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Estimation of default correlation in a loan portfolio of Canadian public firms

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Abstract

There are different methods to calculate default probability of a portfolio. One can name Structural method of Merton, Reduced (non-structural) form, Scoring method and Hybrid model. Although, finding default probability is really important and can help lenders to protect themselves, calculating default correlation between exposures is as critical. Calculation of default correlation enables financial institutions to take into account the effect of diversification and also, gives them a better estimate of overall defaults. The goal of this research is to find the correlation between default risks of publicly traded Canadian companies in an overall loan portfolio. For this purpose, the CreditMetrics method that was described by J.P. Morgan is utilized.

Résumé

Il existe différentes méthodes pour calculer la probabilité de défaut d'un portefeuille. Parmis celles-ci, nous avons la méthode structurelle, la méthode réduite (non structurelle), la méthode de notation et la méthode hybride. Bien que l'évaluation de la probabilité de défaut soit très importante et permet aux prêteurs de se protéger, celle de la corrélation entre les expositions de défaut est cruciale. En effet, le calcul de la corrélation de défaut permet aux institutions financières de prendre en compte l'effet de la diversification en leur donnant une meilleure estimation de la valeur globale du risque. Le but de cette recherche est de trouver la corrélation entre les risques de défaut des compagnies canadiennes cotées en bourse dans un portefeuille de prêts global. À cet effet, la méthode CreditMetrics décrite par JP Morgan est utilisée.

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Chapter 1

Introduction

The dominant role of credit risk in total risk of banks has made financial institutions, researchers and regulators to pay special attention to credit risk. Credit risk is a risk that a debtor may not be able or willing to repay his debt. Usually great percent of banks total risk is explained by credit risk.
As stability of financial and economical system of each country is very dependent to stability of banks in that country, regulators pay special attention to banks. They make banks to calculate their risk and reserve a capital for reverse events. They set limits and conditions but allow banks to choose their own models to calculate their risks. So banks look for an accurate model for calculating their risks to immune them from crises, be accepted by regulators and also allow them to put less capital in reserve. The model does not only precisely estimates the default probability of exposures but also default correlation between exposures. Since banks do not want to simply sum up all the risks without encountering the diversification affects, that allows them to decrease their risks.
The most popular approaches in the financial literature for estimating default probability and default correlation are Structural and Reduced form models (intensity model).
Reduced form models provide statistical representation of the economic system. These models assume that a firm default time is unpredictable and driven by a

default intensity, which is a function of latent state variable. In this approach, default time is the first jump of an exogenously given jump process [9]. According to reduced-form model, multiple defaults are independent, conditional on the sample paths of the default intensities. Therefore, finding the default correlation is equal to finding the correlation between default intensities. Although some researchers such as Fan Yu [22] have worked on this model to show that default correlation can be sensitive to default intensities, some authors such as Hull and White (2001) and Schonbucher and Schubert (2001) argue that the ability of this approach to estimate default correlation is limited.

Structural models are based on a balance notion of solvency. They use market information to calculate default risk. Structural model was introduced for the first time by Merton [19]. The principal idea of Merton model [19] is that payoff to shareholders is very much similar to the payoff an European call option. Merton assumes that shareholders have a call option on the firm's asset value with the strike price equal to the outstanding debt. The Black and Scholes' model (1973) is used to price the option and estimate default probability. In this model default occurs when the asset value is less than debt value of a firm. That means the option will be exercised at maturity only if firm's debt value will be less that firm's asset value. So at exercise time, debt will be reimbursed and then surplus will de shared. There are lots of simplifying assumptions in the Merton model which have been improved by many researchers. One of those assumptions is the time of default which can occur only at maturity of debt.

Brockman and Turtle (2003) are one of those researchers that incorporated important innovation in modeling of default risk of firms. They used barrier option instead of simple European call option. In their model, debtors can put firm in default position as soon as the firm does not respect certain agreements that were defined at the beginning of their contracts. Although in Brockman and Turtle (2003) model, approximating market value of the firm is very important to well estimate default barrier, market value is not evaluated accurately in their paper.

To solve this problem, Wong and Choi [15], Duan, Gauthier and Simonato [8] proposed maximum likelihood method. Wong and Choi (2004) find two first moments of market value distribution and use them to obtain default barrier. Duan, Gauthier and Simonato (2004) utilize maximum likelihood method adapted to Merton model to estimate default barrier.

One of the recent extensions of structural model to find default probability is the work of Jonathan Amar [2]. He first arbitrarily choses a value for mean and volatility of asset value and default barrier. Then he calculates the firm's asset value and inserts it in the model of Duan, Gauthier and Simonato (2004) and finds the optimal value of mean and volatility for default barrier and asset value. After finding the optimal value, he uses Brockman and Turtle (2003) model to find default probability. The default barrier is endogenous in his work.

As mentioned above structural models are one of the most popular models in estimating default correlation. Although there are different approaches for calculating correlation based on Structural models, such as First passage time and Copula, the most popular one is factor based models. Factor based models are the extention of the Merton option theory. In these models, driving default variable (asset value) has two parts, systematic common component and idiosyncratic component. Both are distributed normally with zero mean. The most famous factor based model is CreditMetrics model. In this model companies' asset value can be used to estimate firm's credit migration and default. It defines asset value thresholds for each rating class and compares the value of firms with these thresholds at the end of each year. The purpose is to determine whether credit rating of the firm changed or if firm moved to default position. It proposes that asset returns R are normally distributed with mean μ and standard deviation σ . It establishes a connection between asset thresholds and transition probabilities for each firm.

Since there are more observations in Structural models and they can be generalized and be updated cautiously with the evolution of firm's asset value, they are more flexible than Reduced form models. Therefore in this work we have decided to use Structural model for calculation of default probability and default correlation. We will combine the work of Jonathan Amar with the CreditMetrics model to estimate precise default correlation between Canadian public firms. In this work we have a portfolio of Canadian public firms which are traded on the Toronto Stock Exchange. A motivation for using CreditMetrics model is that no one before applied it on the Canadian data. The other reason is that CreditMetrics has a portfolio approach that we need for our loan portfolio of Canadian firms. Portfolio approach is very popular in the modern finance and is used widely by banks, since it allows them to better capture the impact of diversification in their portfolios. The reason for combining the two works is to get better and more accurate results. CreditMetrics is not enough precise and it has some simplifying assumptions (using same default probability for firms in a same rating class) that prevent it to estimate very accurate correlation.

This document contains five chapters. In first chapter the literature will be reviewed, in second chapter the methodology will be discussed, in third chapter the database will be described, in fourth chapter the results will be presented and fifth and final chapter will conclude this study.

Chapter 2

Literature Review

2.1 Default probability

2.1.1 Structural model of Merton

The structural model of Merton (1974) [19] can be named the first structural model in Credit risk, which allows to estimate risk of corporate bonds and default probability (PD) of corporates. Merton assumes that firm's asset value follows a geometric Brownian motion with a constant volatility and its capital structural consists of common equity and zero coupon debt. He also assumes that market is perfect. In his approach stock holders receive no dividend and the debt holders are paid at the maturity of debt. At the maturity of debt if the asset value of the firm is higher than its debt value then the debtors receive the debt amount and the stock holders receive the amount equal to asset value minus the debt value. However at the maturity if the asset value of the firm is less than the debt value the debtors take over of every thing and stock holders receive nothing. In this model [19] , Merton proposes that payoff to shareholders is very much similar to the payoff an European call option that they would have received if the shareholders bought a call option on the firm's asset value with the strike price equal to the outstanding debt. The Black & Scholes model (1973) is used to price the option and estimate default probability. Therefore at period t:

$$A_t = D(A,T,t) + S(A,t) \tag{2.1}$$

where (D) is the debt of the firm, $S(A,t)$ is equity, A is the firm's asset value and T is the debt maturity. According to Merton firm's value follows the following stochastic process,

$$dA = \mu A dt + \sigma A dz \tag{2.2}$$

where μ is the expected return and σ is the standard deviation of firm's value and z is a Brownian movement.
The Merton model [19] assumes that at maturity, if the firm's value falls below F (the value of debt at time t) firm bankrupts, the bondholders take over the firm and shareholders receive nothing, otherwise the bondholders receive the promised amount of F and the shareholders take the rest. Thus at maturity:

$$\begin{aligned} D(A,0) &= Min(A,F) \\ S(V,0) &= Max(0,A-F) \end{aligned} \tag{2.3}$$

From above equations it can be derived that the shareholders hold a call option on the firm's asset value. The idea that Merton [19] used to calculate the value of S and F by applying the Black & Scholes formula:

$$S(A,t) = A\Phi(a) - Fe^{-r\tau}\Phi(a - \sigma\sqrt{\tau}) \tag{2.4}$$

where $a = \frac{log(A/F) + \left(r + \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}$, $\tau = T - t$ and Φ is the cumulative normal distribution function. By knowing (2.4) and $D = A - S$ one can conclude:

$$D(A,t) = Fe^{-r\tau}\Big\{\Phi[w_2(d,\sigma^2\tau)] + \frac{1}{d}\Phi[w_1(d,\sigma^2\tau)]\Big\} \tag{2.5}$$

$$w_1(d,\sigma^2\tau) = \frac{-\frac{1}{2}\sigma^2\tau - log(d)}{\sigma\sqrt{\tau}}$$

$$w_2(d,\sigma^2\tau) = \frac{-\frac{1}{2}\sigma^2\tau + log(d)}{\sigma\sqrt{\tau}}$$

where $d \equiv \frac{Fe^{-r\tau}}{A}$.

As mentioned above if at maturity of debt the firm's asset value (A) falls below the value of debt (F), firm bankrupts and shareholders receive nothing. Therefore the probability that shareholders receive nothing is equal to the *PD* of the firm:

$$P[A_i(T) < F_i|D_T] = \Phi\left(\frac{\ln(F_i/A_i) - (\mu_i - 0.5\sigma^2)\tau}{\sigma\sqrt{\tau}}\right) \tag{2.6}$$

Merton model [19] has an important role in the evolution of the credit risk since it improves lacunas and limitations of Reduced form models that will be presented later. In this model[19] we do not need the accounting information of the firms and also there is no need for anticipating the future financial status of the firms. However, there are lots of simplifying assumptions in the Merton model [19] which have been improved by many researchers and will be explained in following section.

2.1.2 Evolution of Merton model

1. **Time of default**

   Black and Cox [3] are the first researchers who relax the assumption that firm can default only at the maturity of firm's debt and introduce the concept of default barrier. They show that debtors are willing to put the firm in default as soon as it hits certain barrier even if the firm hits the barrier before maturity of debt. In this model Black and Cox [3] consider that if the firm's asset value improves and goes back on top of the default barrier before maturity, the firm can prevent bankruptcy.

2. **Risk free interest rate**

   Longstaff and Schwartz [18], and Briys and de Varenne [4] improve the model of Black and Cox [3] by proposing stochastic interest rate instead of constant interest rate in their model. Both works assume that interest rate follows Vasicek process (1977). Longstaff and Schwartz [18] also assume that interest rate is correlated with stochastic asset value process. They show that considering correlation between asset and interest rate processes is very important in evaluating firm's liability.

3. **Type of Option**

   Although Black and Cox [3] are the first researchers who introduce the concept of default barrier, researchers continue to use European call option in their structural model for estimating default probability for several years till the work of Brockman and Turtle [5]. Brockman and Turtle [5] integrate barrier options instead of simple European options in their model.
   Two kinds of barrier options are used in their structural model; Down and In $(D\&I)$ and Down and Out $(D\&O)$ options. Barrier options can be activated $(D\&I)$ or deactivated $(D\&O)$ when value of underlying stock hits certain value (barrier).
   Brockman and Turtle [5] assume that shareholders hold a $(D\&O)$ call option on the value of the firm and debtors hold a portfolio that includes risk free debt, short position on a put option and long position on a $(D\&I)$ call option on the value of the firm. If the firm defaults (the firm's value falls under a certain level (default threshold)), the shareholders' $(D\&O)$ option expires

but the debtors' $(D\&I)$ option activates. That means the share holders lose every thing and the debtors take over of every thing and push the firm to bankruptcy. According to Brockman and Turtle [5], Merton model [19] overestimates equity and underestimates debt by amount equal to $(D\&I)$ option.

Model of Bank of England [21] is another study that uses barrier options instead of European call options. In this model [21] authors assume that insolvency can occur the first time the firm's asset value (A) falls below the debt value (D) and uses $(D\&O)$ barrier options which expire as soon as a certain barrier is reached.

To find the default probability (PD) using barrier options, the value of firm's asset (A) and liabilities (F) is supposed to be:

$$\begin{aligned} dA &= \mu_A A dt + \sigma_A A dz \\ dF &= \mu_F F dt \end{aligned} \tag{2.7}$$

where $d_z = \varepsilon\sqrt{d_t}$ and $\varepsilon \sim N[0,1]$.

In this model default occurs when asset-liability ratio$(k = A/F)$ falls below the default point ($\underline{k} = 1$) at any time within the given period (Sensitivity test to the choice of the default point has been carried out but not reported in this work for the sake of brevity (Merton-model approach to assessing the default risk to UK public companies [21], page 15)). Therefore, to estimate (PD) it is essential to model the movement of k by differentiating k and using (2.7) to obtain:

$$dk = (\mu_A - \mu_F)kdt + \sigma_A kdz \tag{2.8}$$

and define $\mu_A - \mu_F = \mu_k$ and $\sigma_A = \sigma_k$. Equation (2.8) is used to derive probability density function of k to estimate μ_k and σ_k by applying maximum likelihood methods. Then these estimated parameters are used to find the probability of default as follow:

$$
\begin{aligned}
PD &= 1-\{[1-N(u_1)]-\varpi[1-N(u_2)]\} \qquad (2.9)\\
u_1 &= \frac{\underline{K}-(\mu_K-\frac{\sigma_k^2}{2})(T-t)}{\sigma_k\sqrt{T-t}}\\
u_2 &= \frac{-\underline{K}-(\mu_K-\frac{\sigma_k^2}{2})(T-t)}{\sigma_k\sqrt{T-t}}\\
\varpi &= \exp\left[\frac{2\underline{K}(\mu_K-\frac{\sigma_k^2}{2})}{\sigma_k^2}\right]\\
\ln\frac{k}{k_t} &= \underline{K}
\end{aligned}
$$

In (2.9), $N(u_1)$ is equal to (PD) obtained using simple European call option as Merton model [19]. The difference between this work and Merton model [19] is $\varpi[1-N(u_2)]$ which estimates (PD) in case of default before maturity (T).

The equation (2.9) might be simplified, however in this section the literature is being reviewed, so we keep all the formulas as they are presented in the original work.

However, the firm's asset value (A) is not observable and so is the k ratio. What can be observed is market capitalization of firm (S) and hence the equity-liability ratio $(y = S/F)$. There is a link between equity-liability ratio and asset-liability ratio as:

$$y(k) = k - 1 - (\underline{k} - 1)\left(\frac{k}{\underline{k}}\right)^{\lambda} \tag{2.10}$$

$$\lambda = \frac{1}{\sigma_A^2}\left(\frac{\sigma_A^2}{2} - \sqrt{\frac{\sigma_A^4}{4} + 2\sigma_A^2\delta}\right)$$

where δ is a constant dividend rate.

By choosing initial values for k, μ_k and σ_k and applying the Newton-Raphson scheme, (2.10) can be solved to find the estimation for k. The estimated parameter is used then to maximize the probability density function of k and to find estimated μ_k and σ_k.

In order to improve the predictability of the estimated (PD), this model applies the hybrid model. Hybrid model is a model that combines accounting information along with information coming from structural model such as Merton model [19].

To implement hybrid model, the probit model is used which considers company accounting data as regressors. The dependent variable is a dummy variable that is equal to one when the firm goes bankrupt and zero otherwise.

The authors of Bank of England model [21] test their model on the English non financing companies in which there were numbers of bankrupted firms. The calculated (PD) predicts default one year before of occurrence.

Other study that uses barrier option along with hybrid model is the work of Dionne et al [5]. They apply the same model as Bank of England [21] on the Canadian public companies which are traded on Toronto stock exchange. As the model of Bank of England [21], they use fix barrier equal to one. They show that predicted default probability is significant once accounting information is added to structural models. They also show that updating the accounting information semesterly can help to better explain the evolution of credit risk in Canadian market.

2.1.3 Estimation of asset value

Asset value (A) of firms are not observable so different models try to find a good proxy for it.
Although most of models use equity value (S) as a proxy, Brockman and Turtle [5] measure asset value (A) as sum of liability (F) and equity value (S) which is not the best proxy in Wong and Choi [15] opinion.
They argue that Brockman and Turtle [5] overestimates default barrier because in their model [5] default barrier is less than debt value of the firm and is not statically significant. For estimating (A) Wong and Choi [15] use Maximum likelihood model to estimate first and second moments of (A) distribution.
KMV of Moody's [20] is another model that proposes a specific approach to calculate (A) and (PD). This model is challenged by Duan, Gauthier and Simonato [8] which will be explained later. In this approach (A) is calculated by stock value (S) and asset value volatility:

$$S_t = g(A_t, \sigma) \tag{2.11}$$

where σ is the asset value volatility and g is a normal distribution function.
To estimate default probability, the authors define a measure called Distance to default which shows the number of standard deviations that the firm's asset value must drop to reach default point which is somewhere between total liabilities.
Duan, Gauthier and Simonato [8] are among the researchers that develop a model for estimating asset value (A).
They compare the Maximum likelihood model developed by Duan (1994) with the method of Moody''s KMV [20]. Duan is the first researcher who adapts Maximum likelihood method to Merton model. The authors show that theoretically Maximum likelihood model [8] and Moody's KMV [20] method should arrive at the same results but practically Maximum likelihood model [8] surpasses KMV

method [20]. The authors show that for structural models with unknown capital structure parameters, such as Brockman ad Turtle [5], Moody's KMV [20] is limited and is not able to generate suitable estimates.

2.2 Correlation

In this section, before reviewing the literature, the definition of the correlation will be defined. We will also explain the fact that correlation exists and how important it is.

2.2.1 Definition

Correlation (ρ), also called correlation coefficient, expresses the strength and direction of a linear relationship between two random variables. In our case the random variables are the two firms' risky asset values or two sectors. The correlation coefficient measures the direction movement of two variables. It varies between 1 and -1. For two random variables x and y the correlation is calculated as:

$$\rho_{xy} = \frac{cov(x,y)}{\sigma_x \sigma_y} = \frac{E((x-\mu_x)(y-\mu_y))}{\sigma_x \sigma_y} \tag{2.12}$$

where *cov* is the covariance of variables and E is the expected value.

Although some have discussed that correlation is not a good measure of dependency, by far it is the most popular measure of dependency in the financial market and most researchers consider it as a good proxy for dependency.

In our study, we are more interested in default correlation that measures whether risky assets default together or separately or whether default of one obligor affects the other ones.

2.2.2 Evidence of existence and importance of correlation

One might claim that there is no correlation between companies because each has its own specific and unique characteristics that are very different from the other companies. Therefore, any change in credit quality of a company would depend only on the specific events happening for that company. If this would be correct and the firms are uncorrelated then there would be no dependency between firms. Movements in a company would not affect the others and the companies should not be concerned by change in the market.

On the other hand, some might claim that companies are perfectly correlated so if one of them defaults, the many other companies would default as well.

However looking at data and checking history of defaulted and non defaulted firms it is observed that the correlation between firms exists indeed and neither of the aforementioned cases is completely accurate.

As we mentioned earlier, portfolio approach has an important role in modern finance and is used widely by financial institutions. This model proposes how rational investors should diversify their portfolios by calculating the correlation between portfolios' components and managing them in an optimal way to decrease their risk. The reason is that, in credit portfolio having many components does not assure a good diversification, because components may be highly correlated to each other and default of one may lead to default of the rest. The concept is called concentration risk in credit risk management.

Another reason is incremental risk. Incremental risk measures portfolio's risk sensibility to any changes in the portfolio's components. So, correlation indicates movement direction of the portfolio's assets with each other and with economic events.

Another reason for obtaining the correlations is to achieve better allocation of assets in the portfolio. Optimal allocation means to minimize the volatility of portfolio which depends on correlation. Any change in the correlation of portfolio changes the optimal asset allocation.

In the next section we will go through different studies on correlation that exist in the literature.

2.2.3 Structural Model

One of the most popular credit risk model is factor based model which, is an extension of Merton (1974) option theory. The main idea behind this model is to assume that asset value, driving factor, falls below some critical threshold that calls firm's liabilities. In this model, driving default variable has two parts, systematic common component CO and idiosyncratic component ε_j, both are distributed normally with zero mean. ε is uncorrelated with CO and with other firm's idiosyncratic component. Lets consider A_j as the driving default variable for firm j so:

$$A_j = \rho_j CO + \sqrt{1-\rho_j^2}\varepsilon_j \tag{2.13}$$

All the variables are standardised so:

$$Var(CO) = Var(\varepsilon_j) = Var(A_j) = 1 \tag{2.14}$$

for all j and

$$corr(A_i, A_j) = cov(A_i, A_j) = \rho_i \rho_j \equiv \rho_{i,j}, i \neq j \tag{2.15}$$

where $\rho_{i,j}$ is the correlation between A_i and A_j.
In this method, x line is divided to $m+1$ levels $(K_{j,u}, u = 1,2,...,m)$. These levels $(K_{j,u})$ are the same for all the firms in the same class and $K_{j,u+1} > K_{j,u}$. These thresholds (levels) are calibrated in a way that the probability between two levels corresponds to the actual probability that firms will end up in a given class. The worst case is default.

1. **CreditMetrics**

   This work [10] is used Merton (1974) option theoretic study to model firms' asset values. It proposes that companies' asset value can be used to estimate credit rate migration and default. It defines asset value thresholds for each rating class and compares firms' value at the end of each year with these thresholds to determine whether there is any change in credit quality of firms. It proposes that asset returns (R) are normally distributed with mean μ and standard deviation σ.

It establishes a connection between asset thresholds and transition probabilities for each firm.

To calculate joint movement of firms with different credit qualities, first, it assumes that the asset's returns are normally distributed and then uses the thresholds to find how those firms move together. It discusses that calculating correlation for each pair of firms in traditional and standard way is impractical and some times impossible, due to the computational complexity. Therefore, it suggests using industry indices correlation to find correlation between firms by mapping each of them to their sector of activities.

Dionne et al [7] inspired by CreditMetrics model [10], use default probability to estimate correlation between the firms in their data set, which includes 824 Canadian public firms. To check validity and robustness of their model, they divide their sample into two subsamples and calculate correlations again. The correlation for each subsample is found to be really close to the correlation of entire sample. Observations are also divided into two subperiods of the same length and it is concluded that default probability explains the presence of correlation. The thresholds for each class of risk is defined to describe the evolution of credit in their data set. However the risk rating of the firms is not present in the sample, hence they generate their own. They classify the firms based on default probabilities and give them a number from 0 to 9. Class zero represents the firm with the worst credit quality.

2. **Fitch Model**

In this model return on equities is taken as a proxy for asset return. According to Hrvatin and Neugebauer from Fitch Rating [16] this is the only data that is observable and available. In this approach, the Merton model (1974) and Monte Carlo simulation are used together to derive default correlation between two companies from asset correlation. In Fitch model [16], for each firm in the portfolio a random variable is drawn that shows the change in the value of portfolio component. Then the variable is compared to default threshold that is firm's liability value. If the variable is less than the threshold then the firm has defaulted. The degree in which the random variables move together represents correlation. The authors have used factor based model to measure equity return correlation. For that, Fitch grouped industries and companies and calculated an average factor loading for each industry-country class. The correlation for the firms in each class is the same. They have found that correlation within industries (intra) is greater than the correlation between industries (inter). To estimate joint default distribution and correlation matrix, they have used Monte Carlo simulation in conjunction with structural model. To demonstrate the importance of correlation, the authors have calculated joint default distribution with the binomial probability distribution, where the correlation is null and have proved that the correlation increases the default probability. They also have concluded that accurate estimation of correlation is important in estimating all risks both on the asset side and liabilities.

3. **First Passage Models**

The study by Chunsheng Zhou [23] is one of the examples of *First passage models* that also uses asset correlation to estimate default correlation. He uses the correlation between firms asset to calculate default correlation and he believes that firms' asset correlation has very critical role in evaluating default correlation. He uses two approaches to estimate firms' asset values; liability structure of firms and variance-covariance matrix. One from firm-specific information such as stock return and value of liabilities, and the other from statistical approach that is based on historical data and credit ratings. He argues that although the statistical approach is easier than the other approach, the other one is more precise and captures the firm-specific information better. In this study the d ynamics of firm value is described by the stochastic process:

$$dln(A_i) = \mu_i dt + \sigma_i dz_i \tag{2.16}$$

Firm will default when

$$A_i(t) < e^{\lambda_i t} K_i \tag{2.17}$$

where $e^{\lambda_i t} K_i$ is a threshold level.
Then he denotes

$$\tau = min_{t \geq 0} \{ t | e^{-\lambda_i t} A_{i,t} \leq K_i \} \tag{2.18}$$

which is the first time that default would occur.
To find default probability, first he assumes that firm's debt has the same expected growth rate as firm's asset so he fixes $\lambda_i = \mu_i$ and then he relaxes the assumption and uses different μ_i and λ_i.
He finds that the difference between μ_i and λ_i has little effect on default probability and so default correlations with one year or two year horizons.
He shows that the default correlation and asset correlation have the same signs however default correlation is smaller over short time horizons.
He also illustrates the relation between default correlation and time. He shows that the time of default correlation depends very much on the credit quality of the firm. Default correlation is dynamic, due to time-varying nature of the credit quality of the firm.

Another example of *First passage models* is the work of Jean-Pierre Fouque, Brian C. Wignall and Xianwen Zhou [12] who model default correlation under stochastic volatilities. They extend first passage model to model correlation in two directions. First by extending model from uni-dimension to multi-dimensions and second by incorporating stochastic volatilities. They consider n defaultable bonds for which, $\{A_t^{(i)}\}_{i=1}^n$ is firm's asset value process and has multi-factor stochastic volatilities:

$$dX_t^{(i)} = \mu_i X_t^{(i)} dt + f_i(Y_t, Z_t) X_t^{(i)} dW_t^{(i)} \tag{2.19}$$

Stochastic volatilities are driven from two Ornstein-Uhlenbeck (OU) processes. Y_t is fast mean reverting and Z_t is slowly mean reverting.
In first part of their work, they consider uncorrelated Brownian movements and find stochastic volatilities correlation. The reasons are to avoid difficulties caused by the interdependency between Brownian movements and to indicate that dependency between Brownian motions is as important as dependency between stochastic volatilities for estimating default correlation precisely. They defined default time for firm i as:

$$\tau_t^{(i)} = inf\{s \geqslant t, X_t^{(i)} \leqslant K_i(s)\} \tag{2.20}$$

where $K_i(s)$ is a default threshold.
Then they define joint survival probability as if a bond defaults before its maturity. To approximate joint survival probability they use partial differential equations (PDE).
In the second part of their study, they relax the restriction on the Brownian motions, assume a correlated Brownian movements and calculate default correlation and probability. They find that for a single maturity the correlation generated from stochastic volatilities with uncorrelated and correlated

Brownian motions are the same. However if we are looking for the term structure of correlation across several maturities, then interdependency between Brownian motions plays an important role.

4. **Asset Return Correlation**

In the study of Hans Gersbach and Alexander Lipponer [13], asset return correlation is used to calculate default correlation. The interesting point about this study is that they use asset return correlation to estimate default correlation in a loan portfolio which is very rare. Most of the other studies have tried to find default correlation between bonds or obligations portfolio. The authors separate their work in two parts. In first part, they present the relation between asset and default correlation and their characteristics and behaviors. In second part, they simulate loan portfolio by Monte Carlo and derive the expected losses and standard deviation of losses. Then they examined how macroeconomics risks affect portfolio diversification and default correlation. They are inspired by Merton model (1974) and assume that firms' asset value are log normally distributed. In other words, the asset returns are normally distributed. They use bivariate normal distribution of firms' asset return to calculate default correlation (ρ_d) , and find upper and lower bound for ρ_d, based on asset correlation (ρ_a) . They also establish a limit for ρ_d and conclude that "If firm has a small default probability, its default correlation with other firms can be approximated by zero," [Gersbach and Lipponer (2000), page 7]. Moreover, they conclude that ρ_a has as important role as default probability in estimating ρ_d. In the event of macroeconomics shocks, the positive relationship between default probability and default correlation has important implications on the behavior of credit risk. To better capture the impact of changing default correlation in the macroeconomics events they fixed the asset correlation. They concluded that when default correlation is high, diversification is rapidly exhausted, and macroeconomics shock causes positive default correlation.

Arnaud de Servigny and Olivier Renault [6] use the asset return correlation to estimate default correlation too, but they use asset return correlation in the portfolio of obligations.

Their work [6] is based on studies of Lucas (1995) and Bahar and Nagpal (2001). In order to calculate joint probabilities Lucas (1995) and Bahar and Nagpal (2001) assume

$$\frac{\text{number of pairs migrating}}{\text{total number of pairs}} = \frac{T_{i,k}(T_{i,k}-1)}{N_i(N_i-1)} \tag{2.21}$$

where N_i is number of elements in group and $T_{i,k}$ is number of migrated bonds to category k. However, Servigny and Renault [6] modify that formula to $\frac{T_{i,k}^2}{N_i^2}$ to prevent negative correlation when defaults are rare.

Factor-based model is applied then to estimate default correlation. Despite, most of studies in factor-based model that use normal distribution, Servigny and Renault [6] use t-distribution. The reason is to find whether a distribution with a fatter tail can catch better default correlation. Their study shows that in almost fifty percent of times t-distribution converges to normal distribution.

These authors [6] then examine the effect of horizon on the default correlation and show that the correlation increases with time.

They also test the effects of business cycle on the default correlation. To do so behavior of joint default probability is studied and results show an increase in probability. They argue that increase can be derived either by an increase in the marginal probabilities of default (univariate) or by increase in correlation (bivariate) or both. To investigate which factor has the most impact, they compute credit value at risk with different confidence level and separate the impact of each factor by performing 50000 Monte Carlo simulations. They find that default probability has the most impact during recession period and its affects mostly the centre of the distribution, however correlation affects mostly the tails of distribution.

5. **Copula**

Copula approach to calculate default correlation is an approach that is used widely in recent years. "A copula function is a function that links univariate marginal to their full multivariate distribution. For m uniform random variables, $U_1, U_2, ..., U_m$, the joint distribution function C_p, defined as:

$$C_p(u_1, u_2, ..., u_m, \rho) = Pr[U_1 \leqslant u_1, U_2 \leqslant u_2, ..., U_m \leqslant u_m] \tag{2.22}$$

which can also be called copula function", (Li (2000), page 12). Sklar (1954) show that any multivariate distribution function DF can be written in the form of copula function and if DF is continuous then copula function is unique. There are different copula functions. The most popular ones are bivariate, meta-elliptics and archimedean families. Meta-elliptics family contains normal and student distribution functions.
Li [17] is one of authors who assesses default correlation by using copula approach. In his paper he first introduces a random variable called time-until-default, which measures the length of time from today until default time, to indicate the survival time of each defaultable entity. Then he defines the default correlation between two entities by correlation between their survival times. He uses market spread information to achieve his goal. To obtain default probability, he constructs the credit curve with survival function and a hazard function. Survival function is the probability that a security reaches age t. He argues that modeling default process is equal to modeling hazard function and uses it to characterize the distribution of survival time; which is called credit curve. Hence, for calculating joint default probabilities, he specifies a joint probability for survival times in a way that the marginal distributions are credit curve and to achieve that he uses a bivariate copula function for $\rho > 0$ and $\rho \leqslant 0$. He discusses that

for estimating ρ a correlation measurement that is independent of marginal distribution is required, therefore Spearman's Rho and Kendall's Tau are used. He also demonstrates that CreditMetrics approach uses bivariate normal distribution though it does not mention it directly. Therefore, he uses the CreditMetrics model to generate survival times. Moreover, he indicates that default correlation increases with time.

2.2.4 Reduced Form Model

As was discussed, most models of default correlation are based on Merton structural approach however there are some others based on other approaches such as reduced-form approach. According to reduced-form models, multiple defaults are independent. Therefore, finding the default correlation is equal to finding the correlation between default intensities. Although some researchers such as Fan Yu [22] have worked on this model to show that default correlation can be sensitive to default intensities, some authors such as Hull and White (2001) and Schonbucher and Schubert (2001) argue that the ability of this approach to estimate default correlation is limited.

According to Yu, the general procedure to calculate default correlation in the reduced-form model is based on existence of two stopping times with two physical intensities. The stopping times are defined as the first time that intensities are greater than a unit exponential random variable and default correlation is defined as correlation between possible stopping times. In his paper, he compares different models from existing empirical studies based on reduced-form approach and concludes that the problem with reduced-form approach is resulted by an insufficient specification of factor structure of the default intensity. In fact if factor structure of intensity could be improved, the reduced-form approach would improve a lot and provides a good approximation of default correlation.

Chapter 3

Methodology

In this section we will describe models that are used in this work.

3.1 Application of the Model of Brockman and Turtle on Canadian public firms

J. Amar study [2] estimates the default probability by applying maximum likelihood model of Duan, Simonato and Gauthier [8] on model of Brockman and Turtle [5]. The Brockman and Turtle's work [5] is the extension model of Merton [19]. As mentioned in literature review, in Merton's model [19], the standard European options are used for calculating default probability for a firm. In his model [19] the time of default is the maturity of European options. The standard European options are not path dependent, they are always alive before their maturity without considering appreciation or depreciation in the value of underlying assets. So choosing an European option for estimating default probability of a firm is not a very good choice because firm's default probability depends on the value of the firm and the path it follows. This is one of the weakness of Merton model[19].

By using barrier options instead of European options, Brockman and Turtle [5]. improve this weakness. Barrier options are path dependent options and they expire as soon as the value of underlying assets reach a certain threshold. Brockman and Turtle [5] use two kinds of barrier options: Down and In $(D\&I)$ and Down and Out $(D\&O)$. According to them shareholders hold a $(D\&O)$ call option on the value of the firm and debtors hold a portfolio of risk free debt, short position on a put and long position on a $(D\&I)$ call options on the value of the firm. When a firm defaults its value falls under a certain level, so the $(D\&O)$ option of shareholders will expire and $(D\&I)$ option of the debtors will activate. This gives right to debtors to put the firm in default. Ignoring the existence of such a threshold, the difference between Merton [19] and Brockman and Turtle [5] models is the amount equal to the value of $(D\&I)$ option. According to Brockman and Turtle [5] the value of equity and the value of $(D\&I)$ option are as follow:

1. Equity value of firm (S) :

$$
\begin{aligned}
S(A,\tau) &= A\Phi(a) - Fe^{-r\tau}\Phi(a-\sigma\sqrt{\tau}) - A\left(\frac{H}{A}\right)^{2\eta}\Phi(b) \\
&\quad + Fe^{-r\tau}\left(\frac{H}{A}\right)^{2\eta-2}\Phi(b-\sigma\sqrt{\tau}) \qquad (3.1)\\
a &= \begin{cases} \frac{\ln(A/F)+(r+(\sigma^2/2))\tau}{\sigma\sqrt{\tau}} & F \geq H \\ \frac{\ln(A/H)+(r+(\sigma^2/2))\tau}{\sigma\sqrt{\tau}} & F < H \end{cases} \\
b &= \begin{cases} \frac{\ln(H/AF)+(r+(\sigma^2/2))\tau}{\sigma\sqrt{\tau}} & F \geq H \\ \frac{\ln(H/A)+(r+(\sigma^2/2))\tau}{\sigma\sqrt{\tau}} & F < H \end{cases} \\
\eta &\equiv \frac{r}{\sigma^2} + \frac{1}{2}
\end{aligned}
$$

where A is the asset value of firm, F presents promised future debt payment at maturity T, H is the default barrier, τ shows the remaining time till maturity of debt, Φ is the cumulative normal distribution and finally r, is the risk free interest rate.

In equation (3.1), a represents the expected firm's value, and b calculates the rebate value received by share holders in case of default before maturity. As explained in this work, H could be set above, below or equal to F. For example for low quality borrower, lender could set H above F. In this case, H is more likely to induce some punitive action such as loan recall rather than force bankruptcy.

2. Down and In $(D\&I)$ option value:

   Option value = Equity value of Merton (standard European option) - Equity value of Brockman and Turtle (barrier option)

   when $\quad F \geq H$

$$A\Big(H/A\Big)^{2\eta}\Phi(b) - Fe^{-r\tau}\Big(H/A\Big)^{2\eta-2}\Phi(b-\sigma\sqrt{\tau}) \tag{3.2}$$

   when $\quad F < H$

$$\begin{aligned} &A\Phi\left(\frac{\ln(A/F)+(r+(\sigma^2/2))\tau}{\sigma\sqrt{\tau}}\right) \\ &-Fe^{-r\tau}\Phi\left(\frac{\ln(A/F)+(r-(\sigma^2/2))\tau}{\sigma\sqrt{\tau}} - A\Phi(a) + Fe^{-r\tau}\Phi(a-\sigma\sqrt{\tau})\right) \\ &+A\Big(H/A\Big)^{2\eta}\Phi(b) - Fe^{-r\tau}\Big(H/A\Big)^{2\eta-2}\Phi(b-\sigma\sqrt{\tau}) \end{aligned} \tag{3.3}$$

When $(H = 0)$ the equations of Brockman and Turtle [5] and Merton [19] are equivalent. Therefore when $(H = 0)$ the Merton [19] model can be used to estimate default probability. However, when $(H \geq 0)$ the Brockman and Turtle [5] model gives more accurate result. Once (H) is estimated the default probability of firm over $[0,T]$ interval is:

$$
\begin{aligned}
PD = \Phi&\left(\frac{(\ln(H)-\ln(A))-(r-(\sigma^2/2))\tau}{\sigma\sqrt{\tau}}\right) \\
&+\exp\left(\frac{2(r-(\sigma^2/2))(\ln(H)-\ln(A))}{\sigma^2}\right) \\
&*\left[1-\Phi\left(\frac{-(\ln(H)-\ln(A))-(r-(\sigma^2/2))\tau}{\sigma\sqrt{\tau}}\right)\right]
\end{aligned} \tag{3.4}
$$

This equation represents the risk neutral probability that the firm's asset value falls below the barrier during the specific period.

As mentioned before the asset value is not observable so to solve this problem, Amar [2] uses the Maximum likelihood method (MLH) of Duan, Gauthier and Simonato [8].

In their model, Duan et al [8] present first the MLH for Merton model [2] and then for Brockman and Turtle model [5]. As mentioned before in Merton model [2], asset value (A) follows Geometric Brownian Motion (GBM):

$$
d\ln(A_t) = \left[\mu - \frac{\sigma^2}{2}\right] d_t + \sigma d_z \tag{3.5}
$$

in which μ represents expected return and σ represents standard deviation of the firm's asset value.

Supposing firm's asset value is observable, Black and Scholes (1973) can be used to derive the MLH of Merton model (1974):

$$
\begin{aligned}
L^a(\mu,\sigma;A_0,A_h,A_{2h},...,A_{nh}) = &-\frac{n}{2}\ln(2\pi\sigma^2 h) \\
&-\frac{1}{2}\sum_{k=1}^{n}\frac{\left(R_k-\left(\mu-\frac{\sigma^2}{2}\right)h\right)^2}{\sigma^2 h}-\sum_{k=1}^{n}\ln A_{kh}
\end{aligned} \tag{3.6}
$$

where $R_k = \ln\left(\frac{A_{kh}}{A_{(k-1)h}}\right)$ and h represent one business day.

This *MLH* model supposes that there is not default during the period in question. Although it is possible to use this equation to estimate μ and σ, Amar [2] maximizes the MLH method of Brockman and Turtle to estimate these parameters along with H.
Because of lack of observability of the firm's asset value, Duan et al [8] take an observable information, market capitalization of the firm (S), to infer the *MLH* of Merton model [19]:

$$\begin{aligned} L^s(\mu,\sigma;S_0,S_h,S_{2h},\dots,S_{nh}) = {} & L^a(\mu,\sigma;\hat{A}_0,\hat{A}_h,\hat{A}_{2h},\dots,\hat{A}_{nh}) \\ & - \sum_{k=1}^{n} \ln(\Phi(\hat{d}_{kh}(\sigma))) \end{aligned} \tag{3.7}$$

where $\hat{A}_{kh}(\sigma) = g^{-1}(S_{kh};\sigma)$ and $\hat{d}_{kh}(\sigma) = \frac{\ln\left(\frac{\hat{A}_{kh}(\sigma)}{F}\right)+\left(r+\frac{\sigma^2}{2}\right)(T-kh)}{\sigma\sqrt{T-kh}}$.

Finally they propose the MLH method of Brockman and Turtle, using the MLH method of the Merton and market capitalization of the firm as:

$$
\begin{aligned}
L^s_{BT}(\mu,\sigma,H;S_0,S_h,S_{2h},...,S_{nh}) = {} & L^a(\mu,\sigma;\hat{A}_0,\hat{A}_h,\hat{A}_{2h},...,\hat{A}_{nh}) \\
& + \sum_{j=1}^{n} \ln\left(1-\exp\left(-\frac{2}{\sigma^2 h}\ln\frac{\hat{A}_{(j-1)h}}{H}\ln\hat{A}_{jh}H\right)\right. \\
& - \ln\left[\Phi\left(\frac{(\mu-\frac{\sigma^2}{2}nh-\ln\frac{H}{\hat{A}_0}}{\sqrt{nh}\sigma}\right)\right. \\
& \left. - \exp^{\left(\frac{2}{\sigma^2}\right)\left(\mu-\frac{\sigma^2}{2}\right)\ln\frac{H}{\hat{A}_0}}\Phi\left(\frac{(\mu-\frac{\sigma^2}{2}nh+\ln\frac{H}{\hat{A}_0}}{\sqrt{nh}\sigma}\right)\right] \\
& - \sum_{j=1}^{n}\ln\left|\frac{\partial g(\hat{A}_{jh}(\sigma,H);\sigma,H)}{\partial\hat{A}_{jh}}\right|
\end{aligned}
\tag{3.8}
$$

In order to estimate default probabilities, Amar [2] first initializes value to μ,σ and H and then uses “fzero” function of matlab to find the firm’s asset value that cancels the difference between (3.1) and market capitalization as follow:

$$
\begin{aligned}
& S-\Big(A\Phi(a)-Fe^{-r\tau}\Phi(a-\sigma\sqrt{\tau})-A\left(\frac{H}{A}\right)^{2\eta}\Phi(b) \\
& +Fe^{-r\tau}\left(\frac{H}{A}\right)^{2\eta-2}\Phi(b-\sigma\sqrt{\tau})\Big)
\end{aligned}
\tag{3.9}
$$

Once A is estimated, the parameters (μ,σ,H) and A are inserted into the *MLH* of Brockman and Turtle to find the best and optimal values for the parameters. Then these parameters are used to calculate default probability of the firm. Although in work of Amar [2] there is no indication about convergence of the *MLH* of Brockman and Turtle, it is possible that *MLH* of Brockman and Turtle does not converge all the time and for all the companies. However, the same methodology is used in this work for estimating default probability and default barrier. When the default barrier is very small the Merton model [19] is used for estimating default probability instead of Brockman and Turtle model [5].

3.2 CreditMetrics Model

The CreditMetrices model [14] is an approach that uses the option theoretic model of Merton (1974) to model a firm's asset value. In this approach a firm's asset value is a stochastic process and the firm's ability to repay its payment obligation depends on its asset value. If the asset value falls below a certain level (default threshold), the firm can not repay its debt and will default as illustrated in figure 3.1.

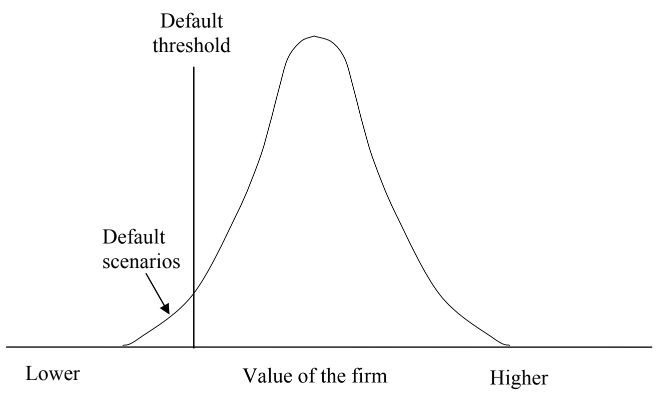


Figure 3.1: Structural Model of the Firm (Source : CreditMetrics - Technical document)

CreditMetrics [14] expands Merton model (1974) to evaluate all credit rating migration in firms. It defines different thresholds which determine the credit rating of firms at the end of specific period. Firms' asset values relative to these thresholds determine their future rating as represented in figure 3.2.

It is possible to establish a link between firm's asset value and these thresholds to model the changes in the asset value and to describe the firm's credit rating evolution. To do so, CreditMetrics [14] assumes that the asset return is normally distributed with mean μ and standard deviation σ. It then establishes a connection between asset's thresholds and the transition probabilities of firms.

In this work, we are only interested in the default correlation, so we only calculate

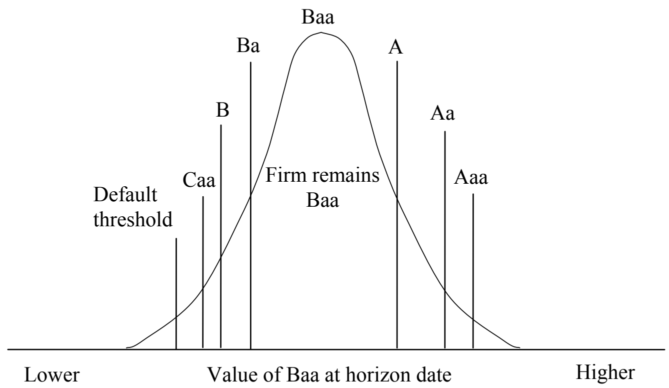


Figure 3.2: Structural model of the Firm account for changes in credit ratings (Source : CreditMetrics - Technical document)

the default probability and default threshold (H). Also weekly equity return is used as proxy for the asset return for the same reason explained in the models above that asset values and returns are not observable. We consider a firm defaults when it goes bankrupt or reorganizes itself.
According to CreditMetrics model [14] for each classes of risk, defined by rating agencies, there is a default threshold, below which, the firm defaults.
In other words,

$$PD_i = Pr\{R < H_0\} = \Phi(\frac{H_0 - \mu}{\sigma}) \Longrightarrow H_0 = \Phi^{-1}(PD_i)\sigma + \mu \tag{3.10}$$

where Φ is the cumulative standard normal distribution, H_0 is a default threshold and PD_i is default probability of firm i.

The default probabilities and default thresholds can be obtained from migration matrix of rating agencies such as Moody's annual migration matrix (Table 3.3).

CreditMetrics [14] assumes the same default probability and default threshold for all companies in the same rating class. For example, according to this model

Initial Rating	Final rating							
	Aaa	**Aa**	**A**	**Baa**	**Ba**	**B**	**Caa**	**Default**
Aaa	93.40	5.94	0.64	0	0.02	0	0	0
Aa	1.61	90.55	7.46	0.26	0.09	0.01	0	0.02
A	0.07	2.28	92.44	4.63	0.45	0.12	0.01	0
Baa	0.05	0.26	5.51	88.48	4.76	0.71	0.08	0.15
Ba	0.02	0.05	0.42	5.16	86.91	5.91	0.24	1.29
B	0	0.04	0.13	0.54	6.35	84.22	1.91	6.81
Caa	0	0	0	0.62	2.05	4.08	69.20	24.06

Figure 3.3: Annual Migration Matrix (Source : Estimation of the default risk of publicly traded Canadian companies [7])

[14] all the BBB rated firms have the same default probability, without considering their idiosyncratic characteristics. We believe this assumption is the simplified assumption and is one of the weakness of this model. Because there are a lot of factors that force a company to default which are different from a company to the other ones. Some of these factors depend on specific characteristic of the companies. So, that leads us to estimate default probability and default barrier for each company in our database. Moreover risk rating is not available for all the Canadian firms in our database.

So far, we are able to detect a movement in the credit rating of each firm but we need joint probability between firms as well in order to be able to calculate default correlation between them. To capture joint movement of two firms, CreditMetrics [14] estimates joint default probability of them. It is assumed that both firms are correlated and normally distributed (CreditMetrcis [14], chapter 8, page 89). To estimate joint default probability, the covariance matrix is required. The covariance matrix of bivariate normal distribution is as follow:

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho_a \sigma_1 \sigma_2 \\ \rho_a \sigma_1 \sigma_2 & \sigma_2^2 \end{pmatrix}$$

where σ_1 is the standard deviation of firm 1 asset return, σ_2 is the standard deviation of firm 2 asset return and ρ_a is the asset correlation.
The joint probability of two firms default simultaneously is:

$$PD_{12} = Pr\{R_1 \leq H_{1,0}, R_2 \leq H_{2,0}\} = \int_{-\infty}^{H_{1,0}} \int_{-\infty}^{H_{2,0}} f(r_1, r_2, \Sigma) dr_1 dr_2 \qquad (3.11)$$

where $f(r_1, r_2, \Sigma)$ is the density function of the bivariate normal distribution.
However, for calculating the joint probability of two firms, we need to know the correlation between the asset returns of them. The reason is if the asset returns of the firms are independent ($\rho = 0$), the joint default probability is the product of the each firm's default probability. For example, the joint default probability for a firm rated BB and a firm rated A and uncorrelated is 0.006. On the other hand if the asset returns are perfectly correlated, the joint default probability of them is the default probability of one of them. For example, for the pair of firms mentioned above the joint default probability is 0.06 (the default probability of the firm rated A) which is 100 times greater than in the uncorrelated case (CreditMetrics [14], chapter 8, page 90). In Chart (3.4) the effect of correlation on the joint default probability is illustrated.

As illustrated in the example for calculating joint default probability, asset correlation is required. On way of calculating asset correlation is to take each pair of firms and find the correlation between them, however because of amount of calculation, it seems unrealistic to use this method for a large portfolio, the case in this study. For example for a portfolio of 100 firms, 4950 correlations have to be calculated. Thus the correlation for each pair of firms will be computed from the correlation between Canadian sectors' returns, which is the equity returns on the Canadian indices.

As mentioned before, we are only interested in the default probability and default correlation of the firms in our study. Once both individual and joint default probability are calculated we can estimate default correlation as:

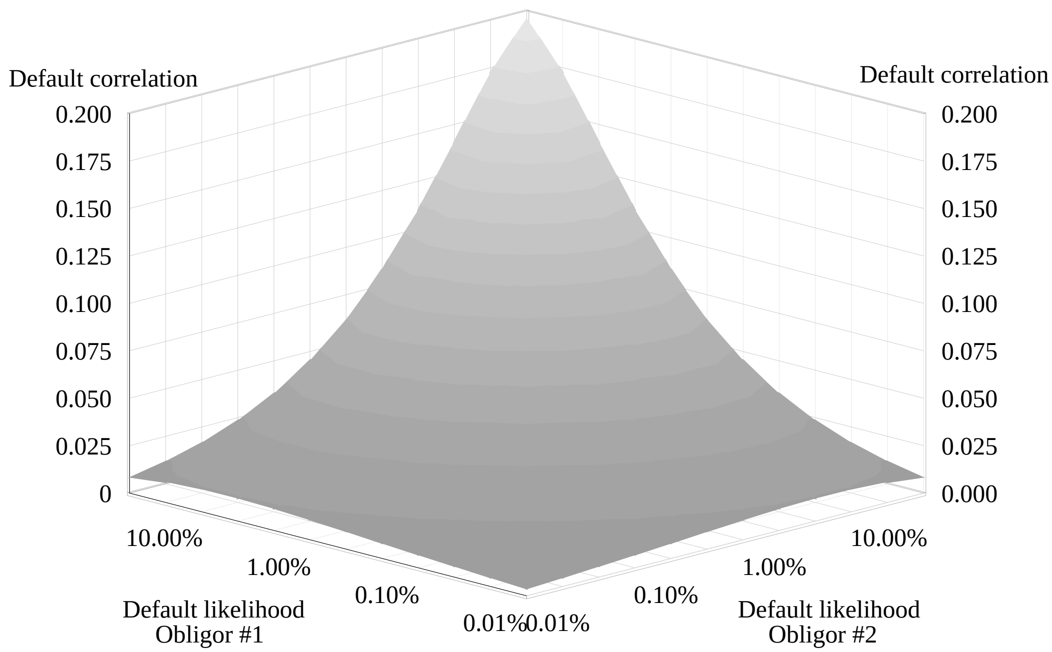


Figure 3.4: Translation of Equity Correlation to Default Correlation (Source : Credit-Metrics - Technical document[14])

$$\rho_d = \frac{PD_{12} - PD_1 PD_2}{\sqrt{PD_1(1-PD_1)PD_2(1-PD_2)}} \tag{3.12}$$

As explained above, the correlations between Canadian sectors returns are used to calculate correlation between firms. So, the firms in the sample are re-grouped into ten sectors of financial and non financial sectors. For each sector and each year, we use 190 weekly standardized equity returns ($\mu = 0$ and $\sigma = 1$) and then calculate the equity return correlation between sectors. Note that the correlation we compute is based on the historical returns because CreditMetrics [14] assumes that the weekly correlations are good reflections of the quarterly or yearly asset movements. These correlations between sectors will be used to calculate the correlation between two specific firms as follow:

- First estimate participation weights of each firm in its sector of activity (in this work, only one sector of activity is considered for each firm). It should be found how much of firm's return volatility is explained by the movement in its sector of activity. As explained in Risk Management and Analysis [1], standardized return of each firm is its weighted sum of the return on its sector of activity and its specific components. So:

$$\begin{aligned} & r_f = w_1 * r_m + w_2 * r_s \\ \Rightarrow \quad & \sigma_f^2 = w_1^2 * \sigma_m^2 + w_2^2 * \sigma_s^2 \end{aligned} \tag{3.13}$$

and

$$w_1^2 + w_2^2 = 1 \tag{3.14}$$

Suppose

$$w_2^2 * \sigma_s^2 = \varepsilon \tag{3.15}$$

then,

$$\sigma_f^2 = w_1^2 * \sigma_m^2 + \varepsilon \tag{3.16}$$

where r_f is company's return, r_m is the sector's return, r_s is the specific return, w_1 shows the sensitivity of firm's return movement to market movement and w_2 presents the sensitivity of firm's return movement to its specific events.
In equation (3.14) it is assumed that σ_m^2 and σ_s^2 are independent.
We run a regression on the above formula to estimate w_1. To calculate w_1, 190 weekly returns are taken from both sectors and companies (in each year there is only one w_1 and only one w_2 for each company). Once the amount of equity price movement of firm that is explained by sector is defined, (3.14) is used to get w_2. w_2 is the percentage of firm's equity price movement that is explained by firm's specific factors.

- Secondly compute the correlation between firms by using sectors' correlations along with the weights of firms that are calculated in first step.

$$\rho(A,B) = w_1 * w_2 * \rho(S_1, S_2) \tag{3.17}$$

where A and B are the companies, w_1 and w_2 are the participation weight of each firm in their sectors and finally S_1 and S_2 are the sectors.

Now that we get familiar with necessary concepts we can expand the idea and compute the correlation between numbers of firms in a portfolio. It is supposed that there are n different firms and m different sectors, and it is desired to compute the correlation between all the firms. Let's name the sectors' correlation matrix as C and $\bar{C}$ as the correlation matrix which includes sectors' and idiosyncratic correlations. This matrix is $m+n$ by $m+n$ and can be represented as below:

$$
\bar{\mathbf{C}} = \left(\begin{array}{c|c}
\mathbf{C} & \begin{pmatrix} 0 & \dots & \dots & 0 \\ \vdots & \dots & \dots & \vdots \\ \vdots & \dots & \dots & \vdots \\ \vdots & \dots & \dots & \vdots \\ 0 & \dots & \dots & 0 \end{pmatrix} \\
\hline
\begin{pmatrix} 0 & \dots & \dots & 0 \\ \vdots & \dots & \dots & \vdots \\ \vdots & \dots & \dots & \vdots \\ \vdots & \dots & \dots & \vdots \\ 0 & \dots & \dots & 0 \end{pmatrix} & \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & \ddots & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \dots & \dots & 1 \end{pmatrix}
\end{array}\right)
$$

The upper left block of the matrix $\bar{C}$ is the matrix C which is the sectors' correlation matrix and the lower right is the identity matrix that shows each firm's idiosyncratic component, which is the correlation of each firm with itself. It is perfectly correlated to itself and has the correlation of one. It is perfectly uncorrelated and independent to the others and has the correlation zero. The other parts are zero that means there is no correlations between idiosyncratic component and the sectors.

The weight matrix W is $m+n$ by n matrix that each column represents a different firm and each row shows weights on the sectors and idiosyncratic components. In the k^{th} column, the first m entries represents the sectors' weights, the $m+n+k$ represents the idiosyncratic component and the rest are zero.

The n by n matrix of firms' correlation is obtained by $W^T\bar{C}W$.

In this study the firms' correlation matrices are calculated for each year (from 1992 to 2004) by using 190 weekly returns.

Chapter 4

Database

In this chapter the raw data and their sources will be presented. We will also explain the way that database is structured.

We use the same database as is used in work of Amar [2] and Dionne et al [5] which was presented to the Bank of Canada.

4.1 Companies

The data includes defaulted and non-defaulted companies. The study period is between January 1998 and December 2004. The original database included 1469 companies among which 130 companies were defaulted.

For each company, market data, market price and market capitalization, is available on daily basis, however accounting data, debt value, is available only yearly.

The information on the defaulted companies are listed in *Financial Post Predecessors & Defunct, Cancorp Financials (Corporate Retriver) and Stock Guide*.

The market data are drawn from *DataStream DEAD.LLT* series. The accounting data are extracted from *Stock Guide and CanCorp Financials*.
During study period 130 companies were defaulted; 112 were bankrupted and 18 were restructured. There are 436 dynamic observations for accounting data, 378 for companies that have defaulted and 55 for the restructured companies. That means in average there are 3.4 accounting observations for each company.
For two reasons some of defaulted companies are eliminated:
First, missing data, accounting or market data, either for estimating default probabilities or for estimating default correlations. For calculating yearly default correlation 190 consecutively weekly equity returns information are necessary and for estimating yearly default probability, two hundreds consecutive daily market data with two accounting data (financial statement) are needed. This required history is not available for all the companies. Also should be mentioned that financial statements are available only four months after the end of previous fiscal year. In this database only available data to investors at time of calculation are used.
Second, big gap between two last financial statements. One reason for this gap is that most of companies do not publish their financial statement one year before bankruptcy. Another reason is time consuming bankruptcy process for some companies. For these reasons, Dionne et al [5] eliminated all the companies that have a gap greater than 18 months.
Moreover, for the companies that bankrupted 12 to 18 months after the last financial statement, Dionne et al [5] move the default date to the date of last financial statement to reconcile it with last observable accounting data (year).

The daily market data for the companies that did not default are extracted from *DataStream FTORO.LLT* series. Accounting data comes from *Stock Guide*.

The same process of clean-up and merging is done for non-defaulted companies as for defaulted ones.
Tables (4.1) and (4.2) present descriptive statistics of final database, which in-

cludes the number of companies in each year and its proportion to original database (1469 firms).

	Year	1992	1993	1994	1995	1996	1997
	Companies' number	169	208	226	255	295	356
	Proportion to original database	12%	14%	15%	17%	20%	24%
Market Capitalisation	**Mean**	756,279	738,117	773,839	787,502	893,195	1,018,828
	Median	54,800	63,060	62,920	65,030	69,490	73,950
	Mode	46,180	2,210	2,210	4,970	34,330	15,570
	Standard Deviation	1,996,996	1,938,478	2,038,500	2,115,424	2,503,670	3,221,509
	Kurtosis	20	18	17	20	26	45
	Skewness	4	4	4	4	5	6
	Number of observations	44,278	54,549	58,760	66,301	77,272	92,916
Debt	**Mean**	3,662,637	3,239,523	3,299,834	3,227,497	3,152,862	3,110,370
	Median	60,298	50,483	42,070	37,051	33,777	32,656
	Mode	1,350,000	-	-	-	-	104,000
	Standard Deviation	16,400,875	16,044,397	17,422,356	18,204,466	19,271,606	20,319,113
	Kurtosis	36	44	52	58	70	83
	Skewness	6	7	7	7	8	9
	Number of observations	44,278	54,549	58,760	66,301	77,272	92,916

Figure 4.1: Firms Yearly Descriptive Statistics (1992 -1997)

4.2 Sector

The information on companies' sector of activity are drawn from *Stock Guide*. However for some companies such as defaulted companies there is no information on the *Stock Guide* so information is extracted from *Sedar.com* for those companies. It left few companies for which there is no information on the *Sedar.com*. Therefore required information are extracted from internet; for example the com-

	Year	1998	1999	2000	2001	2002	2003	2004
	Companies' number	384	428	501	546	602	600	229
	Proportion to original database	26%	29%	34%	37%	41%	41%	16%
Market Capitalisation	**Mean**	1,121,299	1,168,188	1,478,211	1,117,790	1,022,159	1,170,059	989,571
	Median	60,370	48,350	61,790	57,335	68,050	81,310	57,020
	Mode	105,000	9,020	9,020	107,000	103,000	127,000	5,060
	Standard Deviation	3,962,535	5,973,856	12,141,122	4,787,341	3,500,811	3,885,083	4,550,002
	Kurtosis	62	338	484	373	45	40	53
	Skewness	7	15	21	14	6	6	7
	Number of observations	100,215	111,485	130,035	142,506	157,122	156,493	59,770
Debt	**Mean**	3,291,361	3,261,374	2,909,847	2,916,349	2,948,539	3,371,707	5,726,755
	Median	29,433	32,073	33,971	33,971	37,768	41,843	20,872
	Mode	-	-	-	110,000	-	151,000	419,000
	Standard Deviation	22,158,915	23,898,699	21,273,346	21,847,182	23,407,086	24,776,242	39,084,915
	Kurtosis	86	100	112	122	146	131	62
	Skewness	9	10	10	11	12	11	8
	Number of observations	100,215	111,485	130,035	142,506	157,122	156,493	59,770

Figure 4.2: Firms Yearly Descriptive Statistics (1998 - 2004)

pany's own website or governmental websites that provide information on the companies.
In this study *North American Industry Classification System (NAICS)* is chosen to find companies' sector of activity rather than *Standard Industrial Code (SIC)* because, after November 2004, standard changed from the use of *SIC* codes to *NAICS* codes. Also, NAICS codes give greater level of detail about companies' activity, up to 6 digits instead of maximum 4 digits in SIC codes.
Moreover, NAICS codes are based on economic concept and they are developed by United Statess, Canada and Mexico.
However for some companies only *SIC* codes are available, so they are all converted to *NAICS*. To do so, matching table between these codes from *U.S. Census Bureau Correspondence Table: 2002 NAICS Matched to 1987 SIC* is used. NAICS or SIC codes usually provide the companies' primary business activity, however for some companies we find more than one SIC and NAICS codes, which means they are active in several sectors. In that case the code that describes greater part of company's activity is taken because in this work only one sector of activity is considered for each company. The reason for that is the limitation of our database and also complexity of calculation. To find the correspondence table and get more information about *SIC* and *NAICS*, one can consult following addresses: *http://www.naics.com/search.htm* and *http://www.census.gov/epcd/naics02/ N2SIC11.HTM*.

After finding companies' industry codes and their sector of activity, they were regrouped using TSX Stock Guide classification. In this classification there are 10 sectors (Material, Energy, Financial, Information Technology, Utility, Telecommunication service, Consumer Discretionary, Consumer Staple, Industrial, Healthcare). In Appendix the TSX table of classification [11] is presented.

The weekly market values for sectors are drawn from *DataStream, SPTSX* series and their weekly returns are calculated. Below the statistical data of sectors'

Information Technology	
Mean	349.89
Median	211.08
Mode	68.22
Standard Deviation	399.23
Kurtosis	7.69786
Skewness	2.73197

(a) Information Technology

Industrial	
Mean	637.55
Median	566.30
Mode	350.45
Standard Deviation	244.76
Kurtosis	(1.12640)
Skewness	0.41639

(b) Industrial

Figure 4.3: Sectors Descriptive Statistics (1)

HealthCare	
Mean	630.20
Median	663.92
Mode	270.74
Standard Deviation	282.25
Kurtosis	(1.30119)
Skewness	0.14094

(a) HealthCare

Energy	
Mean	715.59
Median	561.23
Mode	392.73
Standard Deviation	378.04
Kurtosis	0.93750
Skewness	1.28043

(b) Energy

Figure 4.4: Sectors Descriptive Statistics (2)

market values are presented.

4.3 Risk free interest rate

One of the inputs for the model that is used to estimate risk neutral default probability is risk free interest rate. The Canadian treasury bond rate is used as the risk free interest rate that are extracted from *International Financial Statistics (IMF)*. The rate is defined as weighted average of income on acquisition of bond with maturity of 3 months. Below rate evolution during study period is shown [2].

Consumer Staple	
Mean	602.03
Median	309.25
Mode	244.14
Standard Deviation	471.91
Kurtosis	(0.54914)
Skewness	0.97285

(a) Consumer Staple

Financial	
Mean	556.57
Median	340.31
Mode	258.07
Standard Deviation	392.79
Kurtosis	(0.77245)
Skewness	0.69381

(b) Financial

Figure 4.5: Sectors Descriptive Statistics (3)

Consumer Discretionary	
Mean	666.23
Median	557.37
Mode	407.51
Standard Deviation	262.62
Kurtosis	(1.48669)
Skewness	0.25003

(a) Consumer Discretionary

Telecommunication	
Mean	357.42
Median	200.53
Mode	101.72
Standard Deviation	279.71
Kurtosis	(0.67471)
Skewness	0.80768

(b) Telecommunication

Figure 4.6: Sectors Descriptive Statistics (4)

Utility	
Mean	702.81
Median	574.55
Mode	320.18
Standard Deviation	360.25
Kurtosis	(0.68148)
Skewness	0.72885

(a) Utility

Material	
Mean	1,061.05
Median	1,043.20
Mode	819.13
Standard Deviation	250.02
Kurtosis	(1.03448)
Skewness	0.26205

(b) Material

Figure 4.7: Sectors Descriptive Statistics (5)

Statistics	Risk free interest rate
Mean	0.0598
Standard Deviation	0.0289
Minimum	0.022
Median	0.0534
Maximum	0.1205

Figure 4.8: Risk Free Interest Rate Descriptive Statistics(1998 - 2004)

Chapter 5

Results

In this chapter we present the results along with the problems that we faced during estimating process.
The results are presented in following order. First, we will go through the estimated results for default probability (PD), default barrier (H) and participation weight of firms in their sector of activity (w_1). Then the estimated default correlation (ρ_d) will be analyzed.
To do that, a year will be chosen and (ρ_d) of firms in that year will be studied in details. Then the evolution of (ρ_d) through time will be analyzed and at the end the (ρ_d) between sectors along with the impact of correlation on the joint movement of companies will be studied.

5.1 Analysis of Default Probability, Default Barrier and Participation weight of companies in their sector of activity

To calculate default correlation default probability (PD) and default barrier (H) of each firm is required. As mentioned in chapter two the study of Amar [2] is used to estimate these parameters.

In Amar [2] study risk-neutral probability is calculated. Risk-neutral measure is usually used in the pricing of derivatives when one assumes that all the assets have the same rate of return (risk free interest rate). This hypothesis is not true in the real world. In the real world price of assets depend on their risk and investors demand payment for bearing uncertainty. However the aim of this study is to calculate the default correlation between assets so we think it is fair enough to use risk-neutral measure because it would not affect the final result.
The estimated (PD) are in line with expectation; defaulted companies have significant and large (PD) and most of non defaulted companies have small and in some cases very small almost zero (PD) $(ex.0.001 * 10^{-20})$. We replace very small (PD), as example, with 0.0001, since formula (3.12) from CreditMetrics model [14] is used to calculate default correlation. This formula is not able to calculate default correlation for companies with very small or very large (PD). As the aim of this study is to find correlation between companies, very small (PD)s are replaced with 0.0001 which is still very small but not zero. Table 5.1 presents more details about replaced $(PD)s$.

When the PD was not available for a year meaning the model couldn't estimate it, the average of available PDs for that company was used. Moreover the model used in Amar's study [2] cannot estimate PDs for Banks therefore, the S&P yearly PDs are used for Banks.

As default probabilities, estimated default barriers (H) are in line with expectation for most of firms. However for few firms the model [2] is not able to estimate (H) and for few others the estimated (H) are very small. In both cases the estimated (H) are replaced by the firms' total debt value. Table 5.1 presents the percentage of replaced (H) in each year.
Below the table 5.1 that represents the percentage of replaced (H) and (PD) in each year and the table 5.2 that represents the descriptive statistics for default probabilities (PD) and default barriers (H) are presented:

Year	% of Modified Default Probabilities (PD)	% of Modified Default Barriers (H)
1992	20%	21%
1993	34%	25%
1994	18%	26%
1995	29%	27%
1996	35%	27%
1997	29%	25%
1998	8%	24%
1999	18%	29%
2000	14%	26%
2001	17%	26%
2002	21%	24%
2003	42%	26%
2004	27%	31%

Figure 5.1: Percentage of Modified Default probabilities and Modified Default barriers

	Default Probability	Default Barrier
Mean	0.2486	3,157,925
Standard Error	0.0124	959,096
Median	0.0905	58,932
Mode	0.0001	10,000
Standard Deviation	0.3042	23,532,083
Skewness	1.0442	12
Range	0.9998	341,090,000
Minimum	0.0001	10,000
Maximum	0.9999	341,100,000
Count	602	602

Figure 5.2: Default probabilities and Default barriers Descriptive Statistics (year 2002)

5.2 Participation weight of companies in their sector of activity

As it is mentioned in the Methodology section, in order to be able to estimate correlation between firms by using the correlation between sectors, we need to know how much of the companies revenue movement is explained by its sector of activity. In other words we need to know the weight of firms (w_1) in their sector of activities.

As mentioned before the firms are regrouped in ten sectors and firms' weekly returns are regressed linearly to their sectors' weekly returns. Regression coefficients (β) are the weight of companies (w_1) in their sector of activity.

After calculating the weights, we found that for some companies the estimated weight is small and it is not significant. It seems those companies are not very dependent to their sector of activities. After investigating we found three reasons for that:

1. First, assigning one sector of activity to each company.
   In our database there are some firms that are active in more than one sector. In some cases even firms' (w_1) in each sector are very close to each other. Since in this study only one sector of activity is assigned to each firm, for those firms the sector that explains most of their activity is chosen. As it is obvious choosing only one sector for those firms may not be sufficient to explain well the volatility of the firms' revenue and that is why we end up with small (w_1) for some companies.
   GemCom can be named as an example. It is a mining software company. In our study it is classified as Information Technology company. As it is clear this company is very sensible to Mining (Industrial) sector too because all its clients are mining companies. So it could be concluded that the volatility of *GemCom* revenue is sensitive to both sectors.
   It should be mentioned that CreditMetrics [14] considers all the sectors of activity of firms. For example if a company is active in Telecommunication

and Information technology, the impact of both sectors on the revenue of the firm is considered.

2. Second, considering only Canadian sectors.
   In the database there are some companies that are not only active in Canada but also in other countries. Some of them are even more active in other countries than Canada. Therefore they are dependent to Canadian sectors and the sectors of other countries where they are active in. So considering only the Canadian sectors may not be very accurate.
   An example is *Extendicare*, which is a long term medical and rehabilitative care centre. Though it is Canadian base company, it is more active in United States. According to their website, in term of care facilities, it has 176 units in United States and 82 unites in Canada. Therefore its revenue is more sensible to American sectors than Canadian.
   In CreditMetrics [14], the authors calculate impacts of all related sectors in all related countries on the revenues of firms in their database. That means if a company is active in a sector in Canada and two sectors in Germany, the impact of all three sectors are considered.

3. Third, data problem.
   As it was mentioned in chapter three, *Stock Guide* is used to get the information about firms' sector of activity. However after estimating (w_1), we found that for some companies the assigned sector is not very accurate. That means the assigned sector does not reflect their main activity.
   For example, the estimated (w_1) for *Labrador Techs (The Association of Engineering Technicians and Technologists of Newfoundland and Labrador (AETTNL))* that is classified as Information Technology company is very small almost zero. After investigating we found that the company is an association of Engineering Technicians and Technologists whose main domain of activity is education and training with specialty in Information Technology. So it should be more classified as Consumer Staple.

Information Technology	
Mean	0.16957
Standard Error	0.01656
Median	0.14289
Standard Deviation	0.14246
Sample Variance	0.02030
Kurtosis	7.80828
Skewness	2.20930
Range	0.88290
Minimum	0.00003
Maximum	0.88293
t Statistics (mean)	3.87039

(a) Information Technology

Industry	
Mean	0.11050
Standard Error	0.01124
Median	0.08221
Standard Deviation	0.10306
Sample Variance	0.01062
Kurtosis	4.49788
Skewness	2.04598
Range	0.51299
Minimum	0.00900
Maximum	0.52199
t Statistics (mean)	2.12159

(b) Industrial

Figure 5.3: Weights Descriptive Statistics (5)

In tables 5.3 to 5.7 the descriptives statistics of weights in sectors are presented. These tables contain also the t-statistics results for regressions. Looking in results average t-statistics values of weights in different sectors vary from 2 to 10 and has been calculated to have p-value of 5% or less. Therefore there is 5% or less chance that the data would come up in a random distribution.

Now that all the issues are addressed, the results for estimated default correlation will be analyzed in next section.

5.3 Default Correlation Analysis

In this section the results for default correlation (ρ_d) will be presented and analyzed.
Since the size of our database is big (up to 600 companies in some years), it is not possible to present the results in a plot and show the relation between firms. Therefore it is decided to present results in different steps.
First, a picture of (ρ_d) for year 2002 will be presented ((ρ_d) for the other years

HealthCare	
Mean	0.17334
Standard Error	0.01904
Median	0.14702
Standard Deviation	0.13463
Sample Variance	0.01813
Kurtosis	2.55768
Skewness	1.38594
Range	0.63895
Minimum	0.00000
Maximum	0.63895
t Statistics (mean)	2.99960

(a) HealthCare

Energy	
Mean	0.17458
Standard Error	0.01884
Median	0.10450
Standard Deviation	0.18263
Sample Variance	0.03335
Kurtosis	1.39244
Skewness	1.43369
Range	0.72780
Minimum	0.00416
Maximum	0.73196
t Statistics (mean)	4.33290

(b) Energy

Figure 5.4: Weights Descriptive Statistics (5)

Consumer Staple	
Mean	0.11971
Standard Error	0.02727
Median	0.08614
Standard Deviation	0.13637
Sample Variance	0.01860
Kurtosis	4.64996
Skewness	1.96903
Range	0.58265
Minimum	0.00106
Maximum	0.58371
t Statistics (mean)	2.17621

(a) Consumer Staple

Financial	
Mean	0.16084
Standard Error	0.02288
Median	0.09250
Standard Deviation	0.18726
Sample Variance	0.03507
Kurtosis	4.02997
Skewness	2.08856
Range	0.80692
Minimum	0.00000
Maximum	0.80692
t Statistics (mean)	3.15269

(b) Financial

Figure 5.5: Weights Descriptive Statistics (5)

Consumer Discretionary	
Mean	0.12094
Standard Error	0.01463
Median	0.08839
Standard Deviation	0.11792
Sample Variance	0.01391
Kurtosis	4.36424
Skewness	1.89145
Range	0.61255
Minimum	0.00000
Maximum	0.61255
t Statistics (mean)	2.20875

(a) Consumer Discretionary

Telecommunication	
Mean	0.25894
Standard Error	0.10154
Median	0.16002
Standard Deviation	0.28719
Sample Variance	0.08248
Kurtosis	7.05014
Skewness	2.61294
Range	0.88024
Minimum	0.07510
Maximum	0.95534
t Statistics (mean)	9.93653

(b) Telecommunication

Figure 5.6: Weights Descriptive Statistics (5)

Utility	
Mean	0.32886
Standard Error	0.07386
Median	0.23429
Standard Deviation	0.26631
Sample Variance	0.07092
Kurtosis	(0.48139)
Skewness	0.69868
Range	0.84428
Minimum	0.01418
Maximum	0.85846
t Statistics (mean)	5.98848

(a) Utility

Materials	
Mean	0.19378
Standard Error	0.01418
Median	0.16740
Standard Deviation	0.15660
Sample Variance	0.02452
Kurtosis	0.29336
Skewness	0.90336
Range	0.71379
Minimum	0.00178
Maximum	0.71557
t Statistics (mean)	2.79280

(b) Material

Figure 5.7: Weights Descriptive Statistics (5)

are presented in Appendix B). Then randomly one of the companies in this year will be chosen and its (ρ_d) with other firms will be studied. In second step, three companies from three sectors will be chosen randomly and their (ρ_d) evolution in time will be examined. In third step, (ρ_d) between sectors will be analyzed for four years. Since the default correlations between sectors are not available directly, the default correlation between them will be presented in two ways. First by choosing a company in each sector as a proxy for that sector and second by calculating average of default correlations between firms in sectors as a proxy for default correlation between sectors. And finally the impact of using default correlation in estimating default probability will be presented.

5.3.1 Default correlation between firms in 2002

The following pictures present the (ρ_d) between companies in 2002, from different angles:

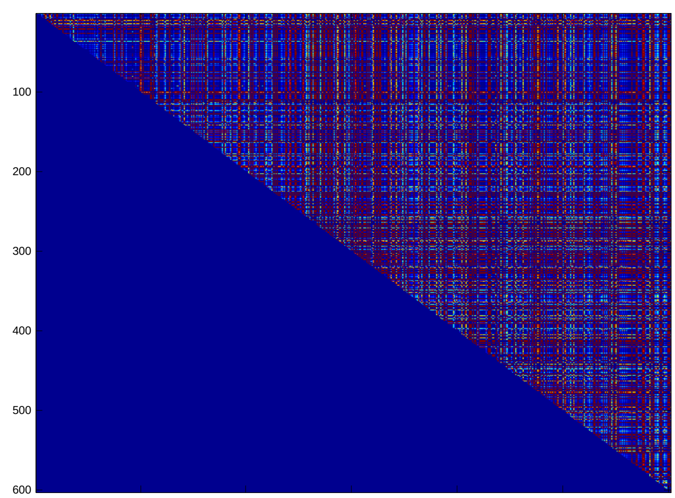


Figure 5.8: Default Correlation Matrix, each column and each row presents a company and each pixel represents a (ρ_d) value between two firms(year 2002)

These pictures demonstrates the (ρ_d) between 600 companies.

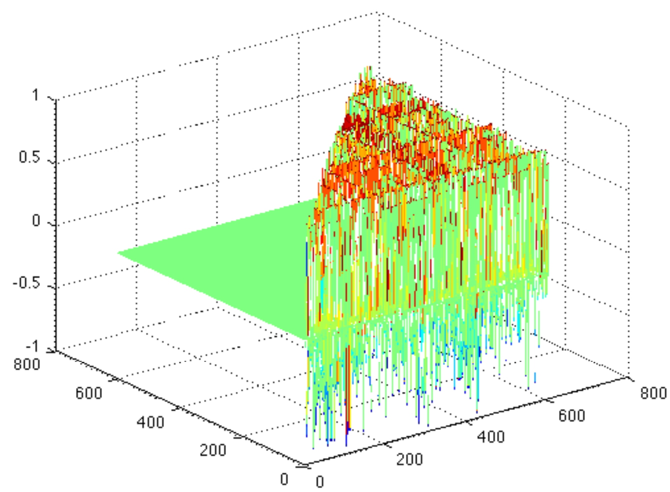


Figure 5.9: Default Correlation Matrix, the X and Y axis present companies and Z axis demonstrates default correlations amounts. This matrix is triangular so green part (flat section) represents zero values. In the other part as we move to red spots the default correlation amounts increase.(year 2002)

Each column and each row in figure 5.8 presents a company. Each pixel represents a (ρ_d) value between two firms. There is color assigned to each value, so the companies with the same (ρ_d) values have the same color. In this figure we can determined some lines. Each line represents (ρ_d) between a company with the rest of companies and shows that company has almost the same (ρ_d) value with other firms.

Figure 5.9 presents the same information as figure 5.8 but from different angle. The resulted volatility can be observed from this angle. Relatively the largest and smallest values can be also noticed in this figure. As we can see in the figure the companies are mostly correlated positively.

After presenting the big picture, in order to be able to analyze the results more precisely, we decided to do case study and break down figure 5.8 and choose a company. As figure 5.10 demonstrates line 22 is chosen.

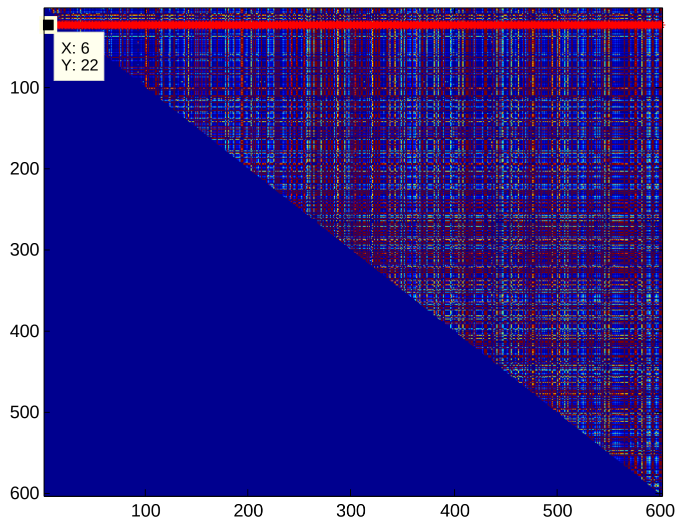


Figure 5.10: Default Correlation Matrix (year 2002)

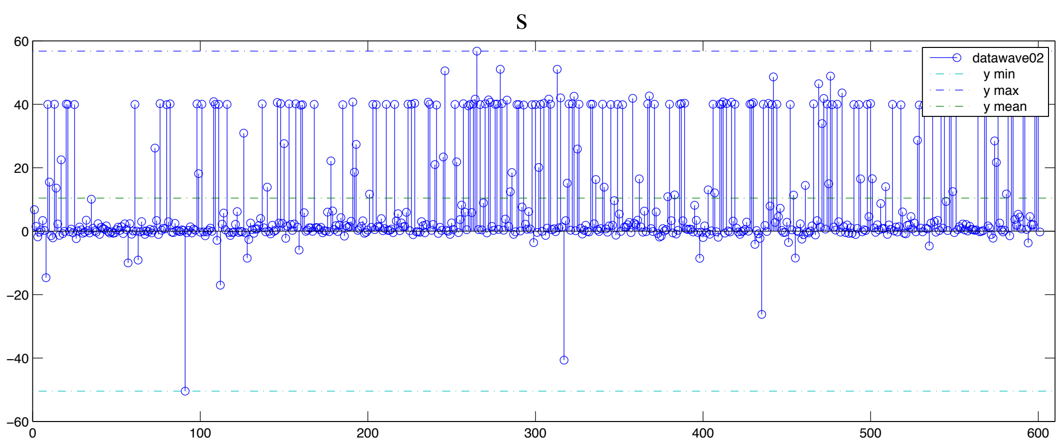


Figure 5.11: Datawave System Default Correlation, the X axis represents companies and Y axis represents the default correlation values between Datawave and the rest of companies. Most values are observed to be around 40%, the description will be given in the text. (year 2002)

Line 22 represents *Dataware System (DW)* which belongs to Information Technology sector. This company sells prepaid internet long distance and cellular phone card plus prepaid internet products and prepaid cash cards.
Figure 5.11 presents (DW) (ρ_d) with the rest of firms in 2002. Circles in figure 5.11 represents the default correlation values. These values vary between -10% and 40% except for the points marked in the figure 5.11. The outstanding points present following companies: *Clearlink Capital Corp (- 50%), Imperial Oil (+ 51%), Petro Canada (+ 57 %), Shell Canada (+ 51%), Loblaws (+ 51%), Raytec Development (- 41%), Mccoby (- 26%), Quebecor World (+ 49%) and Transalta (+ 49%).*
These companies belonging to Industrial, Energy, Consumer Staple and Material sector. In order to analyze the reason for what (DW) default correlations with these companies are standing out, we need more information about (DW), its business, procedure and clients. However in case of *Loblaws* the reason could be explained by the fact that Loblaws is a retail store. So it is possible that *Loblaws* is selling the (DW) products and that is why they are positively and highly correlated together.
To better observe *Datawave* (ρ_d) with other companies, we zoom in sectors and split figure 5.11 by sectors.

As mentioned before DW belongs to Information Technology sector. In figure 5.12 its (ρ_d) with companies in its sector of activity (Information Technology) is presented. The average (ρ_d) is about $+5\%$ and maximum and minimum (ρ_d) are -14% and $+40\%$ respectively. Although it has small or negative (ρ_d) with few companies, with the rest of them it has positive (ρ_d).
As mentioned before, required information to analyze the reason for small, big, positive or negative (ρ_d) is not available. However, we try to analyze the result by verifying estimated default probabilities, joint default probabilities and all other available information we use to calculate (ρ_d). To do that, the biggest, the smallest and zero (ρ_d) are chosen. Among Information Technology companies, (DW)

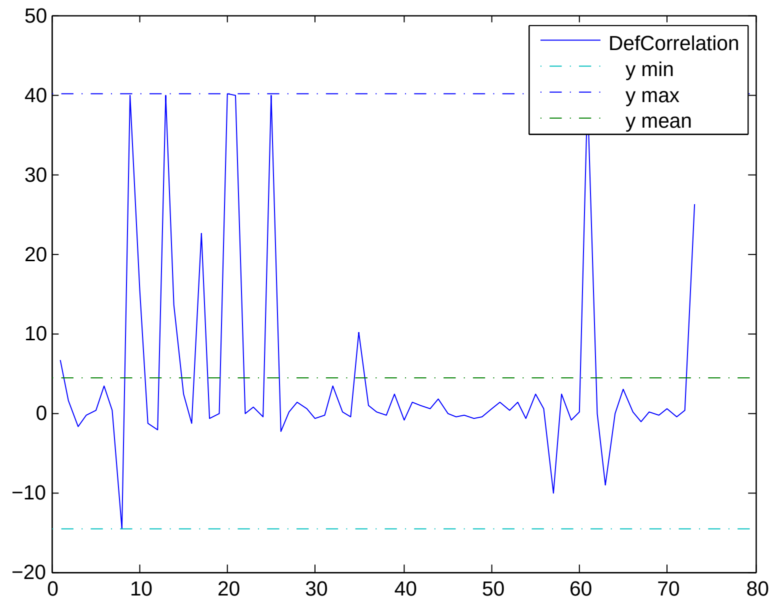


Figure 5.12: Datawave System Default Correlation with Information Technology, the X axis represents companies and Y axis represents the default correlation values between Datawave and IT firms. The minimum value is around -15% and the maximum is at 40%.(year 2002)

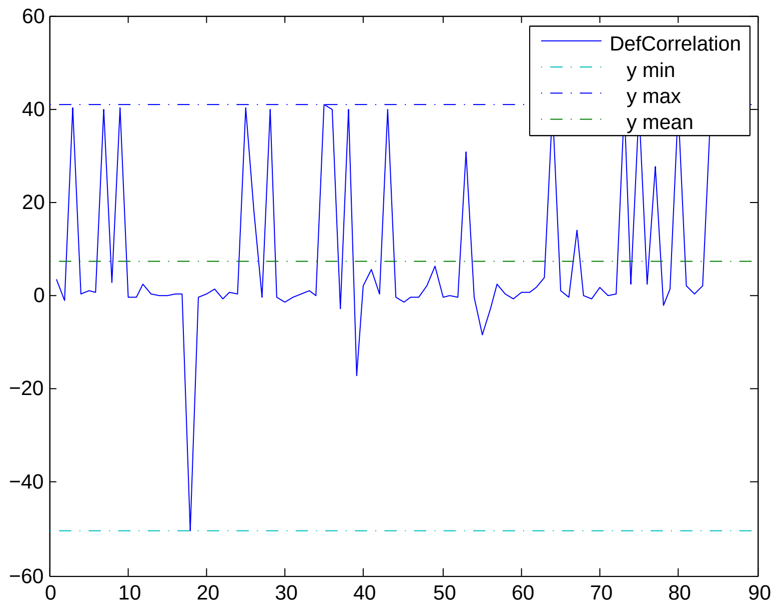


Figure 5.13: Datawave System Default Correlation with Industrial, the X axis represents companies and Y axis represents the default correlation values between Datawave and Industrial firms. The minimum value is around -45% and the maximum is at 40%(year 2002)

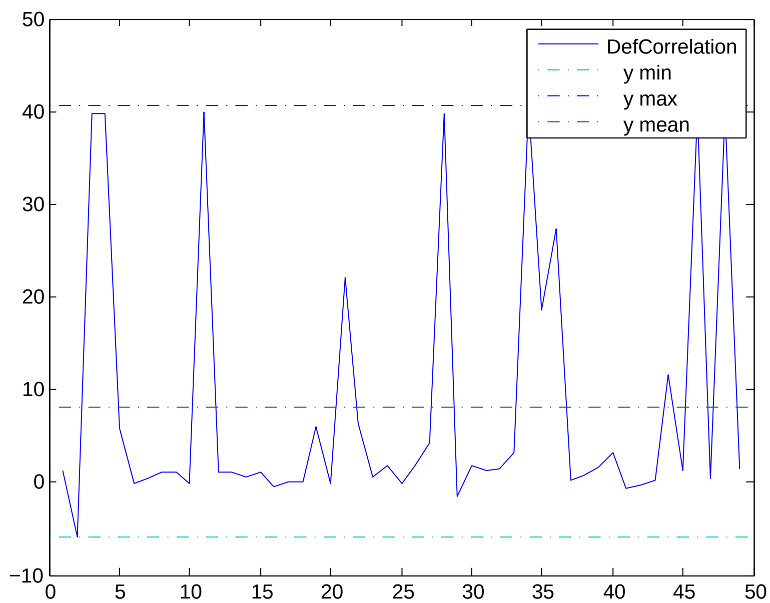


Figure 5.14: Datawave System Default Correlation with HealthCare, the X axis represents companies and Y axis represents the default correlation values between Datawave and firms in HealthCare sector. The minimum value is around -5% and the maximum is at 40%(year 2002)

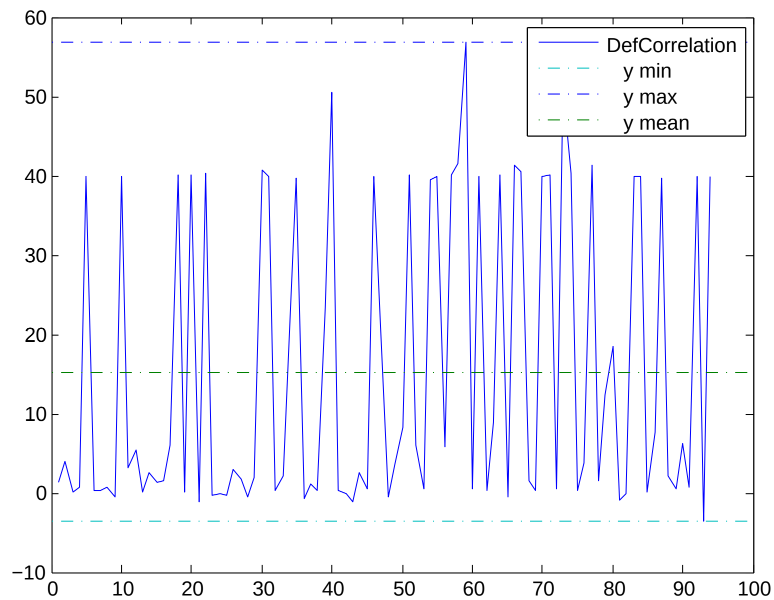


Figure 5.15: Datawave System Default Correlation with Information Technology, the X axis represents companies and Y axis represents the default correlation values between Datawave and firms in Energy sector. The minimum value is around -5% and the maximum is around 58% (year 2002)

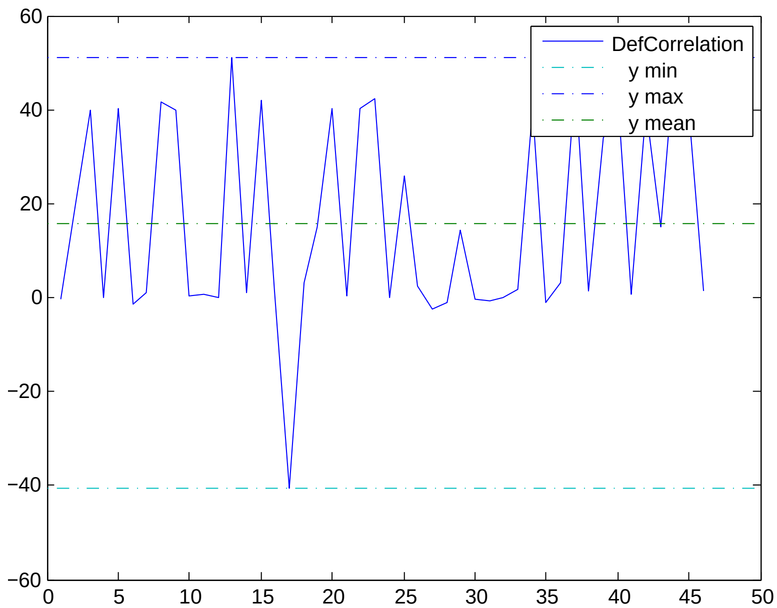


Figure 5.16: Datawave System Default Correlation with Information Technology, the X axis represents companies and Y axis represents the default correlation values. The 25 first firms are belong to Consumer Staple sector, the next eight companies are active in Telecommunication and the rest are working in Utility sector.The minimum value is at -40% and the maximum is around 50%(year 2002)

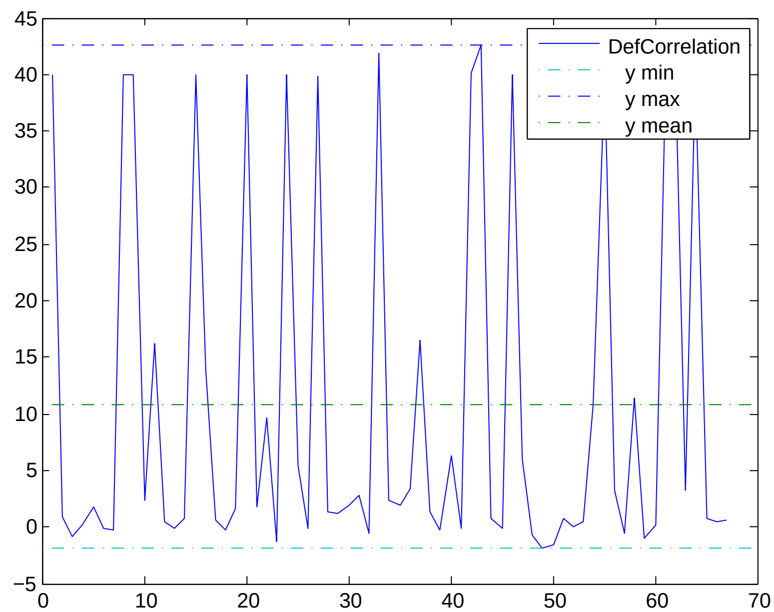


Figure 5.17: Datawave System Default Correlation with Financial, the X axis represents companies and Y axis represents the default correlation values between Datawave and Financial institutes. The minimum value is around 0% and the maximum is around 42%(year 2002)

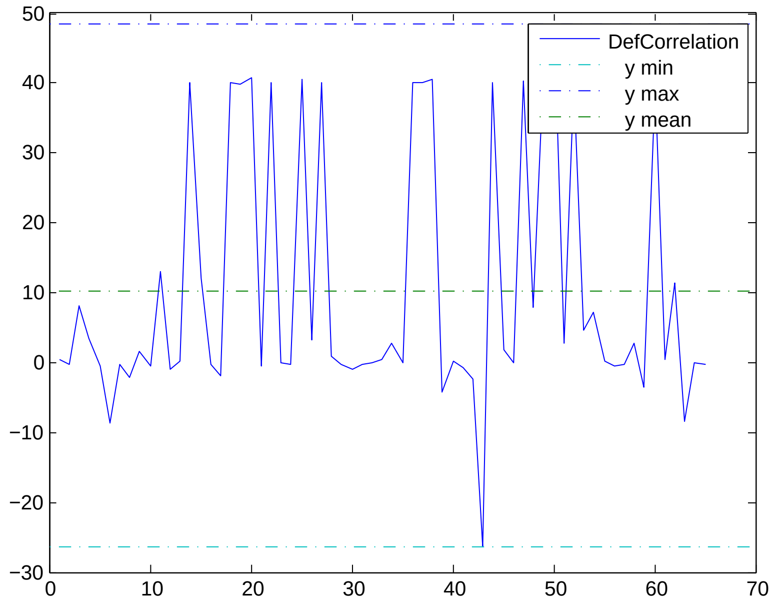


Figure 5.18: Datawave System Default Correlation with Consumer Discretionary, the X axis represents companies and Y axis represents the default correlation values. The minimum value is around -28% and the maximum is at 50%(year 2002)

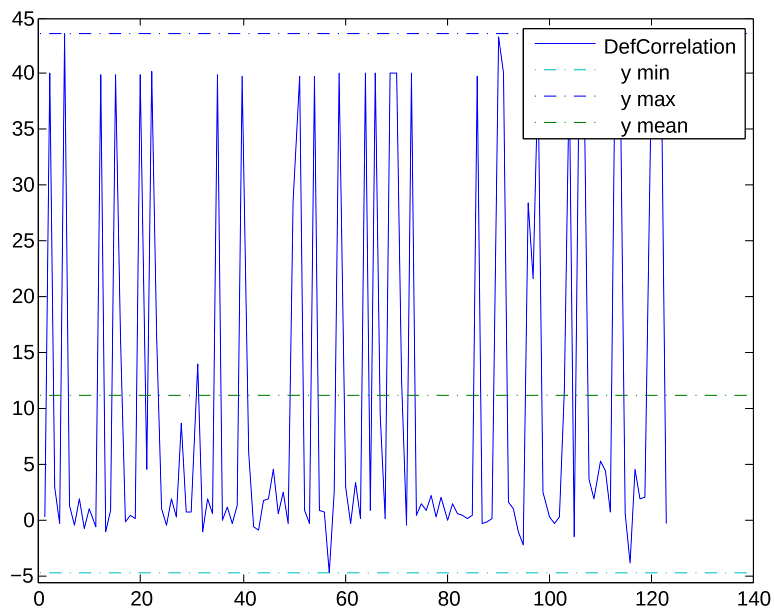


Figure 5.19: Datawave System Default Correlation with Information Technology, the X axis represents companies and Y axis represents the default correlation values. The minimum value is at -5% and the maximum is around 45%(year 2002)

is highly and positively correlated to *Dalsa (DA)* with (ρ_d) of $(+40\%)$, negatively correlated to *Burntsand (B)* with (ρ_d) of (-15%) and seems not correlated to *Tecsys Inc. (T)* with (ρ_d) of (-0.01%).
(DW), (B), (DA) and (T) have default probability of 0.477541, 0.998559, 0.0001 and 0.461314 consecutively. Their joint default probabilities are, (DW, B) = 0.1986, (DW, DA) = 0.2001, (DW, T) = 0.1989. Because product of (B) default probability and (DW) default probability compared to their joint default probability is very big, their (ρ_d) (formula (3.12)) is great and negative. The same logic applies to the other pairs. Product of (DA) and (DW) default probabilities, compared to their joint default probability, is very small so their (ρ_d) (formula (3.12)) is large and positive. However because default probability of (T) is in the same range as default probability of (DW) we end up with small (ρ_d). In conclusion (DW) is highly positively correlated to the companies that are far from default with small (PD), it is highly negatively correlated to companies that are likely to default with great (PD) and it has small (ρ_d) with the rest. This conclusion is valid for all the companies like (DW) that their joint default probabilities are almost stable through companies and their joint default probabilities do not vary a lot from a company to another company. For example joint default probabilities of (DW) with all three companies stated above are 20%.
As mentioned before the (DW) (ρ_d)s mostly vary between -10% and $+40\%$. As it is presented in figure 5.11, (DW) has $+40\%$ (ρ_d) with a lot of companies. The reason is the same as for (DA), the product of (DW)'s (PD) with each of these companies is smaller than their joint default probabilities. These companies have very small (PD)s and their joint default probabilities are very similar. That is why we end up with the same (ρ_d) (+40%) across these companies.
As explained before, estimated joint default probabilities are derived from default barriers and correlations between companies. Correlation between firms are in turn derived from correlations between sectors and weights of firms in their sectors of activity (w_1). (DW) has small (w_1), about 8%, which leads to small correlations with other companies and close joint default probabilities across its

sector.
As explained before, this company provides internet and cellular products. According to its website *www.datawave.com* and *Answers.com (www.answers.com)*, which has useful information about Canadian firms, this company sells its products and services primairly in North America by agreements with telecom companies, financial institutions and retailers. So it is not only dependent to Information Technology sector and not only to Canada but also to Telecom, Financial and Consumer Staple sectors in United States. This can explain the reason for having small (w_1).

Next figure 5.13 represents the (ρ_d) between (DW) and Industrial companies. The average (ρ_d) is $+7\%$ and minimum and maximum values are -50% and $+41\%$ consecutively. In this sector (DW) is highly and negatively correlated to *Clearlink capital corporation* and it is highly and positively correlated with *Husky injection modeling system.*
Husky is a manufacturer of wide range of plastic products such as bottles, medical components and electronic parts. One may say this company could be a supplier for (DW) so that they are correlated positively. This hypothesis can be verified in future studies.

As it is observable in figures 5.14, 5.15, 5.17, 5.19, (DW) has positive (ρ_d) with almost all the companies in HealthCare, Energy, Financial and Material sectors. That means the default of each of them may affect or cause the default of (DW). (DW) has the greatest (ρ_d) with *Petro Canada* among all the companies in all the sectors.

Figure 5.16 represents (ρ_d) between (DW) and companies in three sectors of Consumer Staple, Telecommunication and Utility. The first 25 companies are active in Consumer Staple sector, the next eight companies are working in Telecommunication field and the rest belongs to Utility. As it is presented in this figure the (ρ_d) between (DW) and the companies in these sectors is positive except for *Ratech Design and Development* in Consumer Staple.

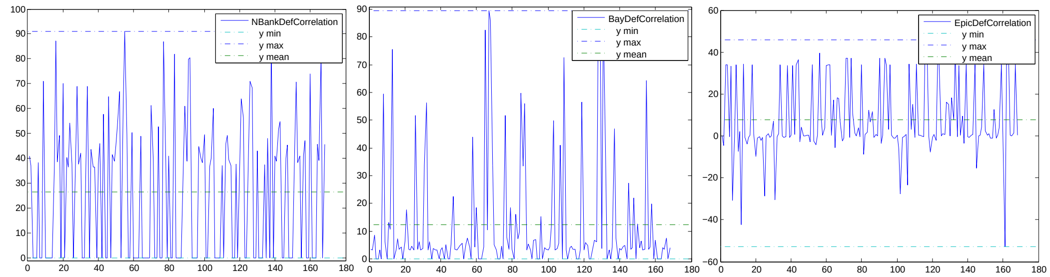
Figure 5.18 shows the relation between *Datawave* and companies in Consumer Discretioary sector. Comparing to the other sectors, this one has the higher number of companies who are negatively correlated to (DW).

As we can see in the above figures (5.12 to 5.19), companies in a same sector can be correlated (positively and negatively) or uncorrelated with a specific company. As for (DW) that is positively and negatively correlated to most of firms in Consumer Discretionary and uncorrelated to few of them. So taking a correlation between a company and a sector as a proxy for correlation of that company with all the firms in that sector is not very accurate. To capture properly the impact of omiting or adding a company to a portfolio, the correlation of that company with each of companies in the portfolio should be calculated.

5.3.2 Yearly evolution of default correlation

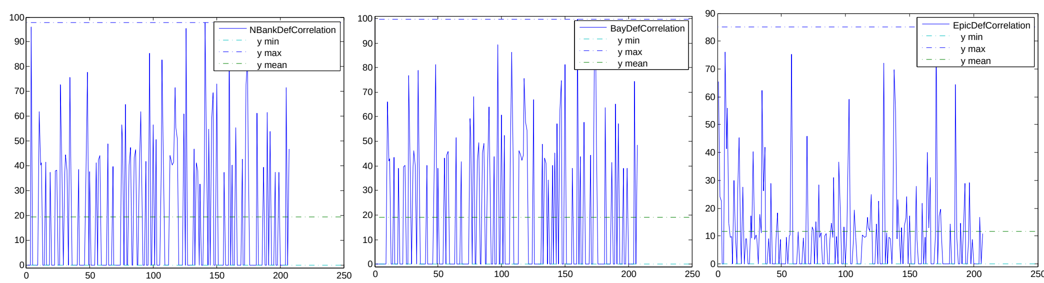
In previous section the time of study was constant and movements of a company (ρ_d) through sectors are analyzed for a specific year. In this section the constraint on constant time will be relaxed and the evolution of a company's default correlation in time will be studied. The evolution of default correlations will be presented in two ways. First the yearly matrix of default correlation for a set of firms *(National Bank, Hudson's Bay, Epic Data)* will be presented and then the average default correlation for another set of firms *(Royal Bank, Gennum, Rothman)* will be calculated and will be presented yearly. So we will be able to follow their average movement through time.

1. In first approach, three companies from three sectors were chosen. First one



(a) National Bank (b) Hudson's Bay (c) Epic Data

Figure 5.20: Default Correlation, year 1992, the X axis represents the companies and the Y axis represents the default correlation values.



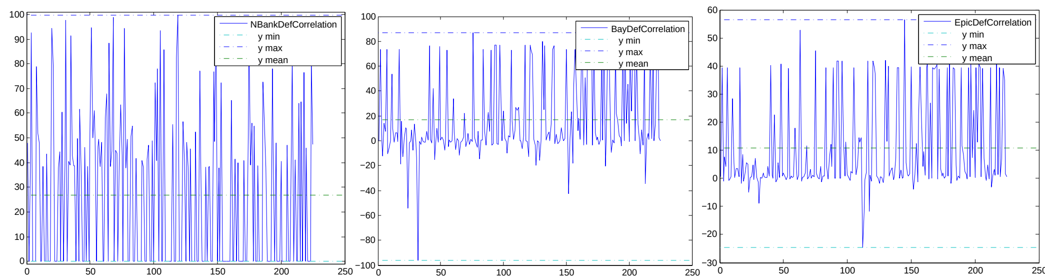
(a) National Bank (b) Hudson's Bay (c) Epic Data

Figure 5.21: Default Correlation, year 1993

is *National Bank of Canada (NB)* from Financial sector, second one is *Hudson's Bay (HB)* from Consumer Discretionary sector and the last one is *Epic Data (EP)* from Information Technology sector. As in the previous section these companies' yearly default correlation will be presented by figures and their default correlation evolution will be studied one by one.

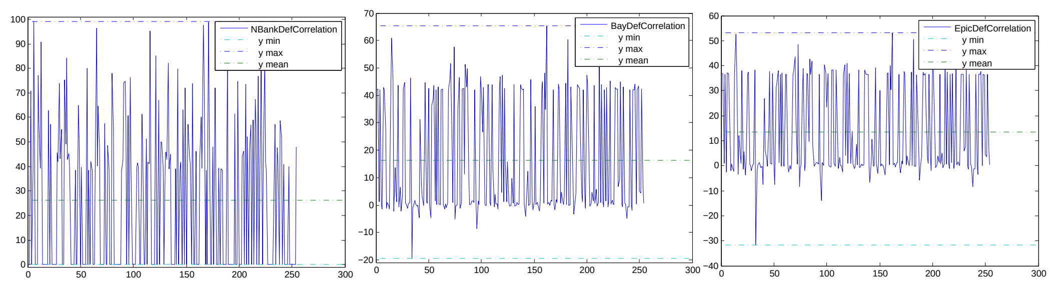
Figures 5.20 to 5.32 are presenting (ρ_d) of the three firms named above from 1992 to 2004. First plot belongs to (NB), second one belongs to (HB) (ρ_d) and the last one shows (ρ_d) of (EP).

As we go through the (NB) plots, we observe that (NB) is either positively correlated to the other firms or it is uncorrelated. After investigating we find



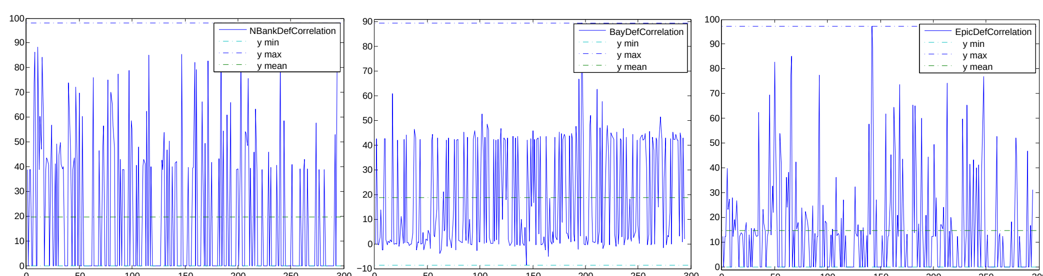
(a) National Bank (b) Hudson's Bay (c) Epic Data

Figure 5.22: Default Correlation, year 1994



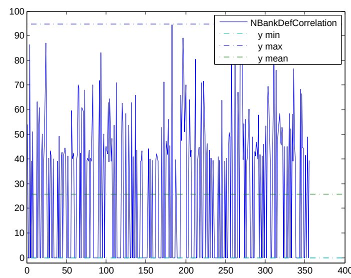
(a) National Bank (b) Hudson's Bay (c) Epic Data

Figure 5.23: Default Correlation, year 1995

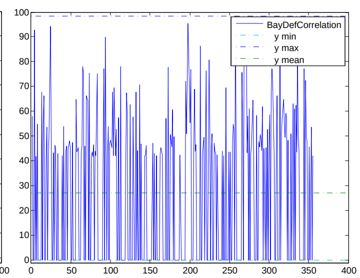


(a) National Bank (b) Hudson's Bay (c) Epic Data

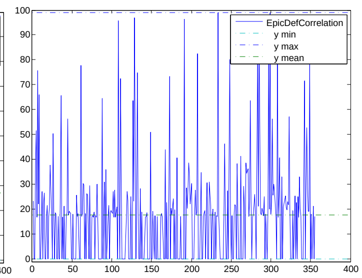
Figure 5.24: Default Correlation, year 1996



(a) National Bank

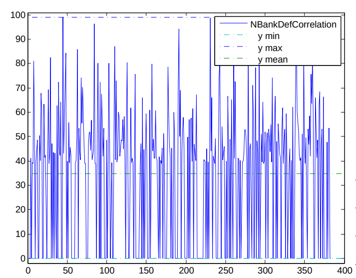


(b) Hudson's Bay

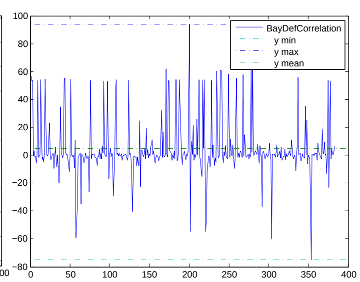


(c) Epic Data

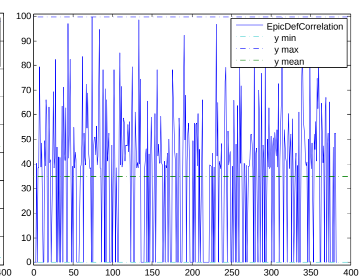
Figure 5.25: Default Correlation, year 1997



(a) National Bank

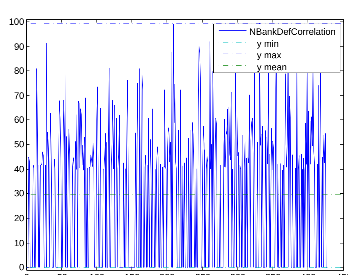


(b) Hudson's Bay

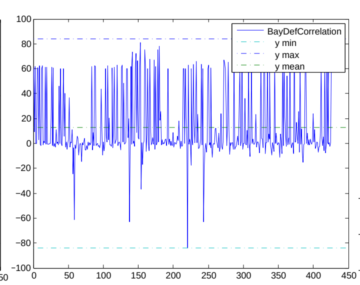


(c) Epic Data

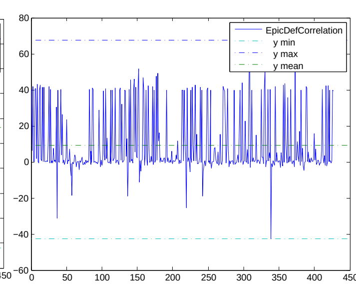
Figure 5.26: Default Correlation, year 1998, companies are observed to be more correlated in this year than previous years which could be explained by beginning of Dot-com bubble event.



(a) National Bank

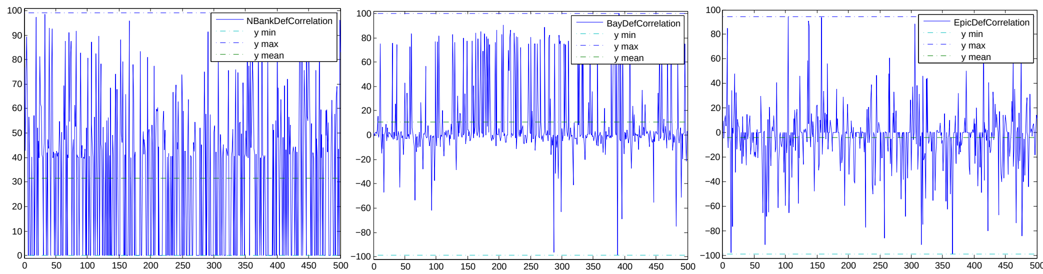


(b) Hudson's Bay



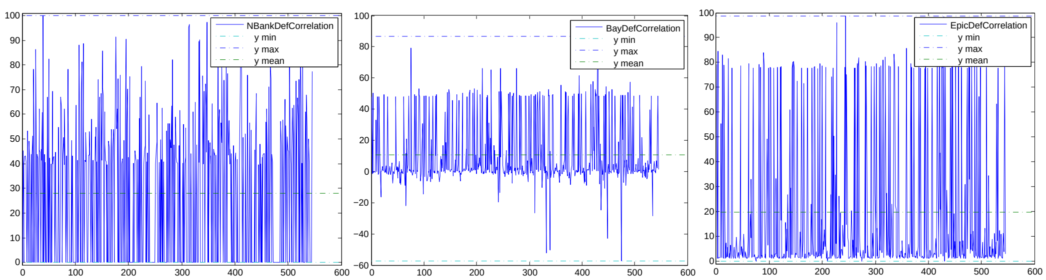
(c) Epic Data

Figure 5.27: Default Correlation, year 1999, the companies are still highly correlated.



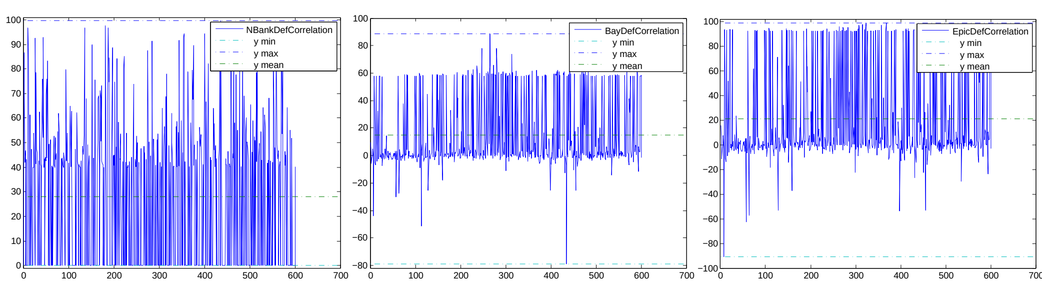
(a) National Bank (b) Hudson's Bay (c) Epic Data

Figure 5.28: Default Correlation, year 2000



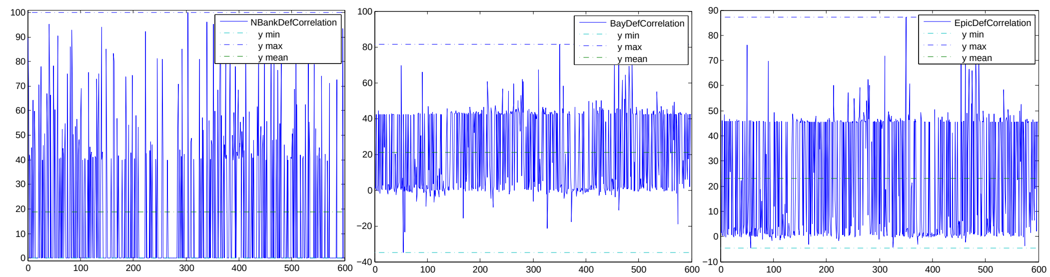
(a) National Bank (b) Hudson's Bay (c) Epic Data

Figure 5.29: Default Correlation, year 2001, the highly correlation between companies are observable in this year too that is explained by the 2001 recession.



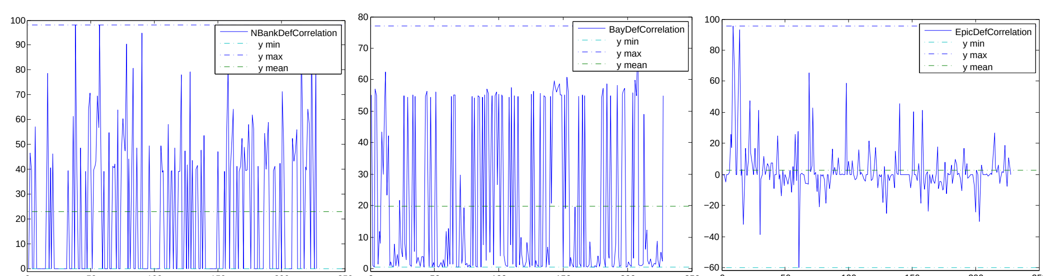
(a) National Bank (b) Hudson's Bay (c) Epic Data

Figure 5.30: Default Correlation, year 2002



(a) National Bank (b) Hudson's Bay (c) Epic Data

Figure 5.31: Default Correlation, year 2003



(a) National Bank (b) Hudson's Bay (c) Epic Data

Figure 5.32: Default Correlation, year 2004

the reason why it is uncorrelated to some firms. (NB) yearly estimated default probability is small, for example in 1992 it has a (PD) equal to 0.0005. The results of our study shows that companies with small default probability (the companies that are unlikely to default) are almost uncorrelated with the companies that are also unlikely to default or with the companies that have very big default probabilities. This result is in line with the result in the study of Hans Gersbach and Alexander Lipponer [13]. They conclude that a company with small default probability has almost zero default correlation with the other companies.
Another observation from plots is that in some years (NB) has great positive (ρ_d) with some firms and even in some cases it is perfectly correlated. For example in 2000 it is almost $+100\%$ correlated to *Nortel Network Corporation*. That means if *Nortel* defaulted in 2000, the (NB) would surely default. However (ρ_d) between these companies drop in the following years (when *Nortel* actually defaulted) to the point that they are almost uncorrelated.

The other interesting observation is that average of (ρ_d) is increased between 1998 and 2002 inclusively. That means that (NB) is more correlated with the other companies, so it is more probable for (NB) to default. This could be explained either by market or by special events that somehow affect (NB). After investigating we find that the reason is market. During that period market faced two events that both negatively affected it; first *Dot-com bubble* that happened in late 1990*s*, second is the 2001 recession and third could be the terrorist attack of September 11, 2001. All seriously affected market everywhere in the world specially in North America.

As mentioned above the other plots represent the movement of default correlations for (HB), (EP) during study period. Despite (NB), they have both positive, negative and zero (ρ_d) with other companies.
These plots show that the correlation is not stable through time. Two com-

panies can be highly positively correlated in one year and next year they will be highly negatively correlated or even uncorrelated. For example, (EP) is perfectly negatively correlated to *Dalsa Corporation* in 2000 but in 2001 they are highly positively correlated with (ρ_d) of $(+79\%)$. So it is very important for a portfolio manager to actively calculate the correlation between the components of its portfolio.

The other result from these figures is that in some years (HB) and (EP) are perfectly and negatively correlated to some companies. For example as we can observe in figure 5.28, (HB) is perfectly negatively correlated with two companies, one of them is *Fortis Inc* in Utility sector. Knowing that and having both companies' shares in a portfolio may let to better diversify the portfolio.

It should be mentioned that same behavior as (NB), is observable in (EP) during 1998 and 2001. (EP) has positive correlation with the rest of companies and its average default correlations increase during this period. The reason is the same as (NB) specially in 1998. Because as we know the *Dot-com bubble* affected mostly the Information Technology sector which is (EP) sector of activity. In case of (HB) there is tendency to increase in (ρ_d) from 2000 to 2001 and this tendency continues through 2002 and 2003. It seems 2001 recession affects Consumer Discretionary in following years.

2. In second approach, the average default correlation for three companies *(Genuum (Information Technology), Rothman (Consumer Staple) and Royal Bank of Canada (Financial))* are calculated for each year. Their average default correlation evolution is presented in figure 5.33.

   The average correlation for these firms varies between 11% to 35%. In general, Royal Bank average default correlation is higher than the other firms and it never hits the minimum value.

   In 1995, 1998 and 2001 three of them show the same behavior and they are all increased. They all hit the maximum value in 1998 and approach the

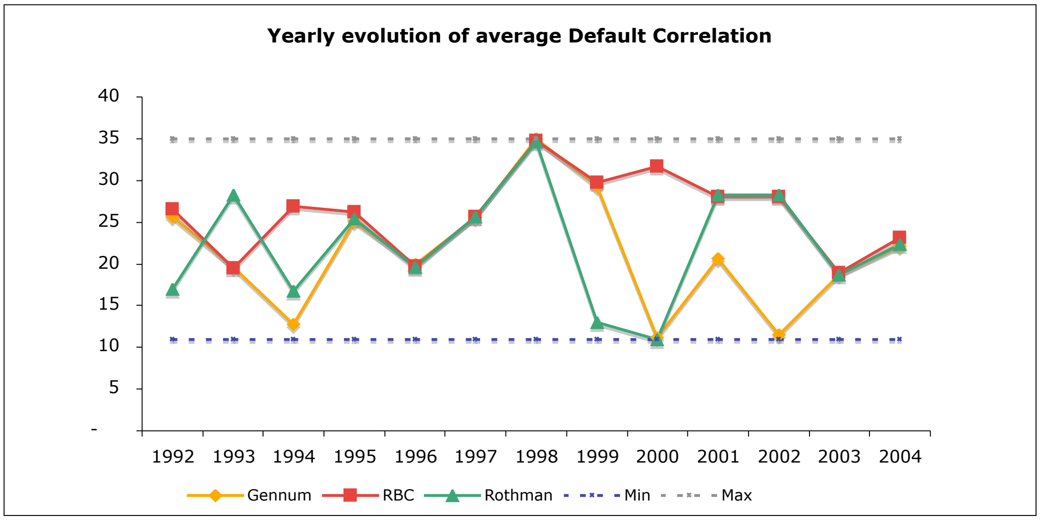


Figure 5.33: Yearly evolution of average Default Correlation, the X axis presents years and Y axis represents correlation values.

maximum value in 2001 when there was a recession.
Gennum and Rothman have their lowest average default correlation in 2000 which is increased in 2001. The average default correlation for Rothman (Consumer Staple) increased more in 2001 than Gennum (Information Technology). It has the same average default correlation as Royal Bank in 2001.

5.3.3 Sectors' default correlation

Since (ρ_d) of sectors are not available, in order to show the default correlations between sectors two methods are used. First, by choosing the largest company in each sector as a proxy for that sector and second, by calculating average of default correlations between firms in sectors as a proxy for default correlation between sectors.
The largest firm is the firm that movement in its revenue (w_1) is mostly explained by movement in revenue of its sector of activity. The weights (w_1) of chosen com-

panies to their sectors are about 90%. However we find out that these companies usually have very good risk quality so their default probabilities are small. As mentioned before the default correlation between companies with small default probabilities are zero. So to solve the problem the firms in each sector are sorted by their (w_1), if the largest firm has a small (PD) the first company after it that has reasonable (PD) (at least 10%) is chosen (Table 5.34). Below the result for four years, 2000, 2001, 2002 and 2003 will be presented.

	List of companies			
Sectors	**2000**	**2001**	**2002**	**2003**
Information Technology (IT)	NORTEL NETWORKS CORPORATION	NORTEL NETWORKS CORPORATION	NORTEL NETWORKS CORPORATION	JDS UNIPHASE CANADA LTD.
Industrial (IND)	BALLARD POWER SYSTEMS INC.	BALLARD POWER SYSTEMS INC.	C A E INC.	BALLARD POWER SYSTEMS INC.
Health Care (HLTC)	Q L T INC.	PATHEON INC.	BIOVAIL CORPORATION	BIOVAIL CORPORATION
Energy (ENR)	NEXEN INC.	CANADIAN NATURAL RESOURCES LTD	TALISMAN ENERGY INC.	ENCANA CORPORATION
Consumer Staple (CSPL)	MAPLE LEAF FOODS INC.	MAPLE LEAF FOODS INC.	VAN HOUTTE INC.	MAPLE LEAF FOODS INC.
Financial (FIN)	ROYAL BANK OF CANADA	POWER FINANCIAL CORPORATION	POWER FINANCIAL CORPORATION	MANULIFE FINANCIAL CORPORATION
Consumer Discretionary (CDIS)	CANWEST GLOBAL COMMUNICATIONS	THOMSON CORPORATION	CANWEST GLOBAL COMMUNICATIONS	CANWEST GLOBAL COMMUNICATIONS
Telecom (TEL)	B C E INC.	B C E INC.	CYGNAL TECHNOLOGIES CORP.	CYGNAL TECHNOLOGIES CORP.
Utility (UTL)	TRANSCANADA CORPORATION	TERASEN INC.	TRANSCANADA CORPORATION	TRANSALTA CORPORATION
Material (MAT)	INCO LIMITED	INCO LIMITED	ALCAN INC.	NOVA CHEMICALS CORPORATION

Figure 5.34: Companies that represent the sectors

The amounts of sectors' (ρ_d) is presented by circles. The filled circles show the positive amounts and empty circles show the negative amounts. The size of circles shows the percentage that two sectors are correlated; bigger circle means higher correlation. For example the big yellow circle that present the (ρ_d) between Financial (FIN) and Telecom (TEL) in figure 5.35 is equal to 90%.
Following points are observed by studying the plots: First, in 2000, Financial

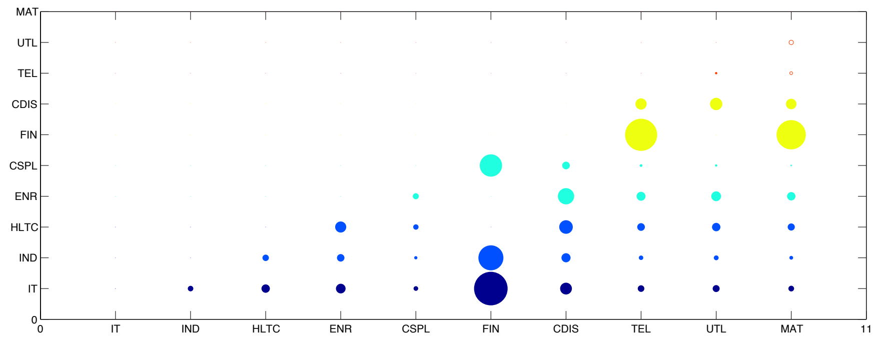


Figure 5.35: Default Correlation between sectors(year 2000)

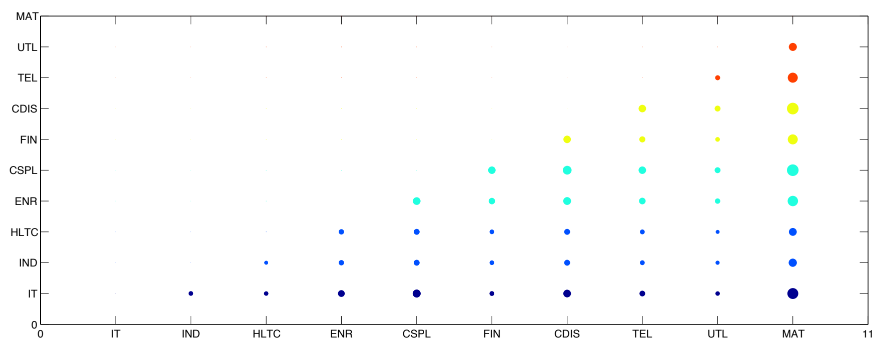


Figure 5.36: Default Correlation between sectors(year 2001)

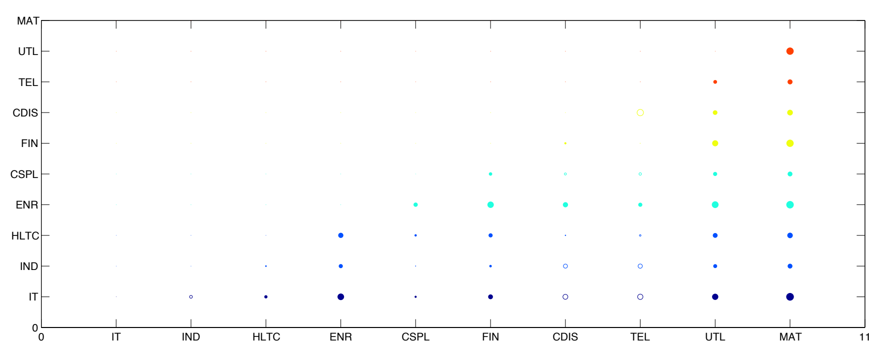


Figure 5.37: Default Correlation between sectors(year 2002)

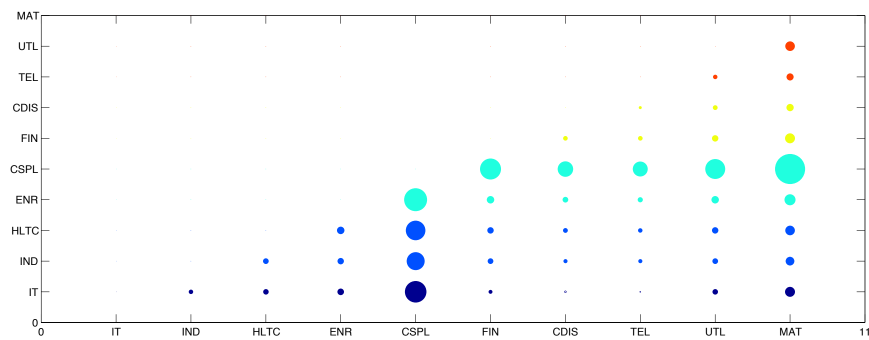


Figure 5.38: Default Correlation between sectors(year 2003)

(FIN) sector is very correlated with Information technology (IT), Telecom (TEL) and Material (MAT) but it is almost uncorrelated with Energy (ENR) and Health-Care (HLTC).

Second, in 2001, all the sectors are correlated which can be explained by recession in 2001.

Third, in 2002 the (ρ_d) between sectors are low and for some sectors such as Consumer Staple (CSPL) and Industrial (IND) are zero. Also in this year some sectors are negatively correlated. For example Telecom and Consumer Discretionary (CDIS) have negative correlation with Information technology (IT) and Industrial (IND). Forth, in 2003, all the sectors are positively correlated. In this year Consumer Staple (CSPL) is highly correlated with other sectors.

As mentioned above the other way of presenting the (ρ_d) between sectors is by using the average of (ρ_d)s between firms as a proxy. The following tables present the results:

The results presented may not be perfectly in lined with the results from the figures because in figures each sector is presented only by one company but in tables the average of all the firms in sectors are used to present the (ρ_d)s of sectors. As tables show there is an increase in (ρ_d) amounts from 2000 to 2001 and sectors are more correlated in 2001. Moving to 2002, sectors are less correlated and

2000	Information Technology	Industrial	Health Care	Energy	Consumer Staple	Financial	Consumer Discretionary	Telecom	Utility	Material
Information Technology	54.84%	8.38%	13.07%	18.19%	11.55%	14.64%	8.56%	9.74%	25.42%	8.49%
Industrial		55.08%	12.77%	18.17%	11.65%	14.26%	8.37%	10.78%	25.38%	8.09%
Health Care			59.38%	18.76%	14.72%	15.59%	12.66%	13.85%	23.85%	12.87%
Energy				66.48%	18.20%	17.41%	17.60%	20.17%	20.57%	17.78%
Consumer Staple					58.70%	15.18%	11.63%	12.78%	23.42%	11.97%
Financial						63.29%	13.97%	17.20%	17.26%	13.64%
Consumer Discretionary							55.66%	10.06%	24.14%	8.51%
Telecom								56.54%	23.79%	8.59%
Utility									71.26%	23.93%
Material										55.59%

Figure 5.39: Default Correlation Between Sectors(year 2000)

2001	Information Technology	Industrial	Health Care	Energy	Consumer Staple	Financial	Consumer Discretionary	Telecom	Utility	Material
Information Technology	58.01%	12.90%	14.14%	13.64%	14.93%	14.81%	11.29%	13.59%	23.38%	12.34%
Industrial		60.36%	16.40%	15.02%	16.76%	15.69%	13.79%	16.99%	22.38%	14.18%
Health Care			64.62%	16.94%	18.59%	18.71%	15.44%	18.38%	25.78%	16.07%
Energy				62.03%	16.08%	16.41%	14.44%	16.95%	20.54%	15.17%
Consumer Staple					63.26%	17.73%	15.83%	19.32%	22.13%	15.81%
Financial						61.67%	15.75%	19.59%	20.81%	15.94%
Consumer Discretionary							57.72%	14.74%	24.22%	13.23%
Telecom								57.55%	28.30%	16.05%
Utility									77.09%	22.02%
Material										60.39%

Figure 5.40: Default Correlation Between Sectors(year 2001)

2002	Information Technology	Industrial	Health Care	Energy	Consumer Staple	Financial	Consumer Discretionary	Telecom	Utility	Material
Information Technology	56.99%	11.76%	13.58%	19.07%	16.05%	13.38%	15.33%	8.08%	24.59%	14.93%
Industrial		58.27%	15.19%	17.63%	15.71%	14.01%	13.51%	11.90%	22.15%	15.05%
Health Care			61.35%	21.03%	18.51%	17.27%	17.44%	13.02%	26.44%	17.60%
Energy				63.76%	17.58%	18.16%	17.09%	19.12%	21.51%	17.74%
Consumer Staple					67.22%	19.58%	18.00%	17.72%	23.97%	19.56%
Financial						62.32%	16.25%	15.34%	22.92%	16.87%
Consumer Discretionary							60.50%	14.04%	20.91%	15.94%
Telecom								53.84%	24.26%	15.96%
Utility									75.87%	22.98%
Material										61.94%

Figure 5.41: Default Correlation Between Sectors(year 2002)

2003	Information Technology	Industrial	Health Care	Energy	Consumer Staple	Financial	Consumer Discretionary	Telecom	Utility	Material
Information Technology	65.67%	18.19%	20.88%	20.20%	19.22%	20.07%	18.33%	20.42%	22.29%	20.14%
Industrial		65.93%	19.89%	19.32%	18.07%	19.58%	18.28%	18.97%	21.62%	19.45%
Health Care			72.80%	17.61%	16.15%	22.07%	21.49%	18.58%	23.27%	18.15%
Energy				71.64%	13.60%	21.02%	21.12%	17.73%	21.83%	16.44%
Consumer Staple					74.33%	19.56%	19.96%	15.00%	19.75%	14.87%
Financial						83.83%	20.11%	20.09%	24.40%	20.70%
Consumer Discretionary							63.20%	20.95%	23.20%	20.60%
Telecom								78.10%	21.36%	17.95%
Utility									75.87%	21.86%
Material										70.44%

Figure 5.42: Default Correlation Between Sectors(year 2003)

there is a decrease in (ρ_d) between sectors except for Energy, Consumer Staple and Consumer Discretionary. In 2003 (ρ_d)s are increased and sectors are more correlated.

5.3.4 Impact of Correlation on Joint Default Probabilities

In this section the impact of correlation on the joint movement of companies will be presented. As mentioned in Methodology chapter, in order to estimate joint default probabilities of two firms, the correlation between those firms is required. It was also mentioned that any change in amount of correlation will change the amount of joint default probabilities between firms. To show the impact the statistic descriptive of joint default probabilities without correlation and with correlation for four years are presented.

Joint Default Probabilities Descriptive Statistics without Correlation				
Years	2000	2001	2002	2003
Mean	0.1156	0.0753	0.0616	0.0291
Std	0.2112	0.1673	0.1409	0.1087
Mode	0.00000001	0.00000001	0.00000001	0.00000001
Median	0.0065	0.0011	0.00050613	0.000027267
10% Quantile	6.1116E-06	1.3403E-06	7.8484E-07	0.00000001

Figure 5.43: Joint Default Probability Descriptive Statistics without Correlation

Joint Default Probabilities Descriptive Statistics with Correlation				
Years	2000	2001	2002	2003
Mean	0.2376	0.2416	0.2462	0.2485
Std	0.067	0.0652	0.064	0.0669
Mode	0.4438	0.4444	0.1996	0.1974
Median	0.2067	0.2118	0.2158	0.2153
10% Quantile	0.1958	0.1976	0.2	0.1977

Figure 5.44: Joint Default Probability Descriptive Statistics with Correlation

As tables 5.43 and 5.44 present the quality of data is increased by using (ρ_d) in estimating joint default probabilities. For example, all years show more concentration of joint default probabilities in second table (Table 5.44) compared to first table (Table 5.43). Also the mean for joint default probabilities using (ρ_d) are greater than the mean for joint default probabilities with zero (ρ_d). For example in year 2000, the mean for joint default probability using (ρ_d) is at 24% and the mean for joint default probability with zero (ρ_d) is at 11%. The same result is observed for the other years as well, therefore one may conclude that joint default probability with no (ρ_d) results in underestimation. These results demonstrate the usefulness of (ρ_d) in calculation of joint default probabilities between companies.

Chapter 6

Conclusion

The objective of this work is to find default correlation between Canadian public firms to better manage loan portfolio of Canadian banks. The CreditMetrics model is used for calculating default correlations. CreditMetrics uses correlation between sectors to derive correlation between firms' asset value. In order to achieve this, CreditMetrics defines sectors of activity for each firm, calculates the weight of each firm according to its sectors and then combines sectors correlations with these weight to estimate firms' asset correlations.
The CreditMetrics methodology is used in this study to calculate assets' correlation. In our database, there are two groups of firms, defaulted and non-defaulted firms. First, a main field of activity for each firm is defined, then the weight of each company to its sector is calculated by running a regression on the companies' returns and sectors' returns. After the combination of weights and sectors correlations is used to derive firms' asset correlations. The default probability and default barrier of each company are also calculated. Firms' asset correlation along with default barriers are utilized to estimate joint default probabilities between firms. At the end, firms' default probabilities and their joint default probabilities are used to calculated default correlations between firms.
The chosen period of study in this work is from 1992 to 2004. The reason for choosing this period is that we can study the evolution of default correlations in

normal and recession years. For example in this period we have the *dot-com bubble* and 2001 recession.

Only one sector is assigned to each firm even though it might be active in several sectors. Also only activity in Canada is considered in this study even though companies might be very active in other countries. The reason for this decision is the limitation of our data and also complexity of calculatiions. As a result of these assumptions we end up with small weight for some companies in our database. The small weights affect the magnitude of default correlations.

Contrary to CreditMetrics, in this study, the risk neutral default probability and default barrier of each firm is calculated. In other words, instead of using the same default probability and same default barrier for all the companies in the same risk class their specific and own default probability and default barrier are calculated. We believe that this improvement should lead to more accurate results.

The results show that default correlation between components of our portfolio increase when market does not perform well or when there is an event that affects market adversely. In this study the effects of such events namely (dot com bubble, 2001 recession and September 11th event) have been considered, and the conclusion is that the correlation between National Bank (Financial) and the rest of companies is increased during these events compared to the other years. The same result as National Bank is concluded for Epic data (Information Technology) during that period.

Also it is observed that companies with very good credit quality are almost uncorrelated to other companies with very good credit qualities (small default probability). Moreover results show that default correlation varies by time, so in order to have an effective diversification, Banks should dynamically calculate the default correlation of their portfolios.

In addition, Banks should calculate default correlation between each pair of firms in their portfolios if they want to diversify their portfolios more accurately.

Finally results show the crucial impact of using default correlation in calculating joint movement of firms (joint default probability). As statistics show the quality of joint default probabilities are improved. A problem is encountered with CreditMetrics formula for calculating default correlation. It is not capable to estimate default correlation for firms with very small or very big default probabilities. Therefore finding a way to improve the CreditMetrics formula would be an interesting future work for this study.

It would be also interesting to calculate physical default probabilities for firms instead of the risk neutral default probabilities that is calculated in this work and compare the impact on the default correlations between firms.

Another extension could be to estimate impacts of all macro-economic factors on the sectors and firms and try to find their relations, like Credit Portfolio view.

Also another measurement of dependency such as non linear relationships could be used to estimate the dependency between sectors or firms. Non linear correlations could better capture the dependency between firms and lead to more accurate results.

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Chapter 7

Appendix

7.1 Appendix A

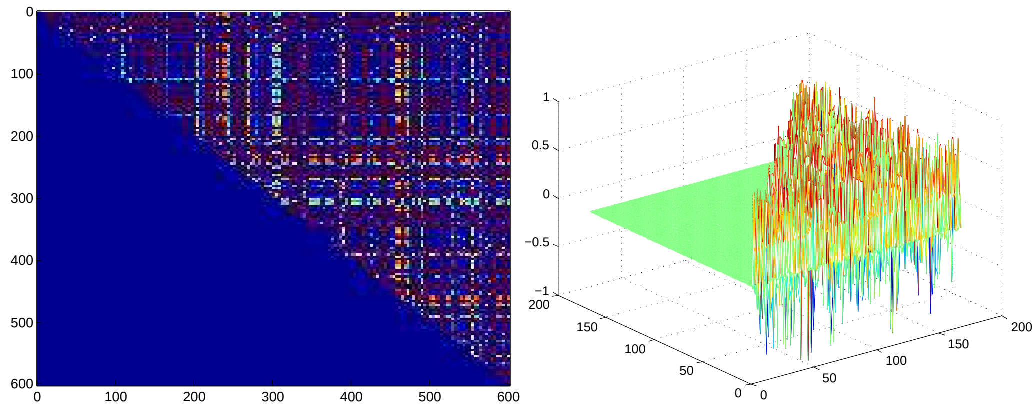
GROUP #	DEFINITION
3	TSX COMPOSITE INDEX
4	TSX – 60 INDEX
7	TRUST UNITS
10	**MATERIALS**
11	METALS & MINING
12	MINING EXPLORATION ONLY
13	GOLD & PRECIOUS METALS
14	STEEL
15	FOREST
16	PAPER
17	CHEMICALS & FERTILIZERS
18	BUILDING MATERIALS
19	CONTAINERS
20	**ENERGY**
21	OIL & GAS – INTEGRATED
22	OIL & GAS – EXPLORATION & PRODUCTION
23	ENERGY – DRILLING
24	ENERGY – EQUIPMENT & SERVICES
26	ENERGY – REFINING, MARKETING, TRANSPORTATION
27	SEISMIC DATABASES
30	**INDUSTRIALS**
31	AEROSPACE & DEFENSE
32	BUILDING PRODUCTS
34	COMMERCIAL PRINTING
36	COMMERCIAL SERVICES
37	CONSTRUCTION, ENGINEERING, FABRICATING
39	DISTRIBUTION & TRADING
41	ELECTRICAL EQUIPMENT
42	ENVIRONMENT
43	INDUSTRIAL CONGLOMERATES
44	INDUSTRIAL MACHINERY
45	OFFICE EQUIPMENT & SERVICES
46	SPECIALTY INDUSTRIES
47	TRANSPORTATION – AIR, LAND, SEA
50	**CONSUMER DISCRETIONARY**
51	AUTO PARTS
52	DEPARTMENT & GENERAL STORES
53	SPECIALTY STORES
54	DISTRIBUTION & WHOLESALING
55	HOTELS, RESTAURANT, LEISURE
56	HOUSEHOLD DURABLES
57	BROADCASTING, CABLE TV
58	MOVIES & ENTERTAINMENT
59	PUBLISHING, ADVERTISING, MARKETING
60	**CONSUMER STAPLE**
61	BREWERS
62	DISTILLERIES
63	BEVERAGES (NON-ALCOHOLIC)
64	DRUGS – RETAIL
65	FOOD – RETAIL
66	FOOD – PROCESSING
67	HOUSEHOLD & PERSONAL PRODUCTS
68	TOBACCO & AGRICULTURE

Figure 7.1: Sectors' Group(Stock Guide, Fundamental Analysis, Appendix C)

69	TRAINING & EDUCATION
70	**UTILITIES**
71	ELECTRIC UTILITIES
72	GAS UTILITIES
73	UTILITIES – OTHER
75	**TELECOMMUNICATIONS SERVICES**
76	TELECOM SERVICES – INTEGRATED
77	TELECOM SERVICES – WIRELESS
79	TELECOM SERVICES – ALTERNATIVE
80	**INFORMATION TECHNOLOGY**
81	COMPUTERS & PERIPHERALS
82	ELECTRONIC EQUIPMENT
83	INTERNET
84	I T CONSULTING
85	NETWORKING
86	SEMICONDUCTORS
87	SOFTWARE
88	TELECOM EQUIPMENT & SYSTEMS
89	OTHER TECHNOLOGIES
90	**FINANCIALS**
91	BANKS
92	FINANCIAL & INVESTMENT SERVICES
93	INSURANCE
94	REAL ESTATE
95	**HEALTHCARE**
96	BIOTECHNOLOGY
97	PHARMACEUTICALS
98	EQUIPMENT, SOFTWARE, SYSTEMS & SERVICES
99	HEALTHCARE FACILITIES

Figure 7.2: Sectors' Group (Stock Guide, Fundamental Analysis, Appendix C)

7.2 Appendix B



(a) 1992 Default Correlations (b) 1992 Default Correlations

Figure 7.3: 1992 Default Correlations

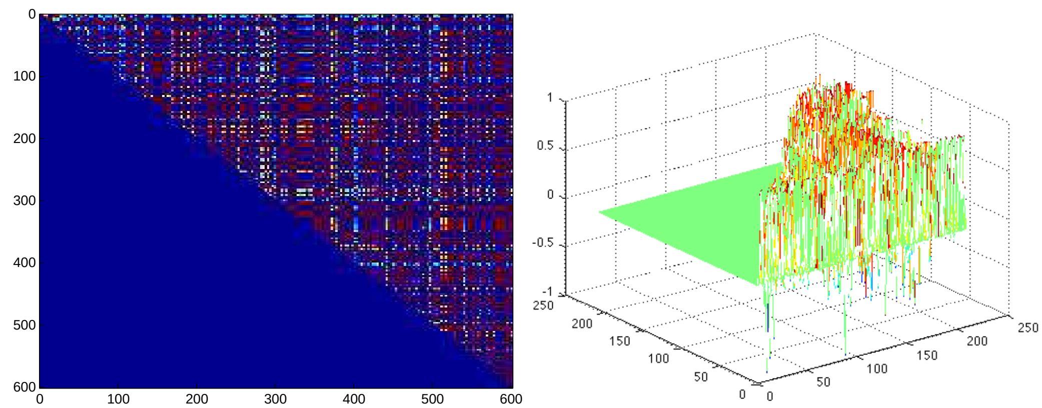


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Figure 7.4: 1993 Default Correlations

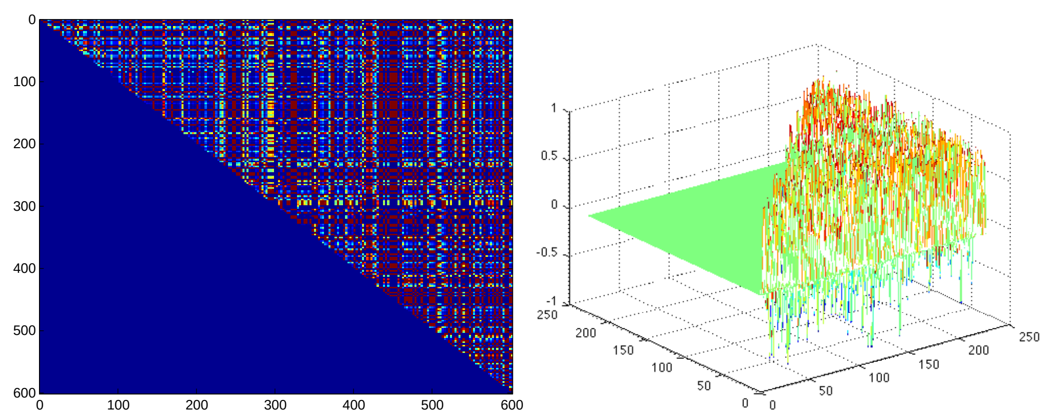

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Figure 7.5: 1994 Default Correlations

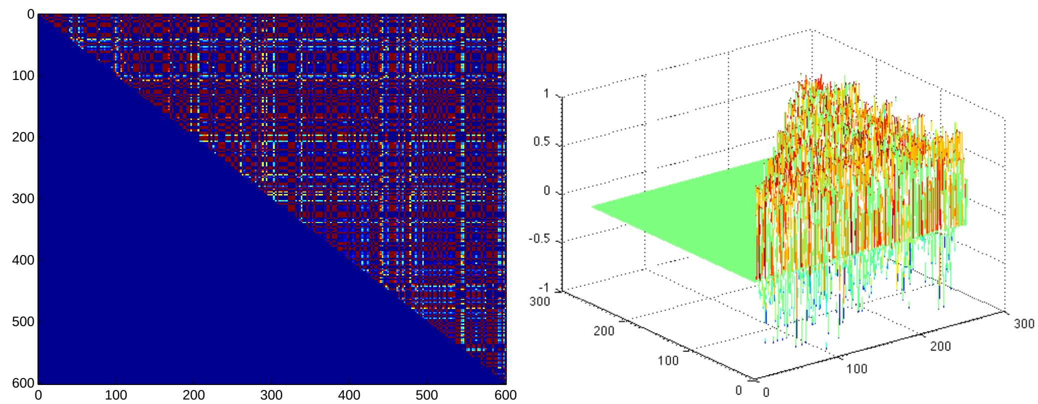

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Figure 7.6: 1995 Default Correlations

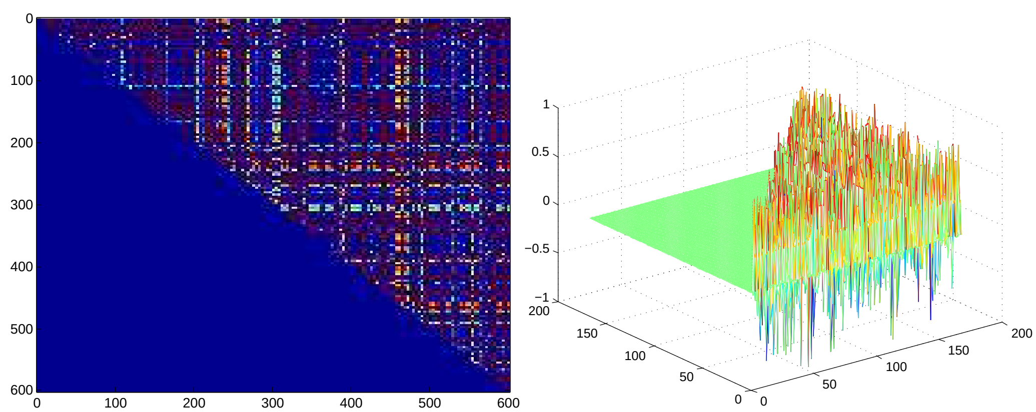

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Figure 7.7: 1992 Default Correlations

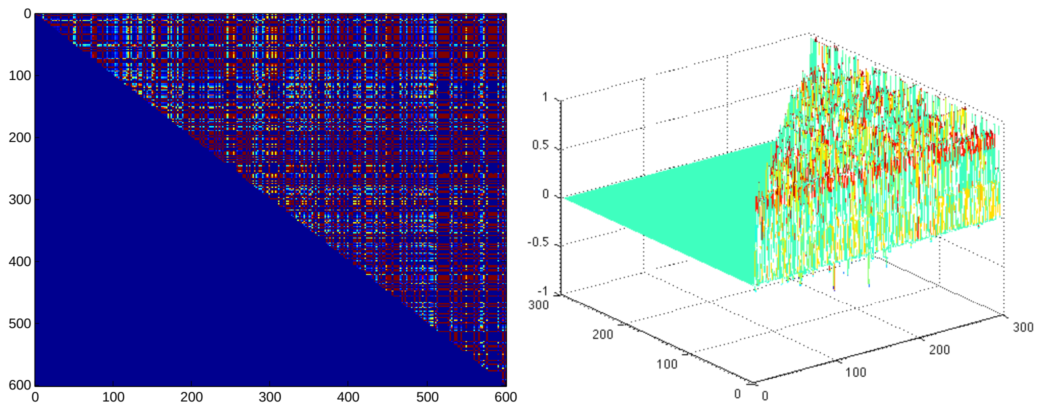

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Figure 7.8: 1996 Default Correlations

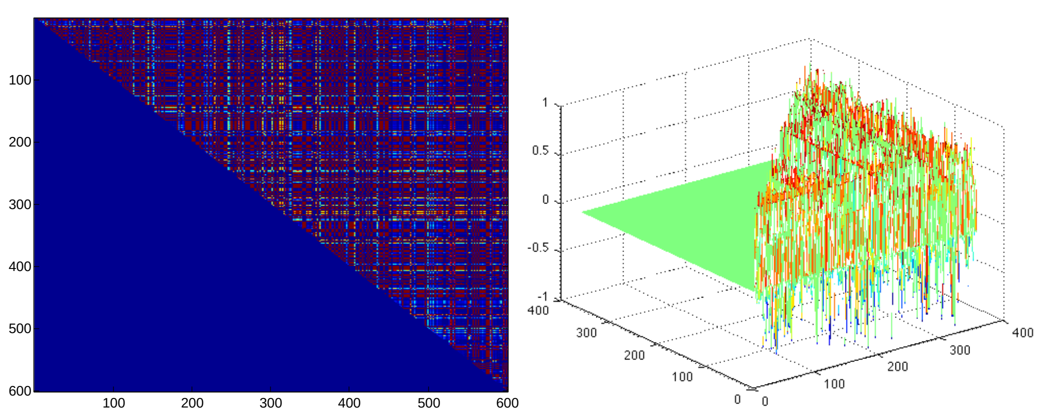

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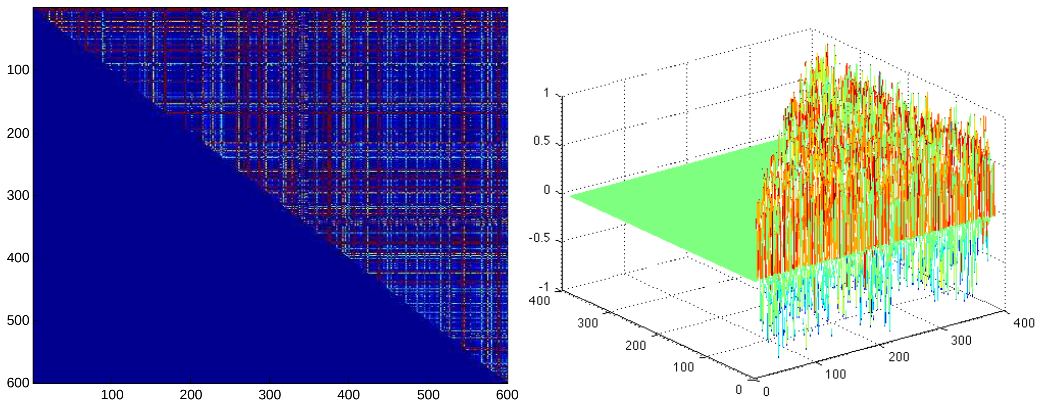
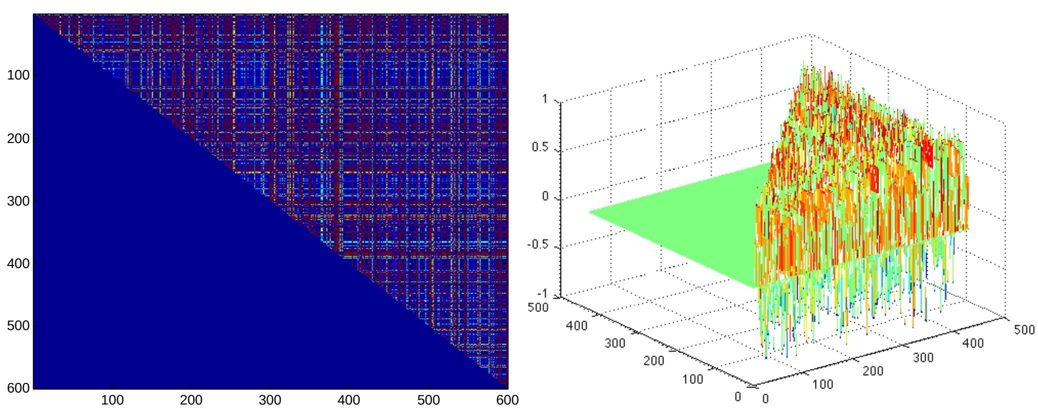
Figure 7.9: 1997 Default Correlations



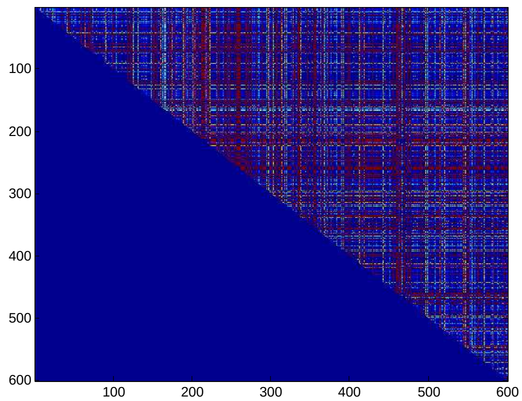
(a) 1998 Default Correlations (b) 1998 Default Correlations

Figure 7.10: 1998 Default Correlations



(a) 1999 Default Correlations (b) 1999 Default Correlations

Figure 7.11: 1999 Default Correlations



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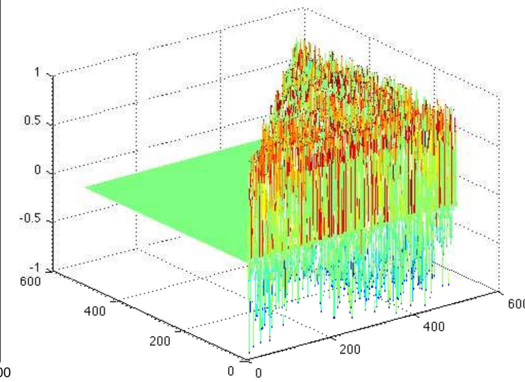
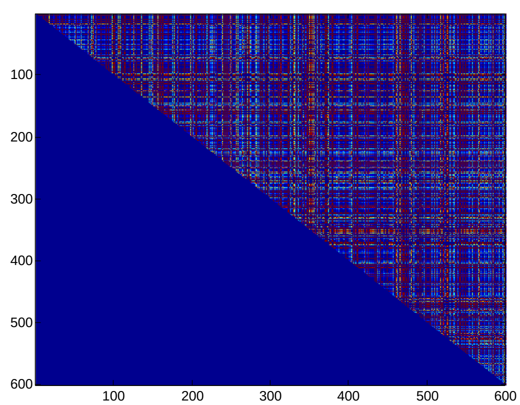


(b) 2000 Default Correlations

Figure 7.12: 2000 Default Correlations



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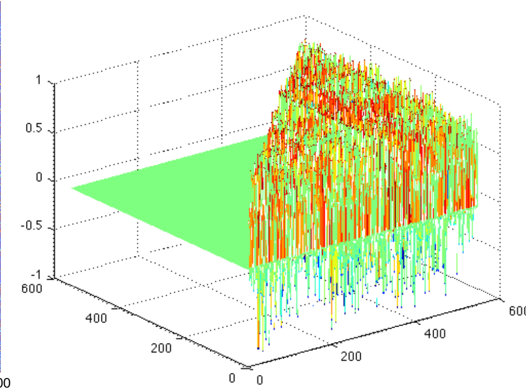

(b) 2001 Default Correlations

Figure 7.13: 2001 Default Correlations

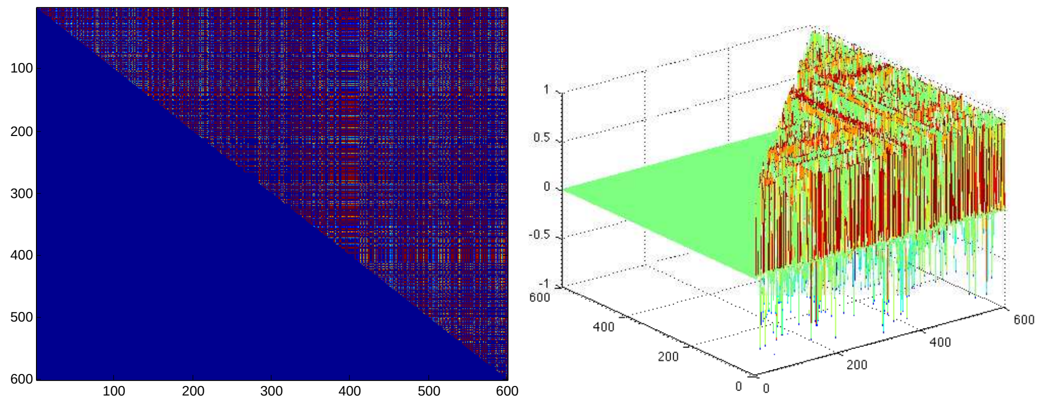

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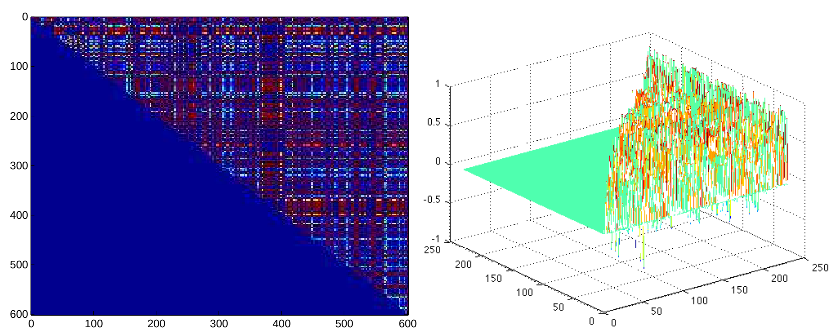
Figure 7.14: 2003 Default Correlations



(a) 2004 Default Correlations (b) 2004 Default Correlations

Figure 7.15: 2004 Default Correlations