + (1-\pi)U'\left(Y - (1+t_p)\pi\frac{B^2}{B-c} + W - T\right) \tag{77}$$

which means that $t_p = 0$. This completes the proof. $\square$