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# Three Essays on Default Risk

par:

Sadok Laajimi

Département de Finance

HEC Montréal

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présentée par: Sadok Laajimi

a été évaluée par un jury compose des personnes suivantes :

Dr. Geneviève Gauthier
Président-rapporteur

Dr. Georges Dionne
Président du comite de surveillance

Dr. Pascal François
Membre du jury

Dr. Jean Marc Bourgeon
Examinateur externe

## Résumé


Depuis les travaux de Beaver (1966, 1968) et Altman (1968), la prédiction des défauts des entreprises a bénéficié d'un intérêt accru dans la littérature financière durant les dernières décennies. En effet, la probabilité de défaut est primordiale pour les créanciers et les actionnaires, ainsi que pour l'évaluation des produits de crédit. De plus, et puisque les défauts corporatifs peuvent mettre en péril les institutions financières, les législateurs et les banques centrales peuvent avoir intérêt à mesurer le risque de défaut comme moyen pour assurer la stabilité du système financier.

La plupart des modèles de prédiction des défauts sont des modèles statistiques, qui utilisent des mesures basées sur des informations comptables pour estimer la probabilité de défaut. Les modèles structurels par contre utilisent les informations de marché. Ils appliquent le cadre de Merton (1974) où les actions de la firme sont considérées comme des options d'achat sur la valeur des actifs de l'entreprise, avec la valeur faciale de la dette comme prix d'exercice. Les modèles structurels sont largement utilisés à la fois dans la recherche et dans l'industrie. Une question importante dès lors est de savoir si les probabilités de défaut structurelles contiennent des informations additionnelles, en termes de prédiction des défauts, par rapport aux modèles statistiques.

Dans le premier chapitre on tente de répondre à cette question en combinant les modèles structurels et, statistiques et de tester si cette combinaison améliore la capacité de prédiction des défauts des entreprises canadiennes. Tandis que cet exercice a été entrepris pour les entreprises américaines par Hillegeist et. al. (2004) entre autres, notre étude est la première à examiner la performance relative de ces deux approches alternatives pour les entreprises industrielles publiques canadiennes, en utilisant un échantillon unique d'entreprises en défaut. Les résultats montrent que la probabilité de défaut structurelle contribue de façon significative à expliquer l'occurrence des défauts lorsqu'elle est ajoutée aux variables comptables et macroéconomiques dans le cadre d'un modèle hybride. De plus les variables comptables et macroéconomiques restent significatives en présence de la probabilité de défaut structurelle. Ainsi, ni les probabilités de défauts structurelles, ni les ratios financiers ne sont des mesures

suffisantes pour prédire les défauts. On peut donc conclure que les deux approches sont complémentaires.

Dans le second chapitre, on s'intéresse à une hypothèse centrale des modèles de risque de crédit, à savoir la barrière de défaut. La plupart de ces modèles supposent que la firme fait défaut quand la valeur marchande de ses actifs tombe en-dessous d'une certaine valeur. On utilise la méthode d'estimation par maximum de vraisemblance de Duan (1994) telle que appliquée par Wong et Choi (2006) pour estimer les paramètres du modèle de Brockman et Turtle (2003) pour notre échantillon d'entreprises canadiennes. Avec cette spécification, la méthode de maximum de vraisemblance permet d'estimer le rendement instantané des actifs, la volatilité ainsi que la barrière implicite à partir des prix de marché des actions.

Un premier résultat indique une barrière de défaut implicite positive et significative dans notre échantillon. On trouve aussi que les modèles de Brockman and Turtle (2003) et KMV-Merton atteignent des performances similaires en prédiction des défauts, en outre la barrière de défaut implicite s'avère proche de la valeur des actifs pour les entreprises en défaut. Les régressions multivariées montrent que l'endettement n'est pas le seul déterminant de la barrière de défaut. Le seuil implicite de défaut est aussi relié positivement avec les coûts du financement, négativement à la liquidité, la volatilité des actifs et la taille de la firme. On trouve aussi que les coûts de liquidation, les entraves à la renégociation et le pouvoir de négociation des actionnaires augmentent le niveau de la barrière implicite. Ainsi, les facteurs non-stratégiques tel que l'endettement, la liquidité, et les coûts de financement externe influence le niveau de la barrière implicite, aussi bien que les facteurs stratégiques qui peuvent inciter les défauts opportunistes des actionnaires.

Dans le troisième chapitre, on revoit la littérature sur les modèles structurels de crédit. Du coté théorique ces modèles d'analyse d'actifs contingents procure un cadre qui permet non seulement la valorisation des actifs de l'entreprise et son risque de défaut, mais aussi les décisions d'investissement et de financement ainsi que leur impact sur la valeur de l'entreprise et ses décisions. Nous présentons les principaux modèles

structurels, leurs hypothèses sous-jacentes, ainsi que les résultats empiriques qui y sont reliés.

**Mots Clés :** Risque de défaut, modèle comptable, modèle hybride, modèle structurel, Bourse de Toronto, méthode de maximum de vraisemblance, modèle de défaut à barrière, option barrière, barrière de défaut, default stratégique.

## Abstract

Since the seminal work of Beaver (1966, 1968) and Altman (1968), the problem of default prediction has been an active issue in the finance literature for decades. Bankruptcy forecast is central to shareholders and creditors, as well as debt instruments valuation. Moreover, since the corporate default affect lenders, legislators and central banks may be interested in measuring the default risk as a device to monitor systematic stability.

While most of the default prediction studies are statistical models, as they rely on accounting based-measure to estimate the default probability, structural models, on the other hand, are market-based methods. They apply the framework of Merton (1974) where the equity of the firm is viewed as a call option on the value of its assets, with a strike price equal to the face value of the debt. Structural models have found their way both among practitioners and academic researches. A major issue then is whether the structural default probabilities estimates contains significant incremental information relative to accounting data.

In the first essay we try to answer this question by assessing how combining structural and statistical model can improve our ability to predict Canadian firm's probability of default. While this exercise were carried for U.S. firms by Hillegeist et. al. (2004) among others, our study is the first to investigate the relative power of these competing approaches in predicting default for non-financial public Canadian firms, using a unique dataset on bankrupted firms. Our results show that the estimated Merton-KMV default probability contribute significantly to explain default occurrence, when it is included alongside the relevant accounting and macroeconomic variables in the hybrid model. Moreover, accounting and macroeconomic variables remains significant in presence of the structural default probability measure. Thus, neither structural nor statistical default probabilities are sufficient measure to forecast bankruptcies. We can conclude hence that the two approaches are complementary rather than interchangeable.

In the second essay, we focus on a key assumption of structural credit risk model, namely the level of the default threshold. Most of these models assume that a firm

defaults when the market value of its assets falls below a given boundary. We use the Maximum Likelihood estimation method of Duan (1994) and applied by Wong and Choi (2006) to estimate the Brockman and Turtle (2003) model parameters for our sample of Canadian public firms. In this setting, the Maximum Likelihood method allows estimating simultaneously the asset's instantaneous return, volatility, and the implied barrier level from equity prices.

A first result indicates a positive and significant implied default barrier in our sample. We find also that the Brockman and Turtle (2003) model with the KMV-Merton approach have similar default prediction accuracy and an implied default barrier close to the estimated asset's value for defaulted firms. Regression analysis shows that the leverage is not the only determinant of the default barrier location. The implied default threshold is also positively related with financing costs, and negatively to liquidity, asset's volatility and firm's size. We also find that liquidation costs, renegotiation frictions and equity holders bargaining power increase the implied default boundary level. Thus, leverage, liquidity and outside financing costs influence the implicit barrier location, as well as strategic factors that have the potential to encourage opportunistic default of equity holders.

In the third essay, we review the literature on credit structural models. Contingent claim analysis offers an appealing theoretical framework allowing not only evaluating firm's claims and default risk, but also financing and investment decisions, as well as determining the impact of policy changes on the firm value and decisions. We present the major structural models, their underlying assumptions as well as the related empirical evidences.

**Keywords:** Default risk, accounting model, hybrid model, structural model, Toronto Stock Exchange, maximum likelihood method, default barrier model.

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# Introduction générale

Durant les dernières décennies, la modélisation du risque de crédit est devenue l'un des sujets les plus importants dans le domaine financier et bancaire. En effet, l'un des risques les plus importants auxquels les banques font faces est le risque de défaut de leurs contreparties. Par ailleurs, le besoin d'une meilleure compréhension et de mesures plus adéquates du risque de crédit ont été mis en exergue par le comité de Bâle sur le contrôle bancaire et constituent un impératif pour être conforme aux accords de Bâle II. En effet, vu la vulnérabilité du tout le système financier face aux défauts bancaires, les organismes de règlementation financière et les banques centrales ont tout intérêt à s'assurer que les banques disposent de fonds nécessaires et de mécanismes permettant de mitiger le risque de crédit promptement.

La débâcle financière du subprime en 2007 aux États-Unis n'a fait que souligner l'importance de mesures appropriées du risque de crédit. L'absence de cadre adéquat de mesure de risque de crédit s'est traduite par des milliers de défauts de ménages et de faillite de plusieurs institutions financières.

L'estimation des probabilités de défaut et la prédiction des défauts est une composante majeure du risque de crédit qui a bénéficié d'un intérêt accru dans la littérature financière. Pour aboutir à une mesure optimale du risque de défaut des entreprises cotées en bourse, deux approches sont généralement considérées. D'une part, les modèles structurels sont basés sur le prix de marché des actions et font appel à la théorie des actifs contingents pour mesure le risque de crédit. Ces modèles présentent l'avantage d'incorporer rapidement les informations sur les conditions financières des entreprises et permettent d'obtenir une mesure prévisionnelle de la solvabilité de l'entreprise. Nous consacrons le premier essai de cette thèse à la revue des développements théoriques et les preuves empiriques des modèles structurels. Toutefois, les prix des actions peuvent ignorer certains aspects relatifs au risque de crédit de l'entreprise et peuvent aussi être contaminés par des frictions.

D’un autre coté les modèles statistiques, identifient les informations comptables et les ratios financiers qui permettent de mieux prédire les défauts corporatifs. Toutefois, l’information comptable donne une image du passé de l’entreprise et manquent de flexibilité, puisque les états financiers sont produits annuellement dans la majorité des cas.

Dans le second essai, et en collaboration avec Dr. Georges Dionne, Sofiane Mejri et Madalina Petrescu, nous comparons ces deux approches. En utilisant les entreprises canadiennes cotées en bourse, nous testons si la combinaison des probabilités de défaut obtenues du modèle structurel avec la valeur contenue dans leurs états financiers permet de mieux prédire la probabilité de défaut des entreprises.

Par ailleurs, les modèles structurels sont basés sur l’hypothèse que le défaut survient lorsque la valeur des actifs descend sous un certain niveau, soit la barrière de défaut. Cette barrière ne correspond pas forcément à la valeur faciale de la dette. Dans le troisième essai, avec Dr. Georges Dionne, et à partir des prix des actions des entreprises cotées en bourse, nous estimons les barrières de défaut à la fois pour les entreprises en défaut et les entreprises survivantes. nous étudions par la suite les caractéristiques des entreprises et les facteurs stratégiques qui déterminent le niveau de la barrière de défaut.

# Chapter 1

# Structural Credit Risk Models: A Review

## Introduction

In this paper, we seek to provide a summary of recent developments in structural credit risk models literature. In recent years credit risk modeling and measures knew increasing interest from both financial institutions and academics. This is due mainly to two reasons. First, the Capital Accord of 2006, or Basel II, allows large banks to use their internal models to assess their capital requirement instead of the more constraining standardized model. Second, the huge increase of off-balance-sheet derivatives and the rising use of the securitization of loans necessitate more developed credit analysis methods.

The last decades showed a growing number of studies modeling the decision to default, or endogenous default models. Our primary goal is to present a taxonomy of these models and a comparison between their underlying assumptions, their results and the related empirical evidence. We also, briefly cover the evolution of the credit risk methodology and distinguish the different categories of models. We point out the forces and limitation of each category. Here, we focus mainly on structural models. Previous reviews covering structural models include Bielecki and Rutkowski (2002), Uhrig-Homburg (2002), Lando (2004) and François (2005).[1].

Despite the appealing theoretical underpinning of structural models, they lack accuracy in explaining the cross-section of credit spreads measured by the yield difference between risky corporate bonds and riskless bonds. The default spread obtained through structural models is far below the credit spread (Eom, Helwege, and Huang, 2004).

[1] Reduced-form models are outside the scope of this review. For reviews on reduced-form models please refer to Duffie and Singleton (1999) or Bielecki and Rutkowski (2002) for instance.

Moreover, structural models underpredict short-term default probabilities (Leland, 2004).

To overcome these limitations, a first trend of the literature propose several extensions to account for more realistic features of financial markets and firm's financing and investment decisions. These developments include specifying stochastic models of risk-free interest rate (Longstaff and Schwartz, 1995; Kim, Ramaswamy, and Sundaresan, 1993; Briys and de Varenne, 1997). Another trend of the literature accounts for the possibility of strategic debt service and debt renegotiation (Hart and Moore, 1994; 1998; Anderson and Sundaresan, 1996; Mella-Barral and Perraudin, 1997; Acharya and Carpenter, 2002). Recent studies points out the difference between private (bank) and public debt in renegotiation (Hackbarth, Henessy and Leland, 2007; Carey and Gordy, 2007). Other researches account for departure from absolute priority rule and renegotiation under Chapter 11 (François and Morellec, 2004; Broadie, Chernov, & Sundaresan, 2007). Another approach considers a dynamic capital structure (Collin-Dufresne & Goldstein, 2001; Goldstein, Ju, & Leland, 2001; Ju, Parrino, Poteshman & Weisbach, 2005), while Mauer and Triantis (1994), Childs, Mauer and Ott (2005) and Sundaresan and Wang (2007) considers endogenous investment. The cash holding management policy is accounted for in Acharya, Huang, Subrahmanyam and Sundaram (2006), Anderson and Carverhill (2007), and Asvanunt, Broadie, and Sundaresan (2007). Leland (1998) allows for optimal dynamic risk management and Sarkar and Zapatero (2003) consider mean reverting cash flows. Zhou (2001), Duffie and Lando (2001) and Giesecke & Goldberg (2004) add a jump component to the value process of assets allowing for "surprise" default at the cost of closed-form solution. Alernatively, Hackbarth, Miao & Morellec (2006) consider jumps in the cash flow process with regime change. Finally, Longstaff (1996), Morellec (2001) and Ericsson and Renault (2006) include a liquidity premia to price corporate debt. We seek to provide a synthesis of the assumptions and the major results of these structural models as well as the related empirical evidence.

As an alternative explanation to the credit spread puzzle, several factors beside default risk explain corporate credit spreads. Indeed, variables that in theory determine credit spreads have limited explanatory power as documented by Collin-Dufresne, Goldstein and Martin (2001) and Campbell and Taksler (2003) among others. Firm specific factors and systematic market risk have substantial explanatory power of credit spread differential. Liquidity is also found to be an important determinant of the credit spread: both bond-specific illiquidity and macroeconomic measures of bond market liquidity explain variations in the observed credit spread (Longstaff, Mithal, and Neis, 2005 and Chen et al., 2007). These evidences suggest that the limited ability of structural models to replicate the observed credit spread is more due to the presence of non-default factors in credit spread rather than their failure to capture the default risk of corporate debt.

Structural models share a common theoretical foundation, namely the classical Merton (1974) model. In this setting, and for a particular diffusion process of asset's value, the firm defaults when its assets reach an exogenous level. Given the central role of the Merton model for all the subsequent structural models, an obvious starting point is to present a short description of this model.

## 1.1. The Merton approach

The Merton model relies on the assumption that default is triggered by the value of the assets, therefore, the starting point is to set the diffusion process of the assets. The value of assets $V$ is assumed to follow a log-normal diffusion process, that is under the physical probability measure:

$$dV_t / V_t = (r - \delta)dt + \sigma dW_t \tag{1.1}$$

where $W_t$ is the a standard Brownian motion, $\mu$ is the instantaneous expected return on assets, $\sigma$ is the constant proportional volatility of the return on the firm value, $\delta$ is the firm's total dividend payout to shareholders. Moreover, the additional assumption of simple capital structure is made. The firm liabilities are represented by a single zero-coupon paying bond maturing at $T$. The value of the firm is the sum of equity, $E$, and the debt value with face value $D$.

The value of the equity represents a call option on the assets of the firm with maturity $T$ and strike price of $D$. The risky zero-coupon bond is equal to its corresponding risk-free zero-coupon bond minus the value of an European put option on the firm's assets $V$, a strike of $D$, and maturity $T$. If the asset value at the maturity of the zero-coupon bond is sufficient to make the necessary payment then the firm remains the property of the shareholders. Otherwise, the firm defaults and the bondholders take possession of the firm's assets and the shareholders receive nothing.

The Merton model assumes that the assets of the firm are traded and the market is sufficiently complete, this allows using risk neutral probability measure, and replaces the expected return in equation (1.1) by the risk-free rate $r$. Hence, the Black and Scholes (1973) formula can be applied to value the equities of the firm as an European call option:

$$E = V\,N(d_1) - De^{-r(T-t)}N(d_2),$$

$$d_1 = \frac{\ln(V/D) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2 = d_1 - \sigma\,\sqrt{T-t},$$

where $E$ is the value of equities, and $N$ is the Normal (0,1) cumulative distribution function.

The simplicity of the Merton model relies on applying the Black and Scholes formula of pricing the European options to value firm's equity and debt. However, this comes at the cost of too simplistic assumptions regarding the asset value process, interest rate, and the capital structure.

The assumption of a single zero-coupon bond for the liabilities of the firm is far from being realistic. Geske (1977) relaxes this assumption and considers the firm's liabilities as a coupon-paying bond, where the equity holders make the needed coupon payment through issuance of new equities. The coupon payments can cause the default of the firm. At each coupon date, the shareholders have the choice either to make the payment to bondholders or to forego the coupon payment causing the default of the firm. In this setting, the coupon bonds are valued as compound options.

The subsequent contributions in the structural models literature are mainly extensions of the Merton basic framework. One conventional way to regroup these pricing models is their assumptions regarding the default trigger. While the exogenous models assume a default trigger determined solely by the capital structure of the firm, endogenous models assume that equity holders/managers decide to default whenever it is optimal for them to stop paying the firm's debt service. Depending on the default trigger, we could classify a model as endogenous or exogenous. In addition to the exogenous /endogenous default classification, we refine these categories by distinguishing the default event assumed in the different models. Indeed, three possible default triggers where identified in the literature. First, the most commonly used is the zero net worth trigger. That is the firm default whenever its asset's value falls below the nominal of debt or some other exogenous trigger. This category includes Merton (1974), Brennan and Schwartz (1978), Longstaff and Schwartz (1995) and Briys and de Varenne (1997) models. These models are thus value-based. The second set of structural models considers that the firm defaults as soon as its cash flow is insufficient to face the debt service requirement. Thus, the default can be triggered by a liquidity shortage in this setting. We refer to these models as cash-based models. This category is represented by the contributions of Kim, Ramaswamy, and Sundaresan (1993), Anderson and Sundaresan (1996) and Ross (2005). The major drawback of this approach is that external financing is assumed unavailable. In addition to the unrealistic feature of this assumption, in presence of external financing costs, cash management becomes possible. Acharya, Huang, Subrahmanyam and Sundaram (2006), Anderson and Carverhill (2007), and Asvanunt, Broadie, and Sundaresan (2007) accounts for financing costs and cash management. These cash-based and value-based models are exogenous models in the sense that default is a result of breaching an exogenous covenant.

Endogenous default models, pioneered by Black and Cox (1976), derive the minimum asset level under which the shareholders maximize their own claim by stopping debt service payment. The default in this setting become a result of a decision making process by the firm's stakeholders. The basic Black and Cox model was extended along several dimensions. For instance, Leland (1994) and Leland and Toft (1996) include tax advantage of debt and bankruptcy costs. Hart and Moore (1994, 1998), Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) and Mella-Barral (1999) include the possibility of strategic defaults of the equity holders in order to obtain debt concessions from creditors. The interest of the endogenous default model reside in their ability to offer a richer modeling of the default decision and to account for stylized facts regarding firm's default and reorganization decision and outcome. Moreover, the contingent claims analysis provide a general framework allowing not only evaluating firm's claims and default risk, but also financing and investment decisions as well as determining the impact of policy changes on the firm's value and decision. In the next sections we present the different categories of structural models.

## 1.2. Exogenous default

The first exogenous default model is the Merton model in which the default barrier is equal to the nominal value of the debt. Kim, Ramaswamy and Sundaresan (1993) extend the Merton model to incorporate both default risk and interest rate risk. The model is cash-based in the sense that the default is triggered by a cash-flow shortage. The asset value of the firms follows a geometric Brownian motion with a proportional dollar payout ratio $\delta V$ to security holders. Moreover, the firm's debt is constituted of a single coupon-paying bond, with a continuous coupon flow, $c$, until the maturity. The asset value model is given by $dV = V(\mu - \delta)dt + \sigma V dz$ .The firm defaults the first time its cash flow falls below the coupon payment. This implies that the default barrier is given by $V_B = c / \delta$.

The short-term interest rate is given by the Cox, Ingersoll Ross (CIR) process, that is

$$dr = a(b - r)dt + \sigma_r \sqrt{r}\, dw$$

The two Wiener process $dw$ and $dz$ are correlated with an instantaneous correlation coefficient $\rho_{Vr}$ The assumption of a CIR short-term interest rate process comes at the cost of a numerical solution for bond price.

The Longstaff and Schwartz (1995) model is similar to the Kim, Ramaswamy and Sundaresan (1993) model in the sense that it considers both the default risk and the interest rate risk to price the corporate debt. A major difference however, is that the short-term interest rate is assumed to follow a Vasicek model, that is:

$$dr_t = a(b - r_t)dt + \sigma_r dw_t$$

where *a* and *b* are constants, while the dynamic of the total value of assets is given by:

$$dV = \mu V dt + \sigma V dz$$

Here again the two standard Wiener processes $dw$ and $dz$ are correlated with an instantaneous correlation coefficient $\rho_{Vr}$.

Longstaff and Schwartz (1995) assume moreover that the value of the firm is independent of its capital structure and assumes a threshold value $V_B$ for the firm at which the financial distress occurs. As soon as the value of the firm value falls below $V_B$, the firm immediately enters financial distress and defaults on all of its obligations. Longstaff and Schwartz argue that this definition of financial distress is consistent with both the cases where the generated cash flows are insufficient to pay its current obligations as well as the violation of the net worth covenant[2]. Briys and de Varenne (1997) criticize this default definition and argue that when the corporate bond reaches maturity the firm can be in a solvent position, i.e, with the value of assets above the default threshold, but with no sufficient assets to pay the face value of the bond at maturity. This is equivalent to a situation where $V_B < V_T < F$ where $V_T$ is the value of assets at maturity *T*, *F* is the face value of the debt and $V_B$ is the default threshold.

[2] Wruck (1990) and Kim, Ramaswamy, and Sundaresan (1992) discuss the difference between the flow-based and the cash-based insolvency.

Longstaff and Schwartz assumes also that when a reorganization occurs the security holders receives $1-w$ times the face value of the security at maturity, where $w$ represents the percentage writedown of security in reorganization. The recovery in their setting is on the treasury value of the security and is assumed to be a fixed constant.

The value of the riskless bond with nominal 1$ and maturity $T$ is given by the Vasicek (1977) model and is central in the derivation of the valuation expressions for risky corporate securities and is denoted by $D(r,T)$ [3].

The price of a contingent claim, that pays 1$ if default doesn't occur during the life of the bond and $1-w$ otherwise, is given by the quasi closed-form solution:

$$P(X,r,T) = D(r,T) - wD(r,T)Q(X,r,T), \tag{1.2}$$

where $X = V/V_B$ and $Q(X,r,T)$ represents the risk neutral probability of default.

The price of risky discount bond is a function of $V$ and $V_B$ through their ratio $X$ only, which can be viewed as a summary measure of the default risk of the firm. By consequence the specification of $V$ and $V_B$ separately is no longer necessary which simplifies the implementation of the model.

The first term in equation (1.2) corresponds to the price of the bond in absence of default risk, while the second term represents a discount for the default risk of the bond. This discount is the product of two components: the first component, $wD(r,T)$ is the present value of the writedown on the bond in case of default, and the second term, $Q(X,r,T)$, is the risk neutral probability of default. This last term is solved recursively by numeric methods.

Nielsen, Saá-Requejo and Santa-Clara (1993) extends the Longstaff and Schwartz model by assuming a stochastic default threshold $V_B$ and also suppose a Vasicek process for. short-term interest rate.

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3 See Vasicek (1977) for the closed-form of the discount bond.

Briys and de Varenne point out that the payment to creditors upon default is independent of the level of the stochastic barrier and the value of assets. This in turn could lead to situations where the bondholders receive more than the assets value at bankruptcy[4].

To overcome the limitations of the Nielsen, Saá-Requejo and Santa-Clara and the Longstaff and Schwartz models discussed above, Briys and de Varenne (1997) propose a model with stochastic interest rate, where a generalized Vasicek model drives short-term interest rate:

$$dr_t = a(t)(b(t) - r_t)dt + \sigma_r(t)dw_t$$

where $a(t)$, $b(t)$ and $\sigma_r(t)$ are deterministic functions, and $\sigma_r(t)$ is the instantaneous standard deviation of $r_t$. They define the exogenous default triggering barrier as

$V_B(t) = \alpha F P(t,T)$ where $0 \leq \alpha \leq 1$, $F$ is the face value of the corporate bond and $P(t,T)$ is the default-free zero coupon bond maturing at *T*.

Under this specification of the default barrier, a closed-form solution to corporate risky zero-coupon bond is obtained.

## 1.3. Endogenous default

For the endogenous models, we just describe in detail the seminal contributions of Black and Cox (1976), Leland (1994) and Leland and Toft (1996). We then review more recent developments and other extensions.

### 1.3.1. The Black and Cox (1976) model

In the Merton model, the timing of the default event is questionable. Indeed, the default time is restricted to the maturity of debt, independently of the evolution of the asset's value before the maturity. Default cannot occur before the maturity of debt. In response to this shortcoming, Black and Cox (1976) pioneered the first passage models, where the

[4] See also Collin-Dufresne and Goldstein. 2001

firm defaults as soon as the value of its assets reaches a non-random default barrier $V_B$. In this case, bondholders get $V_B$ and equity holders get nothing. We now describe in detail the assumptions and the major results of this approach. Black and Cox suppose a perpetual debt, e.g. consol bond, paying constant coupon rate, $c$ ,proportional to the firm value. Under these assumptions, the following process drives the firm's value:

$$dV_t / V_t = (r - \delta)dt + \sigma dW_t \tag{1.3}$$

where, the interest rate $r$ is constant and $\delta \geq 0$ is the payout ratio .

Even if the dividend payments are allowed for in equation (1.3), the shareholders cannot sell assets in order to pay coupons. The coupon payments are possible only through issuance of new equities. However, under a given asset value, $V_B$, the stockholders are no longer willing to issue new equities in order to pay coupons. For a given $V_B$, the optimal default time take the following form: $\tau^* = \inf\{t \geq 0 : V_t \leq V_B\}$. For a fixed default boundary, the price of the consol bond, *D(V),* has to solve the following ordinary differential equation:

$$\frac{1}{2}V^2\sigma^2 D_{VV} + rVD_V - rD + c = 0 \tag{1.4}$$

subject to the lower boundary condition $D(V_B) = \min(V_B, c/r)$ since the value of the bond does not exceed the default free value of the consol bond, that is *c/r*. The upper boundary condition is given by: $\lim_{V \to \infty} D_V(V) = 0$ since the value of the bond tends to its riskless value as the value of the assets tends to infinity. The solution to this differential equation gives the value of debt:

$$D(V) = \frac{c}{r} + \left(V_B^{\alpha+1} - \frac{c}{r}V_B^{\alpha}\right)V^{-\alpha} \qquad \text{where} \quad \alpha = 2r/\sigma^2$$

In order to maximize the value of their equities, stockholders chose the optimal default boundary in such a manner that the debt value *D(V)* is minimized. The optimization problem leads to the optimal default boundary[5]:

$$V_B^* = \frac{c}{r + \sigma^2 / 2} \tag{1.5}$$

Note that the optimal level of the barrier is independent of the current value of the firm. However, the optimal barrier increases with the coupon size and decreases in the asset volatility. Moreover, the barrier is decreasing in the asset's volatility. This result can be explained by a higher value of the option to wait for a recovery of asset's value when its volatility is higher.

We should notice here that both the Merton and Black and Cox models do not allow for debt that is coupon-paying and has finite maturity. They also do not allow analysis of optimal capital structure.

### 1.3.2. The Leland (1994) model

As in the Black and Cox model, Leland (1994) model assumes that the firm issues a consol bond paying a coupon at a rate *c* and the firm defaults when the process *V*, as given by equation (1.3), hits for the first time a lower barrier $V_B$. The major contributions of the Leland model are the introduction of the tax shield of debt and the bankruptcy costs. When the firm defaults, the bondholders receive a recovery payment of $(1-\lambda)V_{\mathbf{B}}$ and the shareholders receive nothing with $0 \le \lambda \le 1$, $\lambda$ being the cost of bankruptcy. The value of bankruptcy cost *BC* is a decreasing convex function of *V*.

Moreover, let $\tau$ be the tax rate. The firm benefits from the tax shield $\tau C$ from debt financing as long as it remains solvent. In case of default, tax benefit cannot be claimed. The tax benefit is modeled as a security that pays a constant coupon $\tau C$. The value of this security, *TB* , is increasing in the value of assets. The total value of the firm, *v*, is then the sum of the firm's assets, *V*, and the value the tax shield of the interest payment, *TB(V),* minus the value of the bankruptcy costs, that is:

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5 This holds when there is no dividend payment.

$$v(V) = V + TB(V) - BC(V) \tag{1.6}$$

with

$$TB(V) = \tau \frac{C}{r}\left(1 - \left(\frac{V}{V_B}\right)^{-\alpha}\right)$$

$$BC(V) = \alpha V_B \left(\frac{V}{V_B}\right)^{-\alpha} \text{ where } \alpha = 2r/\sigma^2.$$

Holding the default barrier level constant, and solving the ordinary differential equation similar to equation (1.4) with the adequate boundary conditions, the debt value is given by:

$$D(V) = \frac{c}{r} + \left((1-\lambda)V_B - \frac{c}{r}\right)\left(\frac{V}{V_B}\right)^{-\alpha} \tag{1.6}$$

The effect of the debt issuance has two contrary effects on the value of the firm. The first effect reduces the firm value since more debt implies higher value of bankruptcy costs. On the other hand, increased interest payment implies more tax shield, due to their deductibility, which in turn increases the value of the leveraged firm.

Leland considers, in a first step, the case of unprotected debt, that is, there is no lower bound imposed on the value of the endogenously chosen default barrier. The equity holders set the default barrier with the objective of maximizing their claims without constraints, that is $E(V) = v(V) - D(V)$ where $v(V)$ and $D(V)$ are given by equations (1.6) and (1.7) respectively. The optimal default barrier is obtained by solving the equity-holders problem:

$$V_B^* = \frac{(1-\tau)c}{r + \sigma^2/2}. \tag{1.8}$$

When the tax benefits are neglected, the default boundary is equal to the one derived in the Black and Cox model. However, we note that the optimal default boundary is insensitive to the bankruptcy costs, even though these costs lower the value of the firm.

The reason is that the maximized equity value is independent of the default level. Indeed, all the reduction in the firm's value related to bankruptcy costs comes from the decreased value of debt value.

With the closed form formulas for the debt and equity values, Leland derives the optimal capital structure of the firm. In addition, the firm determines the optimal coupon rate that maximizes the value of the leveraged firm. By considering the tradeoff between the tax advantage and the bankruptcy costs, a relation is established between bond prices and the optimal leverage to the value of assets, the firm risk, taxes, bankruptcy costs and interest rates.

### 1.3.3. The Leland and Toft (1996) model

The Leland (1994) model relies on the extreme assumption of a perpetual debt in order to obtain a closed form formula for debt, equities and firm value. Leland and Toft (1996) relax this assumption. Instead, they assume that debt is continuously rolled over. That is, the same amount of principal is issued each time an already outstanding bond matures. This modeling of the firm's debt guarantees that, at any time, the outstanding principal, coupons payments and average debt maturity are independent of time, despite the fact that each individual bond has a finite maturity.

More specifically, Leland and Toft begin by considering a single bond with maturity $t$, paying a continuous coupon flow $c(t)$ and principal $p(t)$. In case of default the bondholders receives a fraction $\rho(t)$ of the default-triggering asset value $V_B$. In a risk neutral valuation framework, and for a given exogeneous $V_B$, the value of the bond is given by:

$$d(V;V_B,t) = \frac{c(t)}{r} + e^{-rt}\left[p(t) - \frac{c(t)}{r}\right]\left[1 - F(t)\right] + \left[\rho(t)V_B - \frac{c(t)}{r}\right]G(t), \tag{1.9}$$

where $F(s)$ is the cumulative distribution function of the first passage time to bankruptcy,

$$G(t) = \int_{s=0}^{t} e^{-rs} f(s; V, V_B) ds,$$

and *f(s)* is the density function of the first passage time to bankruptcy.

Leland and Toft also assume that the firm issue new bond at par with maturity $T$ at a rate $p = P/T$ per year, where $P$ is the total principal value of all outstanding bonds. Thus, previously issued bond principal that matures each year is replaced. This allows keeping the total principal of outstanding bonds, $P$, and the coupon payment per year, $C$, constant until $T$, if the firm remains solvent. The total debt service is then equal to $C + P/T$ per year, and is independent of time. Moreover, they assume that the fraction of assets received by bondholders in case of default is independent of the bond maturity in such a way that whenever the default occurs bondholders always receive $(1-\alpha)V_B$.

The value of all outstanding bonds can then be expressed as:

$$\begin{aligned} D(V; V_B, T) &= \int_{t=0}^{T} d(V; V_B, t) dt \\ &= \frac{C}{r} + \left(P - \frac{C}{r}\right)\left(\frac{1 - e^{-rT}}{rT} - I(T)\right) + \left((1-\alpha)V_B - \frac{C}{r}\right) J(T), \end{aligned}$$

where

$$I(T) = \frac{1}{T}\int_{0}^{T} e^{-rt} F(t) dt$$

and

$$J(T) = \frac{1}{T}\int_{0}^{T} G(t) dt.$$

This stationary capital structure allows Leland and Toft (1996) to find an explicit formula for the optimal value of the default barrier, which depends on the maturity of debt. The value of equity is maximized for the optimal default barrier $V_B$:

$$V_B^* = \frac{(C/r)\big(A/(rT) - B\big) - AP/(rT) - \tau Cx/r}{1 + \alpha x - (1-\alpha)B}, \tag{1.10}$$

where

$$A = 2ae^{-rT}N(a\sigma\sqrt{T}) - 2zN(z\sigma\sqrt{T}) - \frac{2}{\sigma\sqrt{T}}n(z\sigma\sqrt{T}) + \frac{2e^{-rT}}{\sigma\sqrt{T}}n(a\sigma\sqrt{T}) + (z-a),$$

$$B = -\left(2z + \frac{2}{z\sigma^2 T}\right)N(z\sigma\sqrt{T}) - \frac{2}{\sigma\sqrt{T}}n(z\sigma\sqrt{T}) + (z-a) + \frac{1}{z\sigma^2 T},$$

$$a = \frac{(r - \delta - (\sigma^2/2))}{\sigma^2},$$

$$z = \frac{\sqrt{a^2\sigma^4 + 2r\sigma^2}}{\sigma^2} \quad and \quad x = a + z.$$

Equation (1.10) shows that the bankruptcy triggering barrier depends on the debt maturity *T*. As the maturity of debt tends to infinity the barrier tends to the one defined by equation (1.8). Moreover, LT note that for long term debt structures the bankruptcy threshold is inferior to the principal value of debt.

### 1.3.4. Strategic default models

The existence of bankruptcy costs may lead to situations where it is optimal to debt holders to concede a part of coupon payment to equity holders through renegotiation of debt. However, such concessions can induce the equity holders to opportunistic default in order to profit from such concessions. Indeed, Asquith, Gertner, and Scharfstein (1994), Franks and Torous (1989, 1993), and Weiss (1990) reports evidence of opportunistic behavior of stakeholders due to the bankruptcy procedure as well as deviations from absolute priority rules. However, renegotiation is not always possible and inefficiency due to bankruptcy and liquidation could not be avoided.

Hart and Moore (1998) consider a two period discrete model. They also assume that there is no asymmetry of information between the debtor and creditor. The returns on the project at the end of the first and second period, $R_1$ and $R_2$, are specific to the debtor /entrepreneur who promises a stream of payment to the debt holder. As long as he makes these payments, the creditor continues to run the project. Otherwise, the creditor can seize the firm and liquidate the project assets. In this case, there is room for renegotiation of the contract because the borrower can extract debt concessions by threatening to withdraw his human capital from the project.

Anderson and Sundaresan (1996) use a discrete time model where all the bargaining power belongs to shareholders. They posit a binomial process for the value of the firm and assume that the firm generates a cash flow proportional to its assets value, at each time point. Moreover, all the involved parties have full information on the state of the nature. The terms of the contract require a constant coupon payment of $CS_t$ out of the generated cash flows at each time point until the maturity of debt $T$. However, if this generated cash flow is not sufficient to make the necessary payment, the firm is not automatically thrown into bankruptcy. The equity holders make a take-it-or-leave-it offer that do not exceed the generated cash flows. In this case, the creditors face a decision node where they have to choose between two options: (1) liquidate the firm and receives the liquidation value less the liquidation costs, or (2) accept the proposed payment. The presence of liquidation costs is an incentive for the creditors to accept the offered payment. In a game theory setting, the equity holders determine the minimum coupon payment above which the creditors are not willing to force liquidation. Thus, Anderson and Sundaresan show that recursive equilibrium is possible and is unique when the liquidation costs are strictly positive. They also demonstrate that accounting for bankruptcy costs leads to credit spreads that are closer to the observed ones, relative to models that do not account for strategic debt service.

While Anderson and Sundaresan (1996) give all the bargaining power to shareholders, Fan and Sundaresan (2000) propose a bilateral bargaining in a game-theoretic setting that can accommodate varying bargaining powers between debt holders and equity holders. They develop continuous-time model that extends Anderson and Sundaresan (1996) approach along several dimensions. The main extension of Fan and Sundaresan (2000) is the inclusion of a tax advantage of debt. In the presence of such advantage, a bargaining on the firm value becomes possible and its value becomes endogenous, since it depends on the optimal reorganization policies.

Indeed, when corporate taxes are considered, the value of assets could differ from the value of the firm. Two bargaining formulations by claimants are then possible. In the first, the borrower and the lender bargain over the value of the assets of the firm. The future tax benefits are assumed to be lost making the value of the assets coincide with the value of the firm. They also consider that the liquidation of assets implies fixed and proportional costs, $\alpha$ and *K*, respectively. Debtors settle for a debt-equity swap in which the lenders exchange their claims for equity for an endogenously determined barrier, which can be seen as a distressed exchange where the absolute priority rule is violated. The firm becomes an all-equity firm in this case, which in turns avoids costly liquidation. The sharing rule, $\theta$, is subject to the Nash bargaining formulation, and its optimal value depends on the relative bargaining power of equity holders, $\eta$:

$$\theta^* = \min\left(\eta\frac{\alpha V_S + K}{V_S}, \eta\right), \tag{1.11}$$

where $V_S$ is the trigger point of the debt equity swap.

The value of equity satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 V^2 E_{VV} + (r-\delta)VE_V - rE + \delta V - c(1-\tau) = 0, \tag{1.12}$$

where $V$ is the asset's value, $\delta$ is the cash payout ratio,

subject to following boundary conditions:

$$\lim_{V\to\infty} E(V) = V - \frac{c(1-\tau)}{r}, \quad \lim_{V\to V_S} E(V) = \eta\alpha V_S \text{ and } \lim_{V\to V_S} E_V(V) = \eta\alpha$$

where the first boundary condition comes from the fact that the debt becomes risk free as the value of asset approaches infinity, and the two last conditions are implied from the bargaining game. Solving for the equity value in equation (1.12) gives the debt-equity swap triggering point:

$$V_S = \frac{c(1-\tau)}{r}\frac{-\lambda_-}{1-\lambda_-}\frac{1}{1-\eta\alpha} = \frac{c(1-\tau)}{r+\frac{\sigma^2}{2}}\frac{1}{1-\eta\alpha}, \tag{1.13}$$

where

$$\lambda_- = \left[0.5 - \frac{(r-\delta)}{\sigma^2}\right] - \sqrt{\left[0.5 - \frac{(r-\delta)}{\sigma^2}\right]^2 + \frac{2r}{\sigma^2}}$$

We can see from equation (1.13) that the triggering asset value found by Leland (1994), is a special case of the Fan and Sundaresan distress exchange triggering asset value. In the Leland framework there is no possibility of renegotiation, that is $\eta = 0$, making the default occurs at a lower level of asset's value. In this framework, stronger equity holders bargaining power, $\eta$, and superior liquidation costs $\alpha$ implies higher default triggering barrier.

In the second bargaining formulation, the borrower and the lender bargain over the value of the firm, $v(V)$, instead of the value of its assets. When an endogenously determined trigger point is reached, $V_S$, borrowers offer a debt service that is less than the contractual amount as an equilibrium outcome of the bargaining process. This allows them to get potential tax benefits in the future when the firm recovers from distress and the present value of these tax benefits is included in the bargaining process.

Fan and Sundaresan derive the value of the firm , $v(V)$, given a trigger point of strategic debt service, $\widetilde{V}_S$. The value of the firm is always greater than the value of the assets because of the present value of the tax shield.

The optimal sharing rule that satisfies the Nash bargaining game in this case, is given by:

$$\theta^* = \arg\max\{\theta v(V)\}^{\eta}\{(1-\theta)v(V) - \max[(1-\alpha)V - K, 0]\}^{1-\eta},$$

which is solved by

$$\theta^* = \min\left(\eta\left(1 - \frac{(1-\alpha)V - K}{V}\right), \eta\right) \tag{1.14}$$

Both the strategic debt servicing amount, $S(V)$, and the trigger level $\tilde{V}_S$ are determined endogenously. Solving the differential equations for the equity value with adequate boundaries, in the same vain than the equity-debt swap case, gives the following strategic debt service trigger point:

$$\tilde{V}_S = \frac{c(1-\tau+\eta\tau)}{r}\frac{-\lambda_-}{1-\lambda_-}\frac{1}{1-\eta\alpha}, \tag{1.15}$$

and the strategic debt service when the value of the assets is lower the trigger point is given by $S(V) = (1-\eta\alpha)\delta V$. Note here that this strategic debt servicing is decreasing in equity holders bargaining power and liquidation costs.

In summary, the basic difference between the two bargaining formulations is that, within the debt-equity swap, claimants bargain over the value of the assets of the firm, but in the second bargaining formulation, the claimants bargain over the whole firm value, that is asset value plus future tax benefits.

The Fan and Sundaresan model shows that debt renegotiation encourage early default and increases credit spreads on corporate debt, given that shareholders can renegotiate in distress to avoid inefficient and costly liquidation. It might be in the interest of debt holders to forgive part of the debt service payments if it can avoid the wasteful liquidations, which can be shared by the two claimants. If shareholders have no bargaining power, no strategic debt service takes place. Furthermore, by introducing the possibility of renegotiating the debt contract, the default can occur at positive equity value. This is in contrast to the Leland's (1994) model in that the default occurs when the equity value reaches zero as a consequence of issuing new equity is costless and the APR is respected.

Mella-Barral and Perraudin (1997) also incorporate strategic debt service by equityholders in a standard, contingent claims asset pricing model. The state variable here is no longer the firm's value, but rather the output price of the firm product. They also assume that there is no informational asymmetries and that agents are risk neutral. They consider a firm that produces a unit of output sold at a price, $p_t$. This output price follows a geometric Brownian motion

$$dp_t = \mu p_t + \sigma p_t dB_t$$

where $\mu$ and $\sigma$ are constant and $B_t$ is a standard Brownian motion. The firms also incur a fixed cost of production $w$ per period in such a manner that its net earning flow is equal to $p_t - w$.

Both direct and indirect bankruptcy costs are included. For the direct cost of bankruptcy, whenever the bankruptcy occurs the new owners can only generate lower earnings $\xi_1 p_t - \xi_0 w$

Where $\xi_1 \leq 1$ and $\xi_0 \geq 1$. Moreover, the liquidation value of the firm is constant and is equal to $\gamma$. The indirect costs of bankruptcy comes from the fact the investment decision can be distorted.

Mella-Barra and Perraudin consider first a case of a firm financed only by equities, and shows that even in absence of debt, liquidation may be optimal. This fact is due to the presence of bankruptcy costs described above. Introducing debt financing creates inefficiencies because of the direct bankruptcy costs it entails and because liquidation ultimately occurs at a lower level of earnings. Indeed, new owners of the firm are assumed unable to maintain the same profitability of the firm's assets compared to initial holders. The authors consider the case where the equity holders can make a take-it-or-leave-it offer to bondholders, that is, all bargaining power belongs to the equity holders, the optimal service debt proposed in this case is below the promised debt service. Thus, equity holders continue to operate the firm despite the lowered debt service payments. Inefficient liquidation is avoided in this context, at least until the liquidation threshold of a purely equity financed firm is reached, which is the efficient liquidation threshold.

When bondholders have all bargaining power, similar results are obtained. Here, bondholders cover operating losses for output prices below the optimal bankruptcy point that would occur without renegotiations. By injecting cash, bondholders keep the firm alive in hands of the equity-holders until liquidation is efficient.

Mella-Barral (1999) extends the previous cited works by allowing for departure from absolute priority rule (APR) in liquidation. This is achieved by dissociating the events of default and liquidation. Moreover, the liquidation price depends on the state variable of the model and liquidation costs are related to the inalienable human capital of the investor. In the first case, when the leverage is high, then liquidation can occur early in an inefficient manner, while for lower leverage the liquidation can occur inefficiently late. In case of low leverage, the creditors have interest in avoiding or postponing an inefficient liquidation by conceding interest payment. In the case of high leverage the investors may have interest in accelerating the default and avoiding inefficient late liquidation by offering to equity holders some of their proceeds from the liquidation, which explain the departure from the absolute priority rule.

### 1.3.5. Bankruptcy procedures

The models discussed above suppose a private workout for renegotiation. Nevertheless, the U.S. bankruptcy laws allow for a Court supervised debt renegotiation under Chapter 11 filing. Francois and Morellec (2004) extend the Fan and Sundaresan (2000) model to incorporate the possibility of Chapter 11 filings. Under this supervised renegotiation, the court grants the survival of the defaulting firm for an observation period. To incorporate this feature, equities are modeled as a Parisian down-and-out option on the firm's asset. The firm is liquidated, i.e. the equity holders' option to repurchase the firm's asset dies, when the value of the firm's assets reaches the default threshold and stays below that threshold for the observation period, denoted by $d$.

The majority of firms in financial distress that fills for the Chapter 11 emerge from the renegotiation process as an ongoing concern. In fact, Gilson, John and Lang (1990) and Weiss (1990) reports evidence of low percentage of firms liquidated under Chapter 7 (Liquidation) after filing for Chapter 11. Thus, two categories of firms can be distinguished: Those that are profitable in general but default in reason of temporary financial distress and which recover under Chapter 11 and firms that continue to have losses during the reorganization process and will be liquidated by the end of the reorganization process.

Similar to Fan and Sundaresan's approach, Francois and Morellec consider a Nash bargaining game between shareholders and equity holders, where their bargaining power is denoted by $\eta$ and $1-\eta$ respectively. They also suppose the firm renegotiates its debt obligations whenever the asset value falls below a constant threshold, $V_B$. However, Francois and Morellec model differs from Fan and Sundaresan's approach regarding the renegotiation costs. They assume that a proportional costs $\varphi$ are incurred by the company during the renegotiation process under Chapter 11, while the renegotiation costs are ignored in the Fan and Sundaresan's approach. Indeed, the financial costs of financial distress are higher in Chapter 11 filing compared to private workouts.

In this framework, the sharing rule upon default, denoted by $\theta$, satisfies the following relation:

$$\theta^* = \arg\max\left\{\left[\theta v(V_B)\right]^{\eta}\left[(1-\theta)v(V_B)-(1-\alpha)V_B\right]^{1-\eta}\right\}, \qquad (1.16)$$

where $v(V_B)$ is the firm value under renegotiation that is shared between both parties and $(1-\alpha)V_B$ is the value of bondholder's claims in case of default.

$\theta v(V_B)$ and $(1-\theta)v(V_B)-(1-\alpha)V_B$ represents the renegotiation surplus for equity holders and bond holders respectively.

The solution to equation (1.16) is given by the following optimal sharing rule:

$$\theta^* = \eta\left(1-\frac{(1-\alpha)V_B}{v(V_B)}\right) \qquad (1.17)$$

Francois and Morellec gives closed-form solutions to the corporate equities and debt for a given renegotiation boundary, and then assess endogenously this renegotiation threshold by maximizing the equity value. The optimal renegotiation threshold is given by:

$$V_{\mathbf{B}} = \frac{\xi}{1-\xi} \frac{c\left[1-\tau+\eta\tau(1-B(d)\right]}{r\left\{1-\eta\left[\alpha(1-C(d))-\frac{\varphi}{\delta}(\delta A(d)-C(d))\right]\right\}}, \tag{1.18}$$

where

$$A(d) = \frac{1}{\lambda}\left(\frac{1}{\lambda+b+\sigma}+\frac{1}{\lambda-b-\sigma}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})}\right),$$

$$B(d) = \frac{\lambda-b}{2\lambda}+\frac{\lambda+b}{2\lambda}\frac{\Phi(-\lambda\sqrt{d})}{\Phi(\lambda\sqrt{d})},$$

$$C(d) = \frac{\Phi(-(\sigma+b)\sqrt{d})}{\Phi(\lambda\sqrt{d})},$$

$\Phi(x) = 1 + x\sqrt{2\pi}\exp\left(\frac{x^2}{2}\right)N(x)$ where $N$ is the standard Normal cumulative distribution function,

$b = \frac{1}{\sigma}(r-\delta-\frac{\sigma^2}{2})$, $\lambda = \sqrt{2r+b^2}$ and $\xi = \frac{1}{\sigma}(b+\lambda)$.

The authors show that the default boundary in equation (1.18) extends both the Leland (1994) and Fan and Sundaresan (2000) models. For the Leland model the liquidation is automatic in case of default. This corresponds to the case where there is no observation period ($d = 0$). On the other hand, Fan and Sundaresan allow only for private workout. This corresponds to the case where liquidation never occurs and renegotiation is costless ($d \to \infty$ and $\varphi = 0$). They also note that for optimal leverage level, the default threshold is increasing with the tax rate, and decreasing with shareholders' bargaining power, liquidation costs, costs of financial distress, firm risk and payout ratio.

The model implies that the introduction of possibility of renegotiation under Chapter 11 increases the credit spread on corporate debt and encourages early default, while its impact on the optimal leverage level is ambiguous.

Moraux (2004) extends the Francois and Morellec framework to account for the total time spent by the state variable, i.e. the firm's asset value, below the default level. He assumes that liquidation is triggered when the accumulated excursion time of the asset's value below the distress threshold exceeds a pre-determined grace period. Thus, the liquidation becomes a result of the entire history of the firm's financial distress, instead of only the last episode of default.

Galai, Raviv and Wiener (2007) point out two additional bankruptcy procedures characteristics:

1. Recent distress events may have greater impact on the decision to liquidate the firm compared to older financial distress episodes.
2. The impact of a financial distress on the decision to liquidate the firm is proportional to its severity.

To account for these bankruptcy procedure features, they introduce the notion of a dynamic grace period, which depends on the severity of the distress period, on its length as well as on its distance from the present. Thus, more severe and more recent distress periods are more likely to cause liquidation compared to older and less profound financial troubles.

Broadie, Chernov, and Sundaresan (2007) develop a model that also distinguishes between default and liquidation. In their model, the optimal debt and equity values are determined in the presence of both Chapter 7 and Chapter 11 under the U.S. bankruptcy code. They explicitly consider two distinct barriers for default and liquidation and consider the optimal choice of these two boundaries.

The authors extend the model of Leland (1994), where only liquidation under Chapter 7 is allowed, by accounting for the key characteristics of the reorganization procedure under chapter 11, such as automatic stay of assets during the grace period, absolute priority, and transfer of control rights from equity holders to debt holders in bad states. The state variable considered in their work is the earning before interest and taxes (EBIT), denoted by $\delta_t$. They assume a geometric Brownian motion for the EBIT under a risk-neutral measure. This in turn implies a geometric Brownian motion for the value of assets of an unlevered firm $V_t$, since $V_t = \delta_t / (r - \mu)$. Moreover, the firm issues a single consol bond to finance its projects. The bankruptcy in their model has no effect on the EBIT process. They model financial rather than economical distress since bankruptcy by itself does not cause poor performance. Therefore, when its earnings are insufficient to make the necessary coupon payment, $c$, the firm leaves the liquid state and enter financial distress.

If the firm's EBIT deteriorates further to reach the bankruptcy boundary, $\delta^B$, the firm stop paying dividends to equity holders and bears a proportional distress cost as long as it remains in the default state. Moreover, the total EBIT is accumulated in a separate account $S_t$ during bankruptcy, while $A_t$ represents the accumulated unpaid coupons plus interest in arrears.

Depending on the evolution of the firm's EBIT after default, three scenarios are possible. First, when the firm recovers from Chapter 11, the debt holders will forgive a fraction of $1-\theta$ arrears, where $0 \le \theta \le 1$, and receive an amount $\theta A_t$. If $S_t$ is not sufficient to repay the arrears, equity holders must raise the remaining at the cost of diluting equity. In contrary, if $S_t > \theta A_t$, the amount of $\theta A_t$ is paid to creditors and the remaining is distributed to shareholders.

The second scenario is when the firm remains in bankruptcy, for a time longer than the grace period. In this case, the automatic stay provision is no longer granted and the firm is liquidated at a cost $\alpha$. Finally, if the firm's earning continues to deteriorate during the grace period, in such a manner to breach a lower liquidation barrier $\delta^B$, then the firm is liquidated.

The main contribution of Broadie, Chernov, and Sundaresan (2007) compared to previous models that distinguishes between default and liquidation, is the possibility of liquidation whenever assets value become too low during the observation period. Thus, liquidation could happen as the firm value either reaches the liquidation barrier or stays under the bankruptcy barrier for longer than the grace period.

In their paper, they focus on the issues of bankruptcy proceedings and the optimal choice of these two boundaries driven by different objectives. They show that the first-best outcome, the total firm value maximization ex-ante upon filing Chapter 11, is different from the equity value maximization outcome. They also show that the first-best outcome can be restored in large measure by giving creditors either the control to declare Chapter 11 or the right to liquidate the firm once it is taken to Chapter 7 by the equity holders. This serves as the threat from debtholders to prevent equity holders from filing for Chapter 11 too soon to get debt relief. Finally, they also find that on average the firms are more likely to default and are less likely to liquidate relative to the benchmark model of Leland (1994).

### 1.3.6. Dynamic capital structure

The models described above assume a static capital structure. The optimal leverage remains constant during the life of the firm. Fisher, Heinkel and Zechner (1989) propose a model where shareholders choose optimal recapitalization in a continuous-time framework. They assume that the firm's investment decisions are exogenous and independent from financing decision. They also assume a geometric Brownian motion for the firm's assets, $A$. Therefore, for a given face value of debt, $B$, the value-to-debt ratio, $y = A/B$, also follows a geometric Brownian motion. The firm issues new debt if its value-to-asset ratio, $y$, increases to an upper boundary, $\bar{y}$, in order to benefit from debt-related tax shields. When $y$ reaches a lower boundary, $\underline{y}$, the firm reduces its debt this time to avoid bankruptcy costs or to be compliant with equity holders limited liability.

In addition, the model relies on two assumptions. First, the value of an optimally levered firm can only exceed its unlevered value by the amount of transactions costs incurred in order to lever it up. This hypothesis is aimed to avoid the possibility of purchasing the sub optimally levered firm, issue additional debt and then sell it for a riskless profit (no arbitrage possibility). Second, a firm that follows an optimal financing policy offers a fair risk adjusted rate of return. Therefore, if leverage is advantageous, then it follows that unlevered firms offer a below-fair expected rate of return.

Fisher, Heinkel and Zechner (1989) characterize the advantage of leverage as:

$$\delta = r(1-\tau_p) - \hat{\mu}$$

where $r$ is the risk free rate, $\tau_p$ is the personal tax rate and $\hat{\mu}$ is the risk-adjusted expected growth rate of the market value of the firm's unlevered assets.

The capital structure equilibrium is defined by the upper and lower recapitalization boundaries, respectively $\overline{y}$ and $\underline{y}$, the face value of debt, $B$, the advantage of leverage, $\delta$ and the coupon rate, $i$, that maximize the value of firm net of recapitalization costs. The maximisation problem can be expressed as:

$$\underset{\underline{y},\overline{y},B,i}{Max}\, V(y_0, B, \underline{y}, \overline{y}) - kB$$

subject to

$$V(y_0, B, \underline{y}, \overline{y}) = By_0 + Bk\,;$$

$$E_y(y = \underline{y}, B, \underline{y}, \overline{y}) \geq 0$$

$$D(y_0, B, \underline{y}, \overline{y}) = B,$$

where $V$ is the value of the firm, $E$ is the value of equity, $D$ is the value of debt[6], $y_0$ is the initial value-to-debt ratio and $k$ is the recapitalization proportional cost. The first condition is a no arbitrage condition. Indeed recall that according to Fisher, Heinkel and Zechner (1989), in absence of arbitrage the value of the firm must be equal to the value of its unlevered assets, $A_0 = By_0$, plus the transaction costs $Bk$. The second constraint grants that the equity value is positive[7] and the last one state that the debt is issued at par.

Numerical solutions for different parameters values show that the resulting optimal dynamic capital structure policy depends on the tax advantage, the bankruptcy costs, the assets volatility, the riskless interest rate and the costs of recapitalization.

Goldstein, Ju, and Leland (2001) argue that the two assumptions advanced by Fisher, Heinkel and Zechner (1989) do not hold in practice. First, the necessary premium to gain control of the firm may deter arbitrage possibility for under-levered firms and second, the market price adjustment allows obtaining fair expected return for firms with publicly traded assets, even if they are unlevered. Goldstein, Ju, and Leland (2001) choose to model the dynamics of EBIT as state variable, instead of the usually used unlevered firm value. They justify this choice by the invariance of the EBIT generating mechanism to the capital structure decision. They notice that using the generated cash flows to pay dividends, taxes or debt services have the same effect on the firm. The advantage of taking the claim on future EBIT is that all contingent claimants, including the tax payment, to future EBIT flows are treated in a consistent fashion. Especially, the tax shelter is no longer treated as a cash inflow in a form of tax benefit, but as a cash outflow in the form of tax. The authors argue that the invariance feature makes the claims on EBIT a well suited framework for investigating multiple capital structure changes and. hence, optimal dynamic capital strategy.

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[6] The value of equity and debt is obtained by solving a PDE similar to equation (11). The interested reader is referred to the original paper for further details.

[7] The authors consider the case of riskless debt. In this case the second constraint becomes $\underline{y} = 1$.

Goldstein, Ju, and Leland (2001) assume a geometric Brownian motion for the EBIT, $\delta$ , under a risk-neutral measure with drift $\mu$ and volatility $\sigma$. This in turn implies a geometric Brownian motion for the value of assets of an unlevered firm $V_t$, since $V_t = \delta_t/(r-\mu)$. They also assume a single consol bond issuance to have time independence of the payout. This grant that any claimant satisfies the following ordinary differential equation:

$$\frac{\sigma^2}{2}V^2F_{VV} + \mu VF_V - rF + P = 0, \tag{1.19}$$

where $P$ is the payout flow.

They define $p_B(V)$ as the present value of a claim that pays 1$ when the firm's value reaches $V_B$, the default boundary. This claim satisfies Equation (1.19) with $P = 0$ because there is no intermediate payout. The solution takes the following form

$$p_B(V) = A_1V^{-y} + A_2V^{-x} \tag{1.20}$$

where $x = \left[\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right]\Big/\sigma^2$,

$$y = \left[\left(\mu - \frac{\sigma^2}{2}\right) - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right]\Big/\sigma^2.$$

where $x$ is positive, while $y$ is negative. The boundary conditions are defined by $\lim_{V\to\infty} p_B(V) = 0$ and $\lim_{V\to V_B} p_B(V) = 1$. Therefore $p_B(V) = \left(\frac{V}{V_B}\right)^{-x}$.

Define $V_{solv}(V)$ a claim entitled to the entire payout $\delta$ as long as the firm remains solvent, i.e., firm value remains above $V_B$. The solution for equation (1.19) takes the form:

$V_{solv}(V) = V + A_1V^{-y} + A_2V^{-x}$ with the following boundary conditions :

$\lim_{V\to\infty} V_{solv} = V$ ( $A_1 = 0$ ) and $V_{solv} = 0$ as $V = V_B$ which gives

$$V_{solv} = V - V_B p_B(V) = V - V_B\left(\frac{V}{V_B}\right)^{-x}.$$

For the claim on the interest payment while the firm is solvent, the solution is in the form of $V_{\text{int}}(V) = C/r + A_1V^{-y} + A_2V^{-x}$ where $C$ is the coupon payment. When $V$ tends to infinity this claim tends to $C/r$, thus $A_1 = 0$ here again and $V_{\text{int}} = 0$ when $V = V_B$. The claim on interest is then given by $V_{\text{int}} = \dfrac{C}{r}[1 - p_B(V)]$.

The separation of value of the continuing operation between debt, equity and government gives:

$$E_{solv}(V) = (1-\tau_{eff})(V_{solv} - V_{\text{int}}) = (1-\tau_{eff})\left[(V - \frac{C}{r}) - (V_B - \frac{C}{r})p_B(V)\right],$$

$$D_{solv}(V) = (1-\tau_i)V_{\text{int}} = (1-\tau_i)\frac{C}{r}[1 - p_B(V)],$$

$$G_{solv}(V) = \tau_{eff}(V_{solv} - V_{\text{int}}) + \tau_i V_{\text{int}}.$$

$\tau_{eff}$ is the effective tax rate, and $\tau_i$ is the tax rate on interest payments.

Both the coupon level $C$ and the bankruptcy level $V_B$ are chosen by management to maximize the equity wealth. The optimal bankruptcy level is obtained by the smooth-pasting condition $\left.\dfrac{\partial E}{\partial V}\right|_{V=V_B} = 0$ which yields:

$$V_B^* = \left(\frac{x}{x+1}\right)\frac{C^*}{r}.$$

The optimal coupon $C^*$ is obtained by maximizing the shareholder wealth, i.e. the value of equity and debt:

$\max_c \{(1-q)D(V, V_B, C) + E(V, V_B, C)\}$ this yields to

$$C^* = V\left(\frac{xr}{x+1}\right)\left[\left(\frac{1}{1+x}\right)\left(\frac{A}{A+B}\right)\right]^{\frac{1}{x}}$$ where

$$A = (1-q)(1-\alpha) - (1-\tau_{eff})$$

$B = \frac{x}{x+1}(1-\tau_{eff})\left[1-(1-q)(1-\alpha)\right]$, $q$ denotes the restructuring costs and $\alpha$ the bankruptcy costs.

In contradiction with models that use the unlevered firm value as state variable, e.g. Leland (1994), the comparative statics shows that the value of equity is decreasing in the effective tax rate. This is due to the fact that a rise of tax rate increases the government claim at the expense of equity, instead of considering the tax benefit as a cash inflow.

Goldstein, Ju, and Leland extend the static model to allow for a dynamic capital structure where the management can adjust the firm leverage upward. As in Fisher, Heinkel and Zechner (1989), they assume that in addition to the threshold $V_B$ where the firm optimally chooses to default, there will be a threshold $V_U$ where the management call the outstanding debt and sell a larger issue. They show by backward induction, that if the EBIT increases by a scale $\gamma$ at each period, then the optimal restructuring and bankruptcy thresholds will increase by the same factor. They find that the optimal initial leverage level with dynamic capital structure is much lower than the one found with static capital structure. This is explained by the option to increase leverage in the future. Also, the bankruptcy threshold decreases when the capital structure is dynamic, the intuition behind this result is that the firm with the option to adjust its capital structure is more valuable, and therefore has more incentive to avoid bankruptcy.

## 1.4. Other extensions

Ju, Parrino, Poteshman and Weisbach (2005) consider a dynamic model of optimal capital structure where the firm financing decision is determined by a balancing between corporate taxes advantage and bankruptcy costs (trade-off theory). The value of the unlevered assets as an exogenous process. They specify a model in which new debt is reissued when old debt matures to keep a given leverage ratio. However, the default boundary is exogenous and has an exponential form. Collin-Dufresne and Goldstein (2001) also consider a dynamic capital structure by modeling a mean-reverting leverage ratio and stochastic interest rate.

Acharya and Carpenter (2002) develop a model with both stochastic interest rate and endogenous defaults. The interest rate is modeled as one-factor diffusion process and the issuer follows optimal call and default rules. Thus, they bridge the gap between endogenous default and stochastic interest rate literatures. They model call and default options as American options written on a non callable, default free bond with fixed continuous coupons. The authors characterize the default region for both callable and non callable bonds and find that this default region is smaller for the callable bond relative to the non callable one. They show that the existence of the call option can encourage the firm to continue servicing its debt when it would otherwise default.

Most of the structural models assume that firm's risk remains constant. Leland (1998) allows the firm to choose its risk strategy and examine the agency problem between equity holders and debt holders related to asset substitution. The model also permits to examine the interaction between capital structure and risk strategy.

Leland (1998) assumes that risk choices are made after the debt is in place, and these choices cannot be constrained through debt covenants or other precommitments. However, he presumes rational expectations, in that both equity holders and the debt holders will correctly anticipate the effect of debt structure on the chosen risk strategy, and the effect of this strategy on security pricing. Thus, he assumes that there is no information asymmetry. In this setting, once the financing decision is set, the stockholders choose the investment policy that maximizes the equity value ex post, but reduce the value of other claimants such as tax, external claimants in default and especially debtholders, creating agency costs due to asset substitution. The initial optimal capital structure made ex ante will balance these agency costs with the tax benefits of debt less default costs.

To measure these agency costs, the firm value with ex post investment decision is contrasted with the situation where both risk strategy and debt structure are made simultaneously ex ante to maximize the firm value. The difference in optimal firm value between ex post and ex ante situations represents the loss in value due to maximization of equity value instead of firm value.

A similar approach is adopted for risk hedging strategy. The firm can decrease its risk level through hedging, and cease hedging at any time. Two environments are considered. In the first, both capital structure and hedging strategy are determined ex ante to maximize market value (ex ante hedging strategy), while in the second, the hedging strategy is established to maximize equity value ex post, i.e. after financing decision is made (ex post hedging strategy). The optimal firm values is compared under ex ante hedging and ex post hedging strategies with the situation where the firm can never hedge and the situation where the firm always hedge. The difference between the value of a firm using optimal hedging strategies and the value of the same firm when hedging is not allowed, represent the benefit of hedging.

Hackbarth, Henessey and Leland (2007) distinguish between bank and public debt. They assume that renegotiation through private workout is only possible for bank debt. This renegotiation possibility makes bank debt more attractive, but limits bank debt capacity for strong firms, e.g. firms with high bargaining power. When the strong firm reaches its bank debt capacity, the firm complements bank debt by public debt to benefit from more tax shield. The model therefore propose an explanation to the seniority of bank debt, and to the fact that small/weak firms relies exclusively on bank debt while mature/strong firms uses a mix of public and bank debt. Bourgeon and Dionne (2007) extend the Hackbarth, Henessey and Leland (2007) model to allow banks to adopt a mixed strategy in which renegotiation is sometimes refused ex-post in order to raise debt capacity ex-ante. Carey and Gordy (2007) suppose that holders of private debt, e.g. banks, with strong covenants control the choice of the bankruptcy threshold. Since the private debt is senior, the bank triggers bankruptcy only when the asset's value falls below the face value of the bank debt. In accordance with their model, they find empirical evidence indicating that the recovery rate is sensitive to debt composition.

Other extensions include Mauer and Triantis (1994), Childs, Mauer and Ott (2005) and Sundaresan and Wang (2007) who considers endogenous investment. The cash holding management policy is accounted for in Acharya, Huang, Subrahmanyam and Sundaram

(2006), Anderson and Carverhill (2007), and Asvanunt, Broadie, and Sundaresan (2007). Sarkar & Zapatero (2003) consider mean reverting cash flows. Zhou (2001), Duffie and Lando (2001) and Giesecke & Goldberg (2004) add a jump component to the value process of assets allowing for "surprise" default at the cost of closed-form solution. Alernatively, Hackbarth, Miao & Morellec (2006) consider jumps in the cash flow process with regime change. Finally, Longstaff (1996), Morellec (2001) and Ericsson and Renault (2006) include a liquidity premia to price corporate debt, while Duffie and Lando (2001) consider accounting information uncertainty.

## 1.5. Empirical Evidence on Corporate Credit Risk

The empirical literature on structural models assesses the ability of different models to predict the credit spread on bonds and CDS. Another trend of the literature assesses the ability of different credit risk models, including the structural models, to predict defaults and the relation between the default risk and equity return.

### 1.5.1. Corporate Credit Risk, Yield Spread and Default Frequency

Jones, Mason, and Rosenfeld (1984), compare the spread predicted by the Merton (1974) model and the empirically observed spreads and find that the credit yield spreads generated by the Merton model are too low. Franks and Torous (1989), find similar results with realistic parameter. Moreover, Anderson and Sundaresan (2000), Lyden and Saraniti (2000) show mixed results on the ability of structural models to explain observed corporate yield spreads.

Elton, Gruber, Agrawal, and Mann (2001) using Fixed Income Database on US corporate and financial institutions bonds from 1987 to 1996, find that default risk accounts for a low portion of the yield spread. Indeed, depending on credit quality and industry, default risk accounts for between 7% and 35% of the yield spread while the tax differential is found to be a major factor in the overall credit spread. Elton et. al. argue that the rest of the corporate bond yield spread represents compensation for systematic risk in corporate bonds. Using linear regressions of bond returns on empirically identified Fama-French factors, the authors show that a large proportion of the yield spread unexplained by default risk and taxes is explained by the three factors of Fama and French (1993). (67% for financial institutions and 85% for industrial). They conclude therefore that the credit risk and tax premium can only partly explain for the difference in corporate spread.

Huang and Huang (2003) use a variety of structural models to examine how much of the historically observed corporate-Treasury yield spread is due to default risk. To explore whether this spread can be explained by implied default probabilities from the structural models. The structural models studied include Longstaff and Schwartz (1995) with stochastic interest rate, Leland and Toft (1996) for endogenous default boundary, Anderson and Sundaresan (1996), Anderson, Sundaresan, and Tychon (1996) and Mella-Barral and Perraudin (1997) for strategic default, and Collin-Dufresne and Goldstein (2001) for mean reverting leverage ratio.

Huang and Huang calibrate each model's parameters to match the observed expected default frequency and the average loss given default for each broad rating category. The average empirical leverage by rating grade is also used as input in the calibration. Since the structural models predict not only bond prices but also equity prices, the authors use equity premium to assess the assets risk (volatility) premium. Thus, the target quantities to calibrate the models are the leverage ratio, the equity premium, the default probability and recovery rate. The time horizons considered are respectively 10 and 4 years.

They find that the calibrated structural models generate similar credit spreads. Moreover they find that the credit risk explains between 20% and 30% of the investment grade treasury yield, while this proportion increases for riskier bonds and accounts for a large portion of the yield spread. However, this fraction decreases as the bond maturity shortens. Indeed, the fact that structural models rely on diffusion process of the value of the firm's assets, makes the credit spread converge to zero for short maturities, which contradicts the empirical observation. The authors conclude that additional factors such as illiquidity and taxes must be important in explaining market yield spreads.

Collin-Dufresne, Goldstein, and Martin (2001) focus on changes in corporate credit spreads. They use the theoretical inputs of structural models as explanatory variables in credit spreads regression. They find a limited explanatory power of these variables, and that a significant part of the residuals is driven by a common systematic factor that is not captured by the theoretical variables. They also find that credit spreads decrease as the market becomes more liquid as measured by the relative frequency of quotes versus matrix prices in the Fixed Income Database (FID). Thus, Collin-Dufresne, Goldstein, and Martin show that the credit spreads of individual bonds react to changes in aggregate liquidity, but do not address changes in liquidity at the individual bond level.

Similar analysis is performed by Campbell and Taksler (2003) using regressions for levels of the corporate bond spread. They conclude that firm specific equity volatility is an important determinant of the bond spread, and that the economic effects of volatility are large. Cremers, Driessen, Maenhout, and Weinbaum (2004) give support to this result and argue that option-based volatility contains useful information for this type of analysis that is different from historical volatility.

These evidences suggest that the observed yield spread contains a large proportion due to liquidity and tax differential. This could explain the weak performance of structural models to reproduce yield spread without a larger jump sizes or larger credit risk premia than in typical calibration.

To circumvent the problem of liquidity and taxes differential in yield spreads, Leland (2004) focus on the ability of exogenous and endogenous structural credit risk models to capture the observed default frequencies across bonds with different ratings. He calibrates the exogenous default models as represented by the Longstaff, Schwartz (1995) model, and the Moody's-KMV variant of the Merton model with common inputs and examines how well these models match the observed default frequencies as reported by Moody's over the period 1970-2000. Leland finds that both models achieve good performance in predicting the shape and the level of default probabilities for horizons exceeding 5 years, but under-predict the default frequencies for shorter time horizons. Since the default frequencies are not affected by bonds liquidity, he concludes that the addition of jumps in the asset value process, as proposed by Zhou (2001) for instance, can solve both the underestimation of the default probabilities and the yield spread.

Eom, Helwege and Huang (2004) also test the ability of five structural models to predict the yield spread of firms with simple capital structure. They find that the Merton (1974) and Geske (1977) models generate spreads that are far below the observed ones on the bond market, in accordance with the previous literature. However, the Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001) models overestimate spreads for riskier bonds (high volatility and leverage) while they underestimate the spreads for less risky bonds.

Longstaff, Mithal, and Neis (2005) make use of the Credit Default Swaps premia to separate the corporate bond yield spread into a default component and a non-default component. They find that the default component increases from an average of 51% of the spread for AA bonds up to 83% for BB bonds. The non-default component in their sample varies substantially with a range of 18.8 to 104.5 basis points and a mean of 65 basis points. Longstaff et al. (2005) find that the non-default component is related to both the degree of asymmetric tax treatment and a proxy for bond liquidity. The non-default component is positively related to the coupon rate of the bond, indicating the market is pricing the differential tax treatment of corporate bonds.

Houweling and Vorst (2005) implement a set of simple reduced form models on market swap quotes and corporate bond quotes. Their paper focuses on the pricing performance of the model and the choice of benchmark yield curve.

Regarding the calibration of structural models, their implementation requires the knowledge of the assets value and volatility. However, these inputs are not observable since only equities are priced by stock markets. Most of the implementations of structural models approximate the value of assets by the market value of equities plus the book value of debt and the assets' volatility using equities' volatility and adjustment for debt in capital structure (Eom, Helwege and Huang, 2004, for instance). Beside this approximation, several methods were proposed in the literature. Jones, Mason and Rosenfeld (1984), Ronn and Verma (1986) use an alternative method that makes use of Itô's lemma to obtain a system of two equations linking the unknown asset values and the asset volatility to the observed equity values and volatility. However, this method was criticized due to the assumption of constant volatility and lack of statistical inference. Crosbie and Bohn (2003) develop an iterative proprietary method based on variance restriction method of Moody's KMV. Duan (1994) and Duan, Gauthier and Simonato (2004) propose a maximum likelihood estimation method, based on equity prices to estimate asset value and volatility. Ericsson and Reneby (2002) conduct a simulation study for different structural models and demonstrate the higher performance of the maximum likelihood estimation compared to the variance restriction method. Li and Wong (2008) empirically examine the proxy, volatility-restriction and maximum likelihood approaches to implement structural corporate bond pricing models, and find also that ML estimation is superior to the other considered methods. Bruche (2005) propose a method that combines different priced assets to estimate asset value and volatility.

Hull, Nelken and White (2004) present an alternative approach to estimate the unobservable asset volatility. Considering the implied volatility of options on the company's stocks, the authors propose a different approach than the variance restriction method, to measure assets volatility. The method is based on Geske (1979) model, which suggests that since the equity of a company can be considered as an option on the firm's assets, an option on the firm's stock is a compound option, and further provides a valuation formula for such compound option. Using Geske (1979) formulation, the authors present a two-equation system that can be solved with two implied volatilities, sampled from stock options.

While testing the proposed alternative with credit default swaps (CDS) spread data, the authors find that this implementation of the Merton model outperforms the traditional methodology.

Schaefer and Strebulaev (2008) study the sensitivity of the corporate bond returns to changes in the hedge ratios and find that structural models provide accurate estimates of hedge ratio. The authors conclude that the limited ability of structural models to accurately predict bond prices is due to non-credit factors.

Davydenko and Strebulaev (2007) identify firm specific strategic factors that affect credit spread. In fact, strategic default models predict lower bond prices when the threat of strategic default is more likely. They proxy for renegotiation frictions, bargaining power in renegotiation and liquidation costs by using debt complexity measure, equity ownership and asset tangibility respectively. They find a significant relationship between these factors and the credit spread, although the economic effect is limited and could not be the reason of the limited performance of structural models to match the levels of credit spreads.

Several studies investigate the sensitivity of credit spread to macro-economic factors. Bakshi, Madan and Zhang (2006) and Elton, Gruber, Agrawal and Mann (2001) show that an important part of corporate bond credit spreads is explained by factors commonly used to model risk premiums for common stocks. Fama and French (1989) find wider credit spreads when economic conditions deteriorate. Similar results are achieved by Duffie, Saita and Wang (2007) who show that macroeconomic variables explain a large portion of yield spread changes and default rates.

Tang and Yan (2006) model relates the firm credit spreads to macroeconomic conditions through the sensitivity of its cash flows to economic factors. A link between market and credit risk is established in their framework. They show that accounting for the macro-economic effect improves fitting the default probabilities and credit spread. David (2008) and Chen (2007) models also predict a decrease of the default probability and credit spreads in macro-economic expansion.

Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn and Strebulaev (2007), Chen (2007), and David (2008) use regime switching models to link credit spread dynamics to macroeconomic conditions and/or the equity risk premium which allows detecting higher impact of economic aggregates on credit spreads.

Both Fama and French (1989) and Koopman and Lucas (2005) find a countercycle behavior of the credit spread. This evidence suggest a distinction between credit cycle and economic cycle ( see Dionne, Maalaoui, François, 2009).

Overall, several factors beside the default risk seem to drive the corporate credit spread, including liquidity, volatility, firm specific factors and market conditions.

### 1.5.2. Structural Models and Default Forecast.

Moody's KMV developed a commercial model derived from the Merton approach, and adjusted to agency ratings and other bond characteristics. The distance-to-default, that is, the normalized distance, measured in standard deviations, of a firm's asset value from its default threshold. Distance-to-default plays a central role in calculating the expected default frequency (EDF) in the Moody's KMV model. Sobehart, Keenan and Stein (2000), and Stein (2002), among others studies, examine the accuracy of the Moody's KMV model. Both studies find the Moody's KMV model to be incomplete. Kealhofer and Kurbat (2002) find opposite results, namely that Moody's KMV model captures all the information contained in agency ratings migration and accounting ratios. Crosbie and Bohn (2003) find that combining market prices and financial statements gives more effective default measurement. The authors empirically test the EDF, derived from the KMV methodology, versus the credit rating analysis, and show that the EDF obtains a better power curve.

The accuracy of default forecasting of the KMV model is studied in Bharath and Shumway (2004). The authors compare the KMV model accuracy with simpler alternative. They find that implied default probabilities from credit default swaps and corporate bond yield spreads are only weakly correlated with KMV-Merton default probabilities. The authors conclude that the KMV-Merton model does not provide a sufficient statistic for default, which can be obtained using relatively naïve hazard models. Hillegeist, Keating, Cram and Lundstedt (2004) and Du and Sou (2005) compare the KMV model to other models, and conclude that the KMV model does not provide adequate predictive power.

However, Duffie, Saita and Wang (2007) discover a significant predictive strength over time within the KMV model. Campbell, Hilscher and Szilagyi (2004) use hazard models to condition the KMV model on other relevant default variables, and find a poor predictive power of the KMV model.

Moody's propose its own commercial implementations of hybrid models. Indeed, Sobehart, Stein, Mikityanskaya, and Li (2000) use a comprehensive proprietary database of over 1,400 US non-financial defaults to assess the performance of Moody's hybrid model in predicting defaults. They combine the structural distance-to-default with other rating, market, and accounting variables. They conclude that neither the structural model nor the financial statements will contain all the relevant information on the firm's credit worthiness. Thus, combining the two methods seems justifiable, since the hybrid model outperforms both the pure structural model and the pure statistical one. However, when Kealhofer and Kurbat (2002) attempted to replicate these findings, they got opposite results. The KMV implementation of the Merton structural approach based on distance to-default shows that the structural model excels other measures of credit risk. Hillegeist, Keating, Cram, and Lundstedt (2004) have documented that the theoretical probabilities estimated from structural models do not capture all available information about a firm's credit risk. They show that traditional risk measures, such as the updated versions of Altman's Z-Score and Ohlson's O-Score, do add incremental information and that the default probabilities estimated from structural models are therefore not a sufficient statistic of the actual probability of default.

### 1.5.3. Structural Models and Stock Returns

The distance to default is widely used in the finance literature as a measure of credit-worthiness. On the other hand, the relationship between financial distress and stock returns was studied in several papers. The financial distress is measured either through accounting based measures, agencies ratings or structural model. Dichev (1998) and Griffin and Lemmon (2002) use Altman's Z-score and Ohlson's O-score to measure financial distress and find evidence of underperformance of distressed stocks. Avramov et al. (2006), rely on credit ratings to detect distressed firms and find similar results.

On the other hand, Garlappi, Shu, and Yan (2008) using default risk measures from Moody's KMV, find that stocks with a high risk of failure tend to have anomalously low average returns. However, Vassalou and Xing (2004) measure the distance to default of listed firms and find that financially distressed stocks earns higher returns contradicting the previous results. This higher return is due mainly to small value stocks. Moreover, Da and Gao (2010) attribute this abnormal return to liquidity factors. Indeed, they find that the liquidity risk rise for distressed stocks and the prices recovers in the following month, which explains the high return of stocks with high default likelihood. Indeed, Campbell, Hilscher and Szilagyi (2008) use both distance to default and logit models to detect financial distress and find evidence of price anomaly since distressed stocks earn lower return. They also find that distressed firms have high market betas and high loadings on the HML and SMB factors of Fama and French (1993,1996).

## 1.6. Conclusion

In this paper, we review the most influential and representative structural models. Structural models offer an intellectually appealing approach to modeling credit risk. They provide a link between the more traditional corporate finance models and the contingent claims analysis. These models study interesting questions of security design, optimal investment and financing decisions, or the incentives resulting from the bankruptcy law.

Most of the structural models provide closed-from expressions of corporate debt as well as the endogenously determined bankruptcy level, which are explicitly linked to taxes, firm risk, bankruptcy costs, risk-free interest rate, payout rates, and other important variables. The behavior of how debt values (and therefore yield spreads) and optimal leverage ratios change with these variables can thus be investigated in detail.

While theoretically elegant, capital structure models do not perform well empirically in risky corporate bond pricing. Researchers have been attempting to resolve the yield spread underestimates by introducing jumps and liquidity premium. On the other hand, the poor performance of structural models may have more to do with the influence of non-credit factors rather than their failure to capture the credit exposure of corporate debt. Growing evidence shows that multiple firm characteristics and market and economic conditions are important determinants of corporate credit spread Moreover, since recent capital structure models put numerous efforts on the event of bankruptcy, structural models are useful for prediction of default probabilities or default events. Finally, some researchers argue that the past poor performance of capital structure models may come from the estimation approaches traditionally used in the empirical studies and we have seen some innovative estimation methods aiming for solving the estimation problem in models employing structural approach.

## 1.7. References

Acharya, *V*., and J. N. Carpenter, 2002, Corporate bond valuation and hedging with stochastic interest rates and endogenous bankruptcy, *Review of Financial Studies 15*, 1355–13833.

Acharya, *V*., J. Huang, M. G. Subrahmanyam, and R. K. Sundaram, 2006, When does strategic debt-service matter? *Economic Theory* 29, 363–378.

Asquith, P., R. Gertner, and D. Scharfstein, 1994, Anatomy of Financial Distress: An Examination of Junk Bond Issuers*, Quarterly Journal of Economics* 436, 625-58.

Anderson, R. W., and A. Carverhill, 2007, Liquidity and capital structure, Center for Economic Policy Research Working paper No. 6044.

Anderson, R. and S. Sundaresan, 1996, Design and valuation of debt contracts. *Review of Financial Studies* 9: 37-68.

Asvanunt, A., M. Broadie, and S. Sundaresan, 2007, Managing corporate liquidity: Welfare and pricing implications, Working paper, Columbia University.

Bakshi, Gurdip S., Dilip Madan, and Frank Zhang, 2006, Investigating the role of systematic and firm-specific factors in default risk: Lesson from empirically evaluating credit risk models, *Journal Business* 79, 1955-1988.

Bharath S. T. and Shumway T., 2004, Forecasting Default with the KMV-Merton Model, Working Paper.

Bielecki, T and Rutkowski, M. (2002). *Credit Risk: Modeling, valuation and Hedging. SpringerFinance,* Springer, New York.

Black, F., and Scholes, M., 1973, The Pricing of Options and Corporate Liabilities, Journal of Political Economy, Vol. 81, p. 637-654.

Black, F., and J. C. Cox , 1976. Valuing corporate securities - some effects of bond indenture provisions. *Journal of Finance* 31(2), 351-367.

Brennan, M. J. and E. S. Schwartz, 1985, Evaluating natural resource investment, *Journal of Business* 58: 135-157.

Bourgeon, J. M. and Dionne, G., 2007, On debt service and renegotiation when debt-holders are more strategic. Working paper, CIRPEE.

Briys, E. and F.de Varenne, 1997, Valuing risky fixed dept: an extension. *Journal of Financial and Quantitative Analysis* 32: 239-248.

Broadie, M., Chernov, M., and S. Sundaresan, 2007, Optimal debt and equity values in the presence of Chapter 7 and Chapter 11. *Journal of Finance* 62(3), pp. 1341-1377

Bruche, M., 2005, "Estimating Structural Bond Pricing Models via Simulated Maximum Likelihood," Working Paper, London School of Economics.

Campbell, J.Y. and Taksler, G.B., 2003, Equity Volatility and Corporate Bond Yields. *Journal of Finance*, 58 (6), 2321-2349.

Carey, M., and M. Gordy, 2007, The bank as grim reaper, debt composition and recoveries on defaulted debt, Board of Governors of the Federal Reserve System Working Paper.

Chen, H., 2007, Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure, Working paper, MIT.

Chen, L., Lesmond, D., and Wei, J., 2007, Corporate Yield Spreads and Bond Liquidity. *Journal of Finance*, 62 (1), 119-149.

Collin-Dufresne, P. and R. S. Goldstein. 2001. Do credit spreads reflect stationary leverage ratios?, *Journal of Finance* 56, 1929-1957.

Collin-Dufresne, P.; R. Goidsiein; and S. Martin, 2001, The Determinants of Credit Spread Changes. *Journal of Finance*, 56, 2177-2207.

Cremers. M.; J. Driessen; P. J. Maenhout; and D. Weinbaum. ,2004, Individual Stock-Option Prices and Credit Spreads. Working Paper No. 04-14. Vale ICF.

Crosbie P. J. and J. R. Bohn, 2003, Modeling Default Risk, Modeling Methodology of Moody's KMV LLC.

Da, Z., and P. Gao, 2010, Clientele change, liquidity shock, and the return on financially distressed stocks, *Journal of Financial and Quantitative Analysis*, 45, 27-48

David, Alexander, 2008, Inflation uncertainty, asset valuations, and the credit spreads puzzle, *Review of Financial Studies*, 21 (6), 2487-2534.

Davydenko, S. A. and I. A. Strebulaev, 2008, Strategic Actions and Credit Spreads: An Empirical Investigation, Journal of Finance 62 (6), 2633-2671.

Dionne, G., François, P., Maalaoui, Chun, O., 2009, Credit Spread Changes within Switching Regimes, Working Paper.

Duan, J.C., 1994. "Maximum Likelihood Estimation Using Price Data of the Derivative Contract", *Mathematical Finance*, 4(2), 155-167. Correction: Duan, J.C., 2000, "Maximum Likelihood Estimation Using Price Data of the Derivative Contract" (Mathematical Finance 1994, 4(2), 155-167), *Mathematical Finance*, 10 (4), 461-462.

Duan, J.,G. Gauthier, and J. Simonato, 2004. On the Equivalence of the KMV and Maximum Likelihood Methods for Structural Credit Risk Models. Working Paper, University of Toronto.

Duffie, D. and *D*. Lando. 2001. Term structures of credit spreads with incomplete accounting information. *Econometrica* 69(3): 633-664.

Duffie, D. and K. J. Singleton,1999, Modeling term structures of defaultable bonds, *Review of Financial Studies* 12, 687-720.

Duffie, D., Saita L. and K. Wang. 2007. Multi-Period Corporate Default Prediction with Stochastic Covariates,." *Journal of Financial Economics* 83, 635-665.

Du Y. and Suo W., 2007, Assessing Credit Quality from Equity Markets: Is a Structural Approach a Better Approach?, *Canadian Journal of Administrative Sciences*, Volume 24 (3), pp 212-218.

Elizalde, A., 2005, Credit Risk Models II: Structural Models, Working Paper, CEMFI and UPNA.

Elton, Edwin J., Martin J. Gruber, Deepak Agrawal, and Christopher Mann, 2001, Explaining the rate spread on corporate bonds, *Journal of Finance* 56, 247-277.

Eom, Y. H., J. Helwege, and J. Huang, 2004, Structural models of corporate bond pricing: An empirical analysis, *Review of Financial Studies* 17, 499-544.

Ericsson, J. and Reneby, J., 2005. Estimating Structural Bond Pricing Models, *Journal of Business* 78, 707–735

Ericsson, J. and O. Renault. 2006. Liquidity and credit risk. *Journal of Finance* 61: 2219-2250.

Fama, Eugene F., and Kenneth R. French, 1989, Business conditions and expected returns on stocks and bonds, *Journal of Financial Economics* 25, 23-49.

Fischer, E., R. Heinkel and J. Zechner. 1989, Dynamic Capital Structure Choice: Theory and Tests, *Journal of Finance* 44, 19-40.

Franks, J R., and W.N. Torous, 1994, A Comparison of Financial Recontracting in Distressed Exchanges and Chapter 11 Reorganizations, *Journal of Financial Economics* 35, 349–370.

François, P., 2005, Corporate Debt Valuation: The Structural Approach, in Hatem Ben Ameur and Michèle Breton (eds), Numerical Methods in Finance, Springer, pp. 1-33

François, P., and E. Morellec, 2004, Capital Structure and Asset Prices: Some Effects Of Bankruptcy Procedures, *Journal of Business* 77, 387-411.

Galai, D. & Raviv, A. & Wiener, Z., 2007, Liquidation Triggers and The Valuation of Equity And Debt, *Journal of Banking and Finance* 31(12), 3604-3620.

Garlappi, L., T. Shu, and H. Yan, 2008, Default risk, shareholder advantage, and stock returns, *Review of Financial Studies* 21 (6), 2743-2778.

Geske, R., 1977, The Valuation of Corporate Liabilities as Compound Options, *Journal of Financial and Quantitative Analysis* 12,541-552.

Geske, R., 1979, The Valuation of Compound Options, *Journal of Financial Economics,* Vol. 7, 63-81.

Giesecke, K., 2004, Credit Risk Modeling and Valuation: an Introduction, Credit Risk: Models and Management, Vol. 2, Edited by *D*. Shimko, Risk Books.

Giesecke, K., 2006, Default and Information. *Journal of Economic Dynamics & Control,* Volume 30, 2281–2303.

Giesecke, K. and L.R. Goldberg, 2004, Sequential defaults and incomplete information. *Journal of Risk* 7: 1-26.

Gilson, S., K. John and L. Lang, 1990, "Troubled Debt Restructurings: An Empirical Study of Private Reorganization of Firms in Default", *Journal of Financial Economics* 27, 315-353.

Goldstein, R., N. Ju, and H. Leland. 2001. An EBIT-based model of dynamic capital structure. *Journal of Business* 74(4), 483-512.

Hackbarth, *D*., Hennessy, C. A. and H. E. Leland, 2007, Can the tradeoff theory explain debt structure?, *The Review of Financial Studies*, 20(5), 1389-1428.

Hackbarth, *D*., J. Miao, and E. Morellec. 2007. Capital structure, credit risk, and Macroeconomic conditions. *Journal of Financial Economics*, 82(3), 519-550.

Hart, O. *D*., and J. Moore, 1994, A theory of debt based on inalienability of human capital, *Quarterly Journal of Economics* 109, 841–879.

Hart, O. *D*., and J. Moore, 1998, Default and renegotiation: A dynamic model of debt, *Quarterly Journal of Economics* 113, 1–41.

Hillegeist S. A., Keating E. K., Cram D. P. and Lundstedt K. G., 2004, Assessing the Probability of Bankruptcy, *Review of Accounting Studies*, Vol. 9, No. 1, pp. 5-34.

Houweling. P., and T. Vorst, 2005, "Pricing Default Swaps: Empirical Evidence." *Journal of Intemational Money and Finance*, 24 (2005). 1200-1225.

Huang, J.Z. and Huang, M., 2003. How much of the corporate-treasury yield spread is due to credit risk? Working Paper, Pennsylvania State University.

Hull, J., Nelken, I., and White, A., 2004, "Merton's Model, Credit Risk, and Volatility Skews",

Jarrow, R. A. and S. M. Turnbull, 1995, Pricing derivatives on financial securities subject to credit risk*,* *Journal of Finance* 50(1), 53-86

Jarrow, R. A., and P. Protter, 2004, Structural versus reduced form models: a new information based perspective, *Journal of Investment Management* 2, 1-10.

Ju, N., R. Parrino, A. Poteshman, and M. Weisbach., 2005, Horses and rabbits? Trade-off theory and optimal capital structure. *Journal of Financial and Quantitative Analysis*, 40(2), 259-281.

Kealhofer S. and Kurbat M., 2002, "The Default Prediction Power of the Merton Approach, relative to Debt Ratings and Accounting Variables," KMV LLC.

Kim, J., K. Ramaswamy, and S. Sundaresan, 1993, Does default risk in coupons affect the valuation of corporate bonds? *Financial Management* 22: 117-131.

Leland, H.E., 1994, Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance* 49 (4), 1213–52.

Leland, H. E., 1998, Agency costs, risk management, and capital structure. *Journal of Finance* 53, 1213-1243.

Leland, H. E. 2004. Predictions of default probabilities in structural models. *Journal of Investment Management* 2, 2004, 1-16.

Leland, H. E., and K. B. Toft, 1996. Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. Journal of Finance 51(3), 987-1019.

Longstaff, F. 1995. How much can marketability affect security values? *Journal of Finance* 50. 1767-1774.

Longstaff, F., S. Mithal, and E. Neis, 2005. Corporate yield spreads: Default risk or liquidity? New Evidence from the Credit Default Swap Market. *Journal of Finance* 60(5), 2213-2253.

Li K.L., Wong H.Y.,2006, Structural models of corporate bond pricing with maximum likelihood estimation, *Journal of Empirical Finance*, 15 (4), pp. 751-777.

Longstaff, F. and E. Schwartz., 1995, A simple approach to valuing risky fixed and floating rate debt. *Journal of Finance* 50: 789-819.

Lyden, S., and D. Saraniti, 2001, An Empirical Examination of the Classical Theory of Corporate Security Valuation, SSRN Working Paper.

Mauer, *D*. and A. Triantis. 1994. Interactions of corporate financing and investment decisions: A dynamic framework. *Journal of Finance* 49, 1253-1277.

Mauer, *D*. and S. Ott. 2000. Agency costs, underinvestment, and optimal capital structure. In M. Brennan, L. Trigeorgis, eds., Project flexibility, agency, and competition: New developments in the theory and application of real options. Oxford, 151-179.

Mella-Barral, P., 1999, The dynamics of default and debt reorganization, *Review of Financial Studies* 12, 535–578.

Mella-Barral, P., and W. R.M. Perraudin, 1997, Strategic debt service, *Journal of Finance* 52, 531–556.

Merton, R. C., 1974, On the pricing of corporate debt: the risk structure of interest rates. *Journal of Finance* 29: 449-470.

Moraux, F., 2004. Valuing corporate liabilities when the default threshold is not an absorbing barrier. Working Paper, Université de Rennes 1.

Morellec, E. 2001. Asset liquidity, capital structure and secured debt. *Journal of Financial Economics* 61: 173-206.

Nielsen, L.; J. Saa-Requejo; and P. Santa-Clara, 1993, Default Risk and Interest Rate Risk: The Term Structure of Default Spreads. Working Paper, INSEAD.

Ross, S. A., 2005, Capital structure and the cost of capital. *Journal of Applied finance,* 15(1), 5-23.

Schaefer S. M. and I.A. Strebulaev, 2008, Structural Models of Credit Risk are Useful: Evidence from Hedge Ratios on Corporate Bonds, *Journal of Financial Economics*, 90 (1), 1-19.

Sobehart J. R., Keenan S. C. and Stein R. M., 2000, Benchmarking Quantitative Default Risk Models: A Validation Methodology, Moody's Investors Service, Global Credit Research.

Stein Roger M., 2002, Benchmarking Default Prediction Models Pitfalls and Remedies in Model Validation, Moody's KMV, Technical Report.

Sundaresan, S. and N. Wang, 2007, Investment under uncertainty with strategic debt service. *American Economic Review* 97 (2), 256-261.

Sarkar, S. and F. Zapatero, 2003, The trade-off model with mean reverting earnings: theory and empirical tests. *The Economic Journal* 113: 834-860.

Tang, D. Y., and H. Yan, 2006, Macroeconomic conditions, firm characteristics, and credit spreads, *Journal of Financial Services Research* 29, 177-210.

Uhrig-Homburg, M., 2002, Valuation of Defaultable Claims - A Survey, *Schmalenbach. Business Review* 54(1),. 24-57.

Weiss, L., 1990, Bankruptcy Resolution: Direct Costs and Violation of Priority of Claims, *Journal of Financial Economics*, 27, 285-314.

Zhou, C, 2001, The term structure of credit spreads with jump risk. *Journal of Banking and Finance* 25(11): 2015-2040.

# Chapter 2

# Estimation of the Default Risk of Publicly Traded Companies: Evidence from Canadian Data

## 2.1. Introduction

Predicting bankruptcy risk is an important component of credit-risk forecasts and has been an area of active research ever since the seminal work of Beaver (1966, 1968) and Altman (1968). This traditional statistical approach to the forecast of default hinges primarily on accounting information. It includes Altman's Z-Score model (1968) and Ohlson's O-Score model (1980). Using multivariate discriminant analysis and regression methodology respectively, these models identify accounting variables and financial ratios that separate more accurately defaulting and surviving firms. More recent accounting-based models are designed either to find additional variables capable of improving estimations of the default probability of firms or to use more advanced econometric methods. For instance, Shumway (2001) criticizes the static nature of one-period logit or probit models. He recommends the use of a duration model and the combination of accounting-based and market-driven variables to improve the forecasting performance (see the recent reviews of Duffie and Singleton, 2003, and Lando, 2004).

Accounting models suffer from the fact that accounting data provide information only about the firm's past and are therefore backward looking. Financial statements lack flexibility, since they are updated only at discrete time intervals, usually on an annual basis. Accounting data could be noisy as well, reflecting a distorted picture of the firm's economic reality. Indeed, based on the disclosure rankings of the Association for Investment Management and Research, Yu (2005) finds that companies disclosing more accurate information tend to have a lower credit spread. Finally, the accounting model is unable to account for the volatility of assets. Campbell and Taksler (2003), using panel data, find that idiosyncratic volatility can explain corporate bond yields; they point out the importance of this parameter, especially when excessive volatility is observed in financial markets (see also Campbell, Lettau, Malkiel, and Xu, 2001).

An alternative approach is to use the structural approach[8]. This approach provides a default probability measure based on the structural relationship between risk factors such as the firm's debt, equity, and asset value. Since equity market prices are used to estimate non-observable assets values, the structural approach is considered a market-based one. In recent years, structural models have found their way as tools for predicting bankruptcies both among academics and practitioners. For example, KMV has developed a widely used commercial structural model to estimate the expected distance to default, and Vassalou and Xing (2004) have used the structural model's default measure to see whether the default risk is priced in equity returns.

Hillegeist, Keating, Cram, and Lundstedt (2004) and Chava and Jarrow (2004) document, however, that the theoretical probabilities estimated from structural models do not capture all the available information relevant to a firm's default risk. Hillegeist et al. (2004) show that traditional risk measures, such as the updated versions of Altman's Z-Score and Ohlson's O-Score, do add incremental information. In other words, the default probability estimated from the structural model is not a sufficient statistic of the actual probability of default. More recently, Du and Suo (2007) and Benos and Papanastasopoulos (2005) have found that the hybrid approach improves the prediction of credit-rating changes.

These results can be explained by the fact that, despite the flexibility and forward-looking nature of the structural model, asset prices may not suffice to estimate the borrower's credit worthiness. Such information is, for instance, unable to predict defaults due to severe liquidity problems. Moreover, asset prices could also be affected by trading noises, which can, in turn, affect the firm's asset volatility (Duan and Fulop, 2009).

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[8] A third means of measuring default risk is to estimate a reduced form model. This model is discussed in Section 2.

Our first goal is to test whether, when being used to estimate the probability of default in a hybrid model, the structural default probability (PD) can add incremental information to the firm-specific accounting data and the macroeconomic factors. To our knowledge, our contribution is the first to apply this hybrid approach to Canadian firms[9]. Our second goal is to compare the standard Merton-KMV model with the three-parameter structural model of Brockman and Turtle (2003). Both models are estimated by using the maximum likelihood method. We find that, when combined with the relevant firm-specific accounting data and macroeconomic factors for estimating default probability, the structural default probability (PD) is significant in predicting the occurrence of defaults. Moreover, the hybrid model outperforms other models. These results are robust to sub-periods, industry estimations, and other robustness tests, such as out-of-sample forecasts and the nature of the structural model. We conclude that the implementation of an early warning system to monitor changes in the credit worthiness of the Canadian public companies studied cannot depend on the structural approach alone. The structural and the accounting approaches are complementary rather than interchangeable.

The remainder of the article is divided as follows. Section 2 reviews the major models found in the literature. Section 3 describes the database used. Section 4 presents the estimation of the structural models, while section 5 covers that of the hybrid model. Robustness tests are also presented in this section. The last section concludes.

## 2.2. Review of Models for Evaluating Default Risk

Default risk models can be grouped into three categories: Accounting models based on firm's financial statements, structural models and reduced form models.

### 2.2.1. Accounting Models

The first models for scoring firms were developed by Beaver (1966, 1968) and Altman (1968). For example, the Z-score model uses five financial ratios to attribute a credit

[9] Our study is not the first one to investigate the performance of hybrid model performance. For instance Du and Suo (2007) study whether the distance-to-default is a sufficient statistic for assessing the credit quality of the debt-issuing firm. However, this study is carried on firms rated by S&P and is not specific to Canadian firms.

score to firms. An extension to this approach has used linear or non-linear regression models to do a direct estimation of the probabilities of default. These models allow several ratios and assorted financial data to be considered simultaneously. Logit and probit models are often used. Typically, the greatest variations in the probabilities of default come from ratios capturing firms' profitability, level of indebtedness, and liquidity. These models can be estimated on cross-sectional or panel data.

The main benefit of accounting models is their precision in estimating probabilities of default. Furthermore, they are easy to use for financial institutions equipped with strong database management systems. On the other hand, these models are not flexible, since they require information from audited financial statements. It thus proves very difficult to update probabilities of default over the course of a year. Some institutions may produce financial statements on a quarterly basis, but these are rarely audited. Another criticism of accounting data is that they have no forward-looking aspect. They reflect the past well, but tell us nothing about the future. Market data are usually more relevant to forecasting probabilities of default.

### 2.2.2. Structural Models

To respond to these criticisms, several structural models based on Merton (1974) were proposed, which allows calculating probabilities of default from market data. This model is a direct application of the Black-Scholes model for valuing European options. Stockholders own call options on the firm's assets, the strike price of which is the debt level. At the horizon date, they exercise the option if the value of the assets exceeds that of the debt; they then reimburse the debt and share the surplus. Otherwise, the firm is in default and stockholders do not exercise their options. Their loss is equal to the initial investment. The probability of default is thus the probability that the option will not be exercised. To evaluate this probability, we need to assign a value to the option. After having computed the mean value of the asset and its standard deviation, we can find the distance-to-default (DD), which is equal to the gap between the mean asset value and the value of the debt, normalized by the standard deviation of the asset value. The shorter this distance, the greater the probability of default (PD). Many versions of the basic structural model have been suggested in the literature, including Moody's KMV that sets the exogenous default barrier at total short debt plus 50% of long-term debt (Crosbie and Bohn, 2003). Finally, Cremers et al. (2008) find that structural models are useful for the analysis and pricing of credit risk.

To improve the basic Merton model, several extensions have been suggested in the literature. The most relevant to our project is the one proposed by Brockman and Turtle (2003). The main criticism of Merton's model is that it does not account for the possibility that the firm may default before the debt matures. Also, only stockholders are involved in exercising the option. Firms will in general default before this horizon date, and the lenders (banks and other creditors) owning options (debt covenants) are in a position to exercise these options if they observe that the latter are in breach of their debt obligations or are simply unable to pay.

To take formal account of these two dimensions, Brockman and Turtle (2003) propose down-and-out options, using Black and Cox (1976) model, but other types can be applied. The down-and-out option makes it possible to bankrupt the firm as soon as the value of its assets reaches the barrier— at any time before, or at, the debt's maturity. The appeal of this option is that it can be adjusted to bankruptcy laws all over the world, including in Canada. It can also account for the various restrictions creditors impose on borrowing firms, restrictions such as maintaining a low debt-to-asset ratio, limiting dividend payments, curtailing merger activity, and not issuing further debt.

Duan, Gauthier, and Simonato (2004) demonstrate that using maximum likelihood estimation (MLE) methodology to estimate the parameters of the Merton model yields results resembling those generated by the iterative estimation method. What makes the MLE method appealing is its openness to statistical inference and to the use of descriptive statistics, such as the value of the firm, in estimating the parameters. Moreover, the MLE method provides an estimate of the asset return drift, $\mu$, while the iterative method does not provide estimate of this parameter. The drift is critical in estimating the physical default probabilities. Another important aspect of the contribution made by Duan, Gauthier, and Simonato (2004) is that, when we insert an additional parameter into the structural model to account for capital structure – as in Brockman and Turtle (2003) – the correspondence between the two estimation methods is not necessarily perfect. In this particular instance, the MLE method provides the best results, since it yields unbiased estimates of the parameters. Wong and Choi (2004) also developed a maximum-likelihood model which uses an endogenous capital structure. In our study, we first apply the MLE method with two parameters. We also conduct a sensitivity analysis by estimating the three-parameter models of Wong and Choi (2004) and Duan et al. (2004) (see Reisz and Perlich, 2007, for another application of the barrier option model, and Ericsson and Reneby, 2004 and 2005, for other contributions showing the superiority of the MLE method in estimating the structural method).

### 2.2.3. Reduced Form Models

In the reduced form approach, the default is treated as an stopping time (see Jarrow and Turnbull, 1995, and Duffie and Singleton, 1999, among others). This is in contrast to the structural model that views default as the outcome of a gradual process of deterioration in asset values. The reduced form approach does not specify the economic process leading to default. By relaxing the structural model's assumption that investors are perfectly informed about asset prices, one makes the default time an unpredictable event and establishes a direct link between the two approaches (Duffie and Lando, 2001; Giesecke, 2006). For our purpose of estimating the default probability of firms listed on a stock exchange, the structural model appears to be more appropriate.

## 2.3. Database

In this section, we present the raw data and their sources; we also explain how we constructed the database used to calculate the probability and the econometric estimations. The study period for the probabilities of default runs from January 1988 to December 2004 for a total of 6309 observation corresponding to 762 firms. To ensure the statistical reliability of the methodology we apply in computing the probabilities of default with the structural model, our data window must stretch back 12 months prior to the estimation period used for predicting the probabilities of default. Thus, the stock exchange and accounting data needed to estimate the structural model were gathered starting from January 1987. We removed all the financial companies from the database, since the structure of their financial statements differs from those of non-financial firms.

Firms that have defaulted are catalogued in Financial Post Predecessors & Defunct, CanCorp Financials (Corporate Retriever), and Stock Guide. Market data, necessary for obtaining daily market capitalization, is extracted from DataStream's DEAD.LLT series. The accounting data comes from Stock Guide and CanCorp Financial with annual frequency. Between 1988 and 2004, 130 firms were identified as being in default[10]: 112 were bankrupt and liquidated while 18 were undergoing reorganization[11].

After merging the accounting data with the daily market data, 77 firms remained in the intermediary database of defaults, i.e., the one intended for the first stage of our study where we compute the probabilities of default using the structural model. This attrition is mostly attributable to the fact that, for some firms, we had only incomplete market data where the available number of daily observations is not sufficient for estimation and, for others, only one year of accounting data – rendering the data unusable for our study in both cases. In fact, application of the structural model requires at least 200 consecutive daily market prices coupled with available accounting data on the book value of debt for defaulted firms. The 200 daily observations requirement reduces the number of defaulted firm but such a window insure the accuracy of the estimated parameter. As in Vassalou and Xing (2004), we use the book value of debt for the new fiscal year starting only four months after the end of the previous fiscal year. The goal is to ensure that we utilize only the data available to investors at the time of calculation. As a result, we need at least two successive financial statements to obtain the 200 estimation observations required.

We looked, in more detail, at the lags separating the bankruptcy or the reorganization dates from the last financial statements of some defaulted firms. Many firms do not publish financial statements during the final years prior to their bankruptcy. We felt

[10] We aknowledge that the default definition is generally wider than the one used here, for instance Basel II Accords (2006)

[11] Among the 18 firms in reorganization, 12 emerged from the reorganization process as an ongoing entity while 6 were liquidated.

obliged to withdraw from the database those for which these lags exceeded 18 months. For the others, i.e., those that had defaulted between 12 and 18 months after their final financial statement, we moved the date of the default up to reconcile it with the last observable accounting year. This filtering reduced the number of defaults retained in our sample to 60 companies. During the second phase of the study, i.e., during the hybrid application, 59 of the defaulting firms remained in the final database. The non-defaulting firms sample includes the non-financial Canadian firms listed on the TSX and contained in the StockGuide database. The financial statements of one firm were incomplete and did not contain the variables required for a multivariate analysis. Table 2.I, Panel A, provides a summary of the filters applied to our database and the number of firms retained at each step.

<Insert Table 2.I here>

The market data on the non-financial firms listed on the Toronto Stock Exchange that did not default are drawn from DataStream's FTORO.LLT series. The frequency of market value data is daily for the period from 1 January 1987 to 31 December 2004. Accounting data are drawn from StockGuide. To begin our estimations, we merged the accounting database from Stock Guide for stocks listed on TSX in 2004 with the market database from DATASTREAM. Our final database includes 762 publicly traded non-financial Canadian firms, 703 of which did not default and 59 of which did. The total number of firm-year observations is 5,744.

Table 2.I, Panel B, provides some statistics for the 762 publicly traded non-financial Canadian firms retained for the study. In total, we have 1,885,707 daily observations for the market value variable. The market capitalization mean, over all firms, is Can$ 820.68 million. The standard deviation is Can$ 4,459.97 million, owing to the existence of very high market capitalization values for some firms.

## 2.4. Estimation of the Probabilities of Default with Structural Models

In choosing a structural model, we first settled on a variant of the Merton (1974) model. In this setting, the equity of the firm is represented as a European call option on the firm's assets. If, at debt maturity, the value of the assets exceeds the book value of debt[12], which corresponds to the strike price of the call option, the equity holders exercise the option and repurchase the firm's assets, otherwise, the firm defaults.

We assume that the firm's capital structure consists exclusively of debt plus equity. Moreover, the debt is assumed to be a single zero-coupon bond. We assume also that the market value of a firm's underlying assets follows a geometric Brownian motion. We consider only two types of liabilities, a single class of debt and a single class of equity. The level of book value of debt is denoted by $X_t$ and represents the strike price of the call option. $(T-t)$ is the time to maturity. Under these specifications, we can apply the Black and Scholes formula for the call option to obtain the market value of equity:

$$V_{E,t} = V_{A,t}N(d_{1,t}) - X_t e^{-r(T-t)}N(d_{2,t}) \tag{2.1}$$

where $V_{E,t}$ is the market value of the firm's equity, $V_{A,t}$ is the market value of the firm's assets at time *t*,

$$d_{1,t} = \frac{\ln(V_{A,t} / X_t) + (r + \frac{\sigma_A^2}{2})(T-t)}{\sigma_A \sqrt{T-t}}, \tag{2.2}$$

$d_{2,t} = d_{1,t} - \sigma_A \sqrt{T-t}$, $r$ is the risk-free interest rate, *N(.)* is the cumulative density function of the standard normal distribution, and $\sigma_A$ is the volatility of the assets.

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[12] In order to ensure that we only use the accounting information available to investors, we use the debt book value 4 months after the end of the fiscal year. For instance, assuming that the total book value of debt is 1,000$ in year y and 1,500$ for a firm with fiscal year ending on December 31[st] . The firm is assumed to have a total debt of 1,000$ until April 30[th]. This is consistent with Vassalou and Xing (2004) for instance.

Since we observe only the market price of equity, the asset's value, expected return, and volatility are unknown. Indeed, we need to infer these parameters from equity-price time series in order to compute the default probabilities. Duan (1994) proposes a procedure which involves estimating the parameters, based on the Maximum Likelihood Estimation (MLE), where the observed equity prices are viewed as a transformed data set with the Black-Scholes formula serving as the transformation. Duan et al. (2004) show that under Merton's (1974) model and assuming that the asset value is directly observable, the log-likelihood function can be written as follows:

$$L_V = -\frac{n}{2}\ln(2\pi\sigma_A^2\Delta t) - \frac{1}{2}\sum_{t=2}^{n}\left[\frac{\ln(V_{A,t}/V_{A,t-1}) - (\mu_A - \sigma_A^2/2)\Delta t}{\sigma_A\sqrt{\Delta t}}\right]^2 - \sum_{t=1}^{n}\ln V_A^t$$

where $\Delta t$ is the time interval between two successive observation dates, expressed in years and $\mu_A$ is the drift of the assets.

However, we do not observe the asset value but the equity values. The Black-Scholes formula in equation (2.1) provides a differentiable relation between asset and equity values. Given the volatility parameter, $\sigma_A$, we can invert equation (2.1) to obtain $\hat{V}_{A,t}$ from $V_{E,t}$. Moreover, since this relation is differentiable, we can obtain the log-likelihood function on the observed equity data[13]:

$$L_E = -\frac{n}{2}\ln(2\pi\sigma_A^2\Delta t) - \frac{1}{2}\sum_{t=2}^{n}\left[\frac{\ln(\hat{V}_{A,t}/\hat{V}_{A,t-1}) - (\mu_A - \sigma_A^2/2)\Delta t}{\sigma_A\sqrt{\Delta t}}\right]^2 - \sum_{t=1}^{n}\ln\hat{V}_{A,t} - \sum_{k=1}^{n}\ln(N(d_{1,t})) \quad (2.3)$$

where the last term represents the sum of logarithm of the derivatives $\frac{\partial V_{E,t}}{\partial V_{A,t}} = N(d_{1,t})$.

[13] Duan et al (2003) show that survivorship issue could bias the asset's drift and volatility estimates. Yet, we don't think that this is the case in our estimation because the studied sample is not restricted to survived firms, but also includes the defaulted firms.

Following Vassalou and Xing (2004) and the KMV implementation of the Merton model, we locate the level of liabilities at short-term debt plus one half of long-term debt, for an option maturity, *T*, of 1 year. While this choice remains arbitrary, Crosbie and Bohn (2003) argue that it is logical and captures adequately the financing constraints of firms. The estimation window is always kept equal to 1 year and the risk-free rate is that of the 1-year Canadian Treasury bill at the beginning of each year. The likelihood function in equation (2.3) is maximized using the Nelder-Mead Simplex Algorithm (Fminsearch in Matlab), where the convergence criterion is set at 1 × 10-6.

After estimating the asset's drift and the volatility, we compute the associated assets value. In so doing, we can obtain the default probability. The probability of default is the probability that the value of the firm's underlying assets will be less than its liabilities at the debt's maturity, that is:

$$DP = \text{Prob}\left\{V_{A,t+T} \leq X_t \middle| V_{A,t}\right\} = \text{Prob}\left\{\ln\left(V_{A,t+T}\right) \leq \ln\left(X_t\right) \middle| V_{A,t}\right\} \tag{2.4}$$

The geometric Brownian motion diffusion process of the firm's assets implies a lognormal distribution. At the time $t+T$, the value of the assets is given by:

$$\ln\left(V_{A,t+T}\right) = \ln\left(V_{A,t}\right) + \left(\mu_A - \frac{\sigma_A^2}{2}\right)T + \sigma_A\sqrt{T}\varepsilon \tag{2.5}$$

where $\varepsilon \sim$ N (0,1).

Combining (2.4) and (2.5), the default probability becomes[14]:

[14] In order to test the effect of the noise in asset drift estimation, we tested the performance of the default probabilities in forecasting defaults of the risk neutral and physical structural probabilities. The latter achieves higher performance in predicting default occurrence; we therefore conclude that the asset drift estimates bring additional information on firm's credit worthiness. Thus, despite the lack of precision in this parameter estimation, it is preferable to use it in the default probabilities computation. This is consistent with Bharath and Shumway (2004) findings.

$$DP_t = N\left( -\frac{\ln\left(\frac{V_{A,t}}{X_t}\right) + \left(\mu_A - \frac{\sigma_A^2}{2}\right)(T-t)}{\sigma_A\sqrt{(T-t)}} \right) = N(-DD_t) \tag{2.6}$$

where $DD_t$ denotes the distance to default. The smaller the distance to default, the greater the likelihood a default will occur. We should note that KMV uses its large historical database, including more than 2,000 defaults, to map the DD level to the default probability. That is, KMV uses the empirical distribution of defaults instead of the theoretical normal density function implied by the Merton (1974) model. Since our database is much more limited, we use the normal density function to compute the default probability. We implement the model with the physical probability instead of the risk neutral probability. Indeed, Bharath and Shumway (2004) find that the drift parameter $\mu_A$ is quite an important element in the distance-to-default computation of the KMV-Merton model.

Table 2.II, Panel A, presents the probabilities of default, computed one year prior to the period of risk exposure, for firms that did default and for those that did not. The mean of the probabilities for defaulting firms is 53.04 %, while that for non-defaulting firms is 13.22 %. As a first way of measuring the accuracy of the Merton approach, we test to see if the difference in the means of PD is statistically significant. The null hypothesis of equality of means is rejected at the 1% level of significance, both with and without assuming the equality of variances between the two groups. The estimated parameters are presented in Table 2.II, Panel B.

<Insert Table 2.II here>

We have also applied the Wong and Choi (2004) and Duan et al. (2004) estimation methods for the three-parameter model. Parameter estimates are reported in Table 2.II, Panel B (details are available from the authors). Both estimation methods yield similar results for the structural model parameters and average *PD*. We shall verify how the nature of the structural model affects the conclusion about the hybrid model.

In the following section, we assess in more detail the structural models' performance in discriminating between the two groups of firms, and we compare the informational content of this measure with accounting and macroeconomic variables using probit estimations.

## 2.5. Estimation of the Hybrid Model

### 2.5.1. Methodology

The probit model estimated accounts for potential correlations between different observations of the same firm at different points in time (different financial statements). It is defined by the following regression:

$$y_{it}^{*} = \beta' x_{it} + \varepsilon_{it}. \tag{2.7}$$

where $y_{it}^{*}$ is an unobservable latent variable, *i* represents the firm, and *t* the time of firm *i*'s financial statement. $x_{it}$ is a vector of explanatory variables such as financial ratios or macroeconomic variables. The observed dichotomous variable is a state variable indicating default and defined as follow:

$y_{it} = 1$ if default or $y_{it}^{*} > 0$

$y_{it} = 0$ otherwise, (2.8)

To account for intertemporal correlation using a random-effects model, the error must be decomposed into two terms $\varepsilon_{it} = v_{it} + \vartheta_i$, where $v_{it} \sim N(0,1)$ is the stochastic-error component and $\vartheta_i \sim N(0, \sigma_u^2)$ is the random firm effect, so that the two error components ($v_{it}$ and $\vartheta_i$) are normally distributed with mean 0 and are independent of each other[15]. The variance of the error term $\varepsilon_{it}$ can then be represented by $\mathrm{var}(\varepsilon_{it}) = \sigma_v^2 + \sigma_\vartheta^2 = 1 + \sigma_\vartheta^2$ and the correlation is equal to $corr(\varepsilon_{it}, \varepsilon_{is}) = \rho = \dfrac{\sigma_\vartheta^2}{1+\sigma_\vartheta^2}$. Thus,

[15] Refer to Greene (2008), page 796, and Wooldridge (1999) for a discussion of these assumptions.

the free parameter is $\sigma_{\vartheta}^{2}=\frac{\rho}{1-\rho}$. This is the parameter that will make it possible to measure the existence of a correlation between the different observations (PDs or financial statements) of a single company over time.

### 2.5.2. Variable Selection

The principal objective of this section is to verify whether combining the statistical and the structural models into a hybrid model will yield a better measure of the default risk. This task is done by comparing the hybrid model's performance in predicting defaults with that of the structural and of the accounting model. To accomplish this, we try to explain defaults by estimating the probit model using three different specifications. For the first model, the structural one, the only explanatory variable is the probability of default (PD) generated by the structural model. The second model, the accounting one, makes use of accounting and macroeconomic variables. Finally, the hybrid model combines the two sets of explanatory variables to explain defaults.

Thus, we test the PD variable's predictive power in explaining corporate bankruptcy by including it in the accounting model. If, after controlling for the effect of the firm's accounting data, we find that the estimated coefficient of the PD variable is statistically different from zero, the probabilities of default yielded by the structural approach will be shown to contain information which supplements that in the accounting and macroeconomic data; and we will then be able to use its coefficient to update the probability of default over time when the PD changes (flexibility).

As to the selection of accounting variables and financial ratios used in the non-structural and hybrid models, we first retained a wide array of variables and financial ratios liable to have an impact on the quality of the firm's credit and for which we were able to obtain satisfactory data. This choice of variables was based on both empirical studies addressing the determinants of default in Canadian firms (Beaulieu, 2003) and on studies conducted in other countries (Bank of England; RiskCalc; Z-score).

To make a sound selection, we started by estimating the probit model on each accounting variable separately. This allowed us to retain the most significant ones and thus reduce the number of missing observations in our final estimation. However, we retained only the variables not heavily correlated with the PDs to avoid multicollinearity problems. Descriptive statistics for the variables retained are presented in Panel A of Table 2.III[16].

<Insert Table 2.III here>

We report the Pearson correlations matrix for the dependent and independent variables retained in Panel B of Table 2.III. We notice that the dependent variable, i.e. the dummy indicating the default, is positively and significantly related to the estimated structural-default probability. Moreover, the default variable is also strongly related to the dummy variables of operating profitability. It would thus seem that defaulting firms tend to have a negative or low operating margin. Finally, the structural PD is significantly correlated with all the independent variables selected, though at a lower level than with the variables dropped. This may indicate that the structural default measure and the accounting variables may contain some common information. We examine these issues in more detail in the multivariate analysis.

## 2.6. Analysis of Probit Results

### 2.6.1. Estimation of Probit Panel Model Using Different Specifications

In this section, we turn our attention to the results from the probit estimation. Table 2.IV allows us to compare the results obtained from regressions on a structural model (Model 1), where the only dependent variable is the structural default probability; on the accounting model (Model 2), where only accounting and macroeconomic variables are included as independent variables; and on the hybrid model (Model 3), where the structural PD is added along with other accounting and macroeconomic variables.

<Insert Table 2.IV here>

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[16] The structural PD variable remains significant when all the other accounting variables are added to the regression. The PD variable is then robust to alternative specification.

We notice that the correlation parameter between different observations of the same company over time $\rho$ is significant only in Model 1, with a p-value < 1%. Hence, we can conclude that the structural default probabilities for the same company are correlated over time. This is not the case in Models 2 and 3, where the correlation parameter is not significant at any of the usual confidence levels, with respective p-values close to 1 in both models. It is well known that a better specification of the model reduces the residual correlation.

Furthermore — and still in Model 1 — the coefficient of the annual mean PD is equal to 2.01 and is statistically significant at the 1% level. We observe, in Model 2, that the coefficients on the firm-specific financial ratios have the expected signs and are significant. Moreover, the real GDP growth is negatively related to the likelihood of default, but is significant only at the 10% level. When we add the structural default probability in the hybrid model (Model 3), the coefficient (1.54) of the PD variable remains significant, while the ratio net value/total liabilities loses its significance. Thus, we can conclude that the information on the likelihood of default contained in the accounting data does not subsume the information content of the structural-default probability.

We also estimated the probit regressions with the barrier model. The results are quite similar to those obtained with the Merton approach. The corresponding parameters for the PD variable are 2.84 (Model 1) and 1.44 (Model 3) with the Wong and Choi (2004) model and 2.05 (Model 1) and 1.06 (Model 3) with the Duan et al. (2004) model. All these parameters are significant at 1%. Details are available from the authors.

We notice from Table 1.4 that the dummy variables accounting for the operating margin are significant at the 1% level in Models 2 and 3 and have the expected signs. That is, if the operating margin is negative, the firm's likelihood of default increases. This may be explained by the fact that many firms have to contend with the non-liquidity of stocks. The hybrid model has a much lower Akaike information criterion (AIC). For instance, Model 3 achieves a 271 Akaike information criterion, whereas Model 1 attains 562. And we observe the value 293 for the accounting model. It is clear here that the hybrid model outperforms both the structural and the accounting models in estimating the occurrence of default. We can therefore conclude that the theoretical probabilities estimated from structural models do not adequately predict the actual default probability. The structural approach's poor prediction of default probabilities for U.S. firms has also been documented by Bharath and Shumway (2004), Hillegeist, et al. (2004) and Stein (2005), among others. In the following section, we conduct additional tests to fine tune our comparison of the performances of the structural and the hybrid approaches in predicting defaults.

### 2.6.2. Different tests

A maximum-likelihood ratio test comparing the first and the third models, in Table 2.IV, leads us to reject the first model at all the usual confidence levels. Indeed:

$$-2(\ln L_{Model\ 1} - \ln L_{Model\ 3}) = 289.76 > \chi^2_{(5)} = 11.07$$

where $L_{Model\ i}$ is the likelihood of the model *i* and, for example, 11.07 is the $\chi^2_{(5)}$ value at the 95% confidence level. On the other hand, the maximum-likelihood ratio test comparing Model 2 and Model 3 attains 24.5, while the tabulated value of the Chi-square with 1 degree of freedom at the 95% confidence level is 3.84. We can thus reject Model 2 in favour of Model 3.

To evaluate the overall explanatory performance of the different models, we use the Receiver Operating Characteristic (ROC) curve, as well as the Cumulative Accurate Profile (CAP) or power curve. The area under the ROC curve or AUC measures the model's performance in predicting actual defaults. Indeed, the ROC reports the percentage of defaults that the model correctly classified as defaults on the y-axis and the percentage of non-defaults that the model incorrectly classified as defaults on the x-axis. The entire curve is obtained by varying the cut-off points, i.e. the predicted probability above which the firm is classified as in default, from 0 to 1. The advantage of the ROC curve as compared to the CAP curve resides in its ability to account for Type II errors. The more accurate the model, the closer its ROC curve approaches the top left corner. The area under the curve measures this performance. A perfect model will have an AUC of 1, while a perfectly naïve model will score 0.5. As shown in Figure 1 (Panel A), the hybrid model greatly outperforms the structural model. Indeed, the AUC for the structural model attains 0.815, compared to 0.977 for the hybrid model. The CAP curves in Figure 1 (Panel B) show the same result. The hybrid models also outperform the accounting model, the latter having an AUC of 0.933. The difference between two AUCs has a Chi-square distribution with one degree of freedom; we can therefore test the null hypothesis of no differences between the AUCs of Models 2 and 3. The associated Chi-square statistic is 5.1, which corresponds to a p-value of 0.0238. We can reject the null at the 5% confidence level. We therefore conclude that, compared to the accounting variables, the PD measure does offer additional informational content.

We also the performance of the three models is examined against the 9$^{th}$ decile of the predicted probabilities (Table 2.V). The firm/year observations are classified according to their predicted probabilities generated for each model. We then examine how many defaults belong to the highest decile[17] for each model. The preceding conclusion is confirmed. Indeed, the hybrid model succeeds in classifying 96.61% of the defaults correctly, whereas the structural model succeeds in the correct prediction of only 61.02% of the defaults. We can therefore conclude that the hybrid model outperforms the structural model in predicting corporate failures. Thus, structural-default probabilities do not act as substitutes for accounting variables and financial ratios, but rather as complements to them.[18]

<Insert Table 2.V here>

### 2.6.3. Robustness Tests

We conducted sensitivity tests to analyze the robustness of the main results. First we replicate our probit panel estimation for sub-samples of firms by year. The first period

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[17] The choice of the percentile is subjective, nonetheless the 10% percentile is usually used, refer for instance to Brockman and Turtle (2003).

[18] These results are robust to Jackknifed regressions and the deduction of the industry mean for accounting variables.

goes from 1988 to 1995 and contains 1,441 firm-year observations, among which 27 are defaults. The second data subset covers the 1996-2004 period; it contains 4,303 observations, including 32 defaults.

Table 2.VI presents the estimation results. We observe that, for the two sub-periods, the coefficients on the structural default probability are quite similar in magnitude. Moreover, they do not differ much from the estimates for the whole sample. In unreported results, we worked with another estimation period —1988-1996 instead of 1997-2004— with qualitatively the same results. Hence, our estimation results are not period specific and are robust to sub-period estimations.

<Insert Table 2.VI here>

In Table 2.VII, Panel A, we compare the models' performance in predicting defaults for the two sub-periods. Here again we are interested in the percentage of hits and false alarms in the 9$^{th}$ decile of the model's predicted default probabilities. The hybrid models perform better in the second period. For instance, the Type I error for Model 3 is 7.41% in the first period, as compared to 0% in the second period. Again, the hybrid models outperform the structural model for the two sub-periods, confirming that, despite its importance as a predictor of the occurrence of defaults, the Merton model is weak as an approach to the probability of default. We also used the parameters estimated for the 1988-1995 period to perform an out-of-sample evaluation in the 1996-2004 period (see Reisz and Perlich, 2007, for details on the out-of-sample methodology). The results, presented in Panel B of Table 2.VII, are consistent with the previous ones. The estimation statistics are almost the same for Model 2 and Model 3, but both models outperform Model 1 (maximum-likelihood ratio test values of 151 and 144 respectively). Model 3 outperforms the other two models in terms of predicting defaults (Type I and Type II errors). The same conclusions hold with the AUC test.

<Insert Table 2.VII here>

We move on to check for industry effect; we want, in particular, to test whether the incremental information of the accounting variable is specific to some industries. The industries most heavily represented in our sample are the manufacturing and the mining groups, with respectively 2,415 and 1,318 total observations, containing respectively 19 and 12 defaults.

<Insert Table 2.VIII here>

We repeat our analysis for the manufacturing group, comparing the results obtained to the remaining companies in our sample. Table 2.VIII reports the probit panel regression results and shows that the structural-default probability remains significant at the 1% level in the hybrid models for the two groups of firms. In terms of Type I and Type II errors, Table 2.IX shows that the hybrid models make fewer misclassifications of defaulted and non-defaulted firms. We conducted the same test to compare the mining industry to the remaining firms in our sample. The findings are similar to those for the manufacturing group. The results for the PD variable are reported in the note of Table 2.VIII.

<Insert Table 2.IX here>

Vassalou and Xing (2004) find that the Fama-French factors SMB and HML contain some default-related information. For this reason, we check to see whether the structural PD is correlated with the market capitalisation of equity, the book-to-market ratio, and with the leverage ratio. We find no significant Pearson correlation PD coefficient with these variables. We go further in exploring the effect of the BTM and market capitalization variable: we divide our sample according to the medians of BTM and market cap variables, in order to see whether the ability of these models to predict defaults depends on the firm's size and BTM. The structural PD coefficients estimated in Models 1 and 3 are reported along with the Type I and Type II errors in Table 2.X. When the sample is divided by size, the structural model performs better for small firms. For larger firms, the structural PD performs less well both in terms of statistical significance in the probit model and in terms of Type I and II errors. On the other hand, the PD variable is significant at the 1% level for both low and high book-to-market firms.

<Insert Table 2.X here>

Moreover, the Jackknifed regressions reported in Table 2.XI shows that the coefficients are similar with those obtained with the whole sample. We can therefore conclude that the results are not driven by a particular default observation.

<Insert Table 2.XI here>

Finally, other accounting variables have been found to be significant in explaining credit spreads. For instance, Campbell and Taksler (2003) and Chen et al. (2007) identify the pretax interest coverage ratio and the total debt-to-capitalization as factors affecting credit spreads. We estimated Model 3 augmented by these ratios. We find, in unreported results, that the added variables are not significant in explaining the incidence of defaults in our sample. Moreover, the structural PD coefficient remains unchanged after adding these variables.

### 2.6.4. Update of the Predicted Probabilities of Default

Another advantage of the hybrid model is its flexibility. Measures of probabilities of default can be obtained at a much higher frequency with the structural approach than with accounting data. It is also possible to update the probabilities of default predicted with hybrid models by incorporating PD variables computed on a quarterly, monthly, or even daily basis. We conducted this exercise for some of the firms in our sample. We updated the probabilities of default predicted with the hybrid models by including a quarterly PD. Our results in Figure 2 show that the probability of default can increase dramatically in a year. The same analysis can be applied at a greater level of aggregation, for example, to a given group of firms or sectors.

To improve our assessment of the gains made by monthly updates of the default probabilities obtained from the hybrid model with the structural PD, we compute Type I and II errors by predicting defaults $t$ months ahead, where $t = 1, 2…, 12$. Unfortunately, these updates did not improve the accuracy of the models. Details are available from the authors.

## 2.7. Conclusion

### 2.7.1. Summary

The goal of this research is to determine how a continuous evaluation of the probabilities of default for Canadian firms publicly traded on the stock exchange might improve the prediction that a firm may default. One way of accomplishing this goal is to estimate a hybrid model in which the estimated probability of default from the structural model is introduced into the accounting model as explanatory variable.

We conducted this exercise for non-financial publicly traded companies whose shares are traded on the Toronto Stock Exchange. Our results indicate that the structural-default probabilities (PDs) contribute significantly to default predictions when they are included alongside the accounting and macroeconomic variables selected. However, other variables remain significant and maintain substantial predictive power in the presence of the structural-default probability. Thus, the structural-default probabilities act as complements to the accounting and macroeconomic data rather than as substitutes for them.

### 2.7.2. Implication for Theory and Applications

The appealing idea of using a hybrid model which combines the structural PD with accounting and macroeconomic data has recently been put into practice. Moody's has developed its own commercial implementations of the hybrid model. Indeed, Sobehart et al. (2000) combine the structural distance-to-default with other rating, market, and accounting variables. They conclude that neither the structural model nor the financial statements contain all the relevant information on the firm's credit worthiness. Thus, combining the two methods seems justifiable, since the hybrid model outperforms both the pure structural model and the pure accounting one (see Tudela and Young, 2003, Kealhofer and Kurbat, 2002, and Standard and Poors' web site for other applications of the hybrid model). Finally, Saunders and Allen (2002) offer a discussion on both the KMV implementation of the structural model (Expected Default Frequency) and the Moody's hybrid approach. They point out the limitations of the structural approach. Our results support the conclusions of Sobehart et al. (2000) and Saunders and Allen (2002). They were obtained with two versions of the structural model: the Merton model and the default barrier model. Both models were estimated with the maximum likelihood method.

### 2.7.3. Limitation and Future Research Directions

There are several possible extensions to this analysis. First, a method could be developed for aggregating the analysis over industrial sectors or over financial institutions' portfolios. This aggregation should account for correlations between the probabilities of default of the firms included. Ultimately, this model could be used to help banks construct more diversified loans portfolios

A second extension pertains to estimation of the PD using the structural model. Applying a data-filtering algorithm, like the one in Duan and Fulop (2009), can reduce bias in the estimates of standard deviations caused by trading noises on stock exchanges which distort the one-to-one relationship between asset values and firm values.

Finally, it would be very useful to adapt this method to the purposes of economic policy. This requires finding the relevant aggregates and choosing the periods in which the aggregates must be continuously updated, so as to disseminate the information to the financial institutions affected.

## References

Altman, E.I. (1968). Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy. *Journal of Finance*, 23 (4), 589–609.

Beaulieu, S. (2003). Modèle d'évaluation des probabilités de défaut un an des firmes privées canadiennes : approche probit non-structurelle. Master's thesis, HEC Montréal.

Beaver, W. (1966). Financial Ratios as Predictors of Bankruptcy. *Journal of Accounting Research*, 6, 71–102.

Beaver, W. (1968). Market Prices, Financial Ratios, and the Prediction of Failure. *Journal of Accounting Research*, 8, 179–192.

Benos, A. and Papanastasopoulos, G. (2005). Extending the Merton Model: A Hybrid Approach to Assessing Credit Quality. Working paper. University of Piraeus, Piraeus, Greece.

Bharath, S.T and Shumway, T. (2004). Forecasting Default with the KMV-Merton Model. Working paper, University of Michigan.

Black, F., and J. C. Cox , 1976. Valuing corporate securities - some effects of bond indenture provisions. *Journal of Finance* 31(2), 351-367.

Brockman, P. and Turtle, H.J. (2003). A Barrier Option Framework for Corporate Security Valuation. *Journal of Financial Economics*, 67 (3), 511–529.

Campbell, J.Y., Lettau, M., Malkiel, B.G., and Xu, Y. (2001). Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk. *Journal of Finance*, 56, 1-43.

Campbell, J.Y. and Taksler, G.B. (2003). Equity Volatility and Corporate Bond Yields. *Journal of Finance*, 58 (6), 2321-2349.

Chava, S. and Jarrow, R. (2004). Bankruptcy Prediction With Industry Effects. *Review of Finance*, 8, 537-569.

Chen, L., Lesmond, D., and Wei, J. (2007). Corporate Yield Spreads and Bond Liquidity. *Journal of Finance*, 62 (1), 119-149.

Cremers, M., Driessen, J., and Maenhout, P. (2008). Explaining the Level of Credit Spreads: Option-implied Jump Risk Premia in a Firm Value Model. *Review of Financial Studies* 21 (5), 2209-2242.

Crosbie, P. and Bohn, J. (2003). Modeling Default Risk. Mimeo, Moody's/KMV.

Du, Y. and Suo, W. (2007). Assessing Credit Quality from the Equity Market: Can a Structural Approach Forecast Credit Ratings? *Canadian Journal of Administrative Sciences*, 24, 212-228.

Duan, J. C. (1994). Maximum Likelihood Estimation Using Price Data of the Derivative Contract. *Mathematical Finance*, 4, 155-157.

Duan, J.C. and Fulop, A. (2009). Estimating the Structural Credit Risk Model when Equity Prices are Contaminated by Trading Noises. *Journal of Econometrics,* volume 150, 2 (June), 288-296.

Duan, J.C., Gauthier, G., and Simonato, J.G. (2004). On the Equivalence of the KMV and Maximum Likelihood Methods for Structural Credit Risk Models. Mimeo, HEC Montréal.

Duan, J.,G. Gauthier, J.G. Simonato, and S. Zaanoun, (2003). Estimating Merton's Model by Maximum Likelihood with Survivorship Consideration. Working Paper, HEC Montréal.

Duffie, D. and Lando, D. (2001). Term Structure of Credit Spreads with Incomplete Accounting Information. *Econometrica*, 69, 633-664.

Duffie D. and Singleton J.K. (1999). Modelling Term Structure of Defaultable Bonds. *Review of Financial Studies*, 12, 687-720.

Duffie D. and Singleton J.K. (2003). *Credit Risk*. Princeton Series in Finance, Princeton University Press, 396 pages.

Ericsson, J. and Reneby, J. (2005). Estimating Structural Bond Pricing Models. *Journal of Business*, 78 (2), 707-735.

Ericsson, J. and Reneby, J. (2004). Implementing Structural Credit Risk Models Using both Stock and Bond Prices – An Empirical Study. *Journal of Fixed Income*, 13 (4), 38-49.

Giesecke, K. (2006). Default and Information. *Journal of Economic Dynamics & Control,* Volume 30, 2281–2303.

Greene, W. H., 2008, Econometric Analysis, 6th Edition, *Stern School of Business, New York University*

Hillegeist, S., Keating, K.E., Cram, P.D., and Lundstedt, G.K. (2004). Assessing the Probability of Bankruptcy. *Review of Accounting Studies*, 9, 5-34.

Jarrow, R. and Turnbull, S. (1995). Pricing Derivatives on Financial Securities Subject to Credit Risk. *Journal of Finance*, 50 (1), 53-86.

Kealhofer, S and Kurbat, M. (2002). The Default Prediction Power of the Merton Approach, Relative to Debt Ratings and Accounting Variables', KMV LLC. Mimeo.

Lando, D. (2004). *Credit Risk Modeling*. Princeton Series in Finance, Princeton University Press, 310 pages.

Merton, R.C. (1973). Theory of Rational Option Pricing. *Bell Journal of Economics and Management Science*, 4 (1), 141–183.

Merton, R.C. (1974). On the Pricing of Corporate Debt: the Risk Structure of Interest Rates. *Journal of Finance*, 29 (2), 449–470.

Ohlson, J. (1980). Financial Ratios and the Probabilistic Prediction of Bankruptcy. *Journal of Accounting Research*, 19, 109-131.

Reisz, A.S. and Perlich, C. (2007). A Market-based Framework for Bankruptcy Prediction. *Journal of Financial Stability*, 3, 85-131.

Saunders, A. and Allen, L. (2002). *Credit Risk Measurement*. John Wiley & Sons, Inc., New York.

Shumway, T. (2001). Forecasting Bankruptcy More Accurately: A Simple Hazard Model. *Journal of Business*, 74 (1), 101-124.

Sobehart, J.R., Stein, R., Mikityanskaya, V., and Li, L. (2000). Moody's Public Risk Firm Risk Model: A Hybrid Approach to Modeling Short Term Default Risk. Moody's Investor Service, Global Credit Research, Rating Methodology.

Stein, R.M. (2005). Evidence on the Incompleteness of Merton-type Structural Models for Default Prediction. Moody's KMV Technical Paper 1-2-1-2000.

Tudela, M. and Young, G. (2003). A Merton-model Approach to Assessing the Default Risk of UK Public Companies. Working Paper No. 194, Bank of England.

Vassalou, M. and Xing, Y. (2004). Default Risk in Equity Returns. *Journal of Finance*, 59 (2), 831–868.

Wong, H. and Choi, T. (2004). The Impact of Default Barrier on the Market Value of Firm's Asset. Working Paper, Chinese University of Hong Kong.

Wooldridge, J. M., 1999. Distribution-free estimation of some nonlinear panel data models, *Journal of Econometrics*, Elsevier, vol. 90(1), pages 77-97.

Yu, F. (2005). Accounting Transparency and the Term Structure of Credit Spreads. *Journal of Financial Economics*, 75 (1), 53-84.

*Table 2.I: Data Description*

Panel A: Sample Selection

| | Non Default sample | | Default sample | |
|---|---|---|---|---|
| | # of observations | # of firms | # of observations | # of firms |
| Total non-financial firms | 9,436 | 882 | 436 | 130 |
| After merging with market data | 6,782 | 820 | 150 | 77 |
| Less than 18 months interval | 6,782 | 820 | 150 | 60 |
| With accounting data for the probit model(1) | 5,606 | 703 | 138 | 59(2) |

(1) The annual observations from the default sample prior to the year of default (79; 138-59 observations) were added to the non default sample for the analysis. The total number of firm-year observations is 5,744 for 762 individual firms.

(2) Unusable firm observations proportion for non default firms is due to a lack of data is 20%, while it is 55% for defaulting firms. This is due to more defaulting firms without available stock prices but also more defaulting firms do not produce balance sheets 18 months before default date.

Panel B: Descriptive Statistics for the Firms Retained for the Analysis (in millions of Canadian dollars)

| Statistic | Market capitalisation |
|---|---|
| Mean | 820.68 |
| Median | 61.28 |
| Mode | 1.76 |
| Standard deviation | 4459.97 |
| Skewness | 34.33 |
| Kurtosis | 1,760.87 |
| Range | 25,432.00 |
| Interquartile range | 321.65 |
| Number of daily observations | 1,885,707 |
| Number of firms | 762 |

*Table 2.II: Average Default Probabilities and Structural Model Parameters*

Panel A: Average Default Probability of for All Firms, Computed One Year Prior to Risk Exposure Using Merton-KMV Model[19]

| Year | Probability of default for firms that did not default | Probability of default for firms that did default |
|---|---|---|
| 1988 | 11.38% | 62.09% |
| 1989 | 12.83% | 63.84% |
| 1990 | 23.77% | 47.77% |
| 1991 | 15.11% | 41.90% |
| 1992 | 15.28% | 59.60% |
| 1993 | 5.84% | 83.77% |
| 1994 | 11.42% | 23.80% |
| 1995 | 9.07% | 64.54% |
| 1996 | 6.58% | 51.96% |
| 1997 | 12.48% | 60.91% |
| 1998 | 19.67% | 68.07% |
| 1999 | 13.68% | 65.76% |
| 2000 | 20.96% | 55.78% |
| 2001 | 15.66% | 46.89% |
| 2002 | 14.50% | 32.28% |
| 2003 | 6.31% | 19.65% |
| 2004 | 10.16% | - |
| Mean | 13.22% | 53.04% |
| Number of firms | 703 | 59 |

Panel B: Average Structural Models Parameters

| Model | Estimated parameter | | | |
|---|---|---|---|---|
| | $\mu_A$ | $\sigma_A$ | Barrier/Va | $\overline{\text{PD}}$ |
| Merton-KMV | 0.25 | 0.81 | - | 0.13 |
| Barrier Model (Wong and Choi) | 0.22 | 0.53 | 0.29 | 0.11 |
| Barrier Model (Duan et al.) | 0.15 | 0.53 | 0.30 | 0.14 |

[19] The presented numbers are the equally-weighted average of default probabilities.

*Table 2.III: Descriptive Statistics and Correlation of Explanatory Variables*

Panel A: Descriptive Statistics for the Explanatory Variables

| Variable | Mean | Median | Standard deviation | Minimum | Maximum |
|---|---|---|---|---|---|
| Merton –KMV PD | 0.13 | 0.01 | 0.22 | 0 | 1 |
| Profit < 0%(1) | 0.28 | — | 0.45 | 0 | 1 |
| Profit 0-6%(2) | 0.11 | — | 0.31 | 0 | 1 |
| Debt/Asset | 0.52 | 0.47 | 1.2 | 0 | 78 |
| Net value/total liabilities | 9.54 | 1.04 | 42.49 | -0.98 | 1,214 |
| Real GDP growth | 0.03 | 0.03 | 0.02 | -0.02 | 0.06 |

The accounting variables are in millions of Canadian dollars.

(1) A dummy variable assuming the value 1 if the margin of profit (EBITDA/sales) is negative, 0 otherwise.

(2) A dummy variable assuming the value 1 if the margin of profit (EBITDA/sales) is between 0 and 6%, 0 otherwise. The reference category is more than 6% profit.

Panel B: Pearson Correlation Matrix of the Dependent and Independent Variables

| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 1 | Default | 1 | | | | | | |
| 2 | Merton –KMV PD | **0.17** | 1 | | | | | |
| 3 | Profit < 0% | **0.55** | **0.16** | 1 | | | | |
| 4 | Profit 0-6% | **0.29** | **0.05** | -0.01 | 1 | | | |
| 5 | Net value/liabilities | -0.02 | **-0.08** | -0.03 | -0.01 | 1 | | |
| 6 | Debt/asset | **0.16** | **0.15** | **0.08** | 0.01 | -0.07 | 1 | |
| 7 | Real GDP growth | -0.03 | **0.05** | -0.02 | 0.02 | 0 | -0.03 | 1 |

Numbers in bold are significant at the 1% level.

*Table 2.IV: Probit Panel Estimates of the Bankruptcy Models*

The dependent variable is the dummy variable which takes 1 if the firm defaults and 0 otherwise. Profitability < 0% is a dummy variable which takes 1 if the operating margin (EBITDA/Sales) is negative and 0 otherwise. 0% ≤ Profitability < 6% is a dummy variable which takes 1 if the operating margin ranges between 0% and 6%. The *p*-values are shown in parentheses below the coefficient. *, **, *** significant at the threshold of 10%, 5% and 1% respectively. AUC = Area Under Curve (ROC Curve), AIC = Akaike Information Criteria.

| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Constant | -3.18*** | -2.49*** | -3.03*** |
| | (0.00) | (0.00) | (0.00) |
| Merton–KMV PD | 2.01*** | | 1.54*** |
| | (0.00) | | (0.00) |
| Profit < 0% | | 2.75*** | 1.55*** |
| | | (0.00) | (0.00) |
| Profit 0-6% | | 3.25*** | 3.13*** |
| | | (0.00) | (0.00) |
| Net value/ total liabilities | | -0.25** | -0.18 |
| | | (0.04) | (0.13) |
| Debt/ asset | | 0.04** | 0.03* |
| | | (0.04) | (0.08) |
| Real GDP growth | | -7.92* | -9.03** |
| | | (0.06) | (0.05) |
| $\rho$ | 0.28*** | 0 | 0 |
| | (0.00) | (>0.99) | (>0.99) |
| -2 Log Likelihood | 544.86 | 279.6 | 255.1 |
| AIC (smaller is better) | 562.1 | 293.6 | 271.2 |
| AUC | 0.815 | 0. 933 | 0.977 |
| N | 5,744 | 5,744 | 5,744 |

*Table 2.V: Performance in Predicting Defaults*

Type I and II errors when the threshold is the 9th decile of predicted probabilities. The firm/year observations are classified according to their predicted probabilities generated for each model. We then examine how many defaults belong to the highest decile for each model. The percentages represent the proportion of correct and incorrect classification for defaulted and surviving firms.

| Model prediction | Actual defaults | Actual non-defaults |
|---|---|---|
| Hybrid model (Model 3) | | |
| Defaults | 57<br>96.61% | 517<br>9.09% |
| Non-defaults | 2<br>3.39% | 5,168<br>90.91% |
| Total | 59 | 5,685 |
| Accounting model (Model 2) | | |
| Defaults | 51<br>86.44% | 523<br>9.2% |
| Non-defaults | 8<br>13.56% | 5,162<br>90.8% |
| Total | 59 | 5,685 |
| Structural model only (Model 1) | | |
| Defaults | 36<br>61.02% | 538<br>9.46% |
| Non-defaults | 23<br>38.98% | 5,147<br>90.53% |
| Total | 59 | 5,685 |

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*Table 2.VI: Probit Panel Regression of the Default Probability by Sub-period*

The dependent variable is the dummy variable which takes 1 if the firm defaults and 0 otherwise. Profit < 0% is a dummy variable which takes 1 if the operating margin (EBITDA/Sales) is negative and 0 otherwise. Profit 0-6% is a dummy variable which takes 1 if the operating margin ranges between 0% and 6%. The *p*-values are shown in parentheses below the coefficient. *, **, *** significant at the threshold of 10%, 5% and 1% respectively.

| | Period I: 1988-1995 | | | Period II: 1996-2004 | | |
|---|---|---|---|---|---|---|
| | Model 1 | Model 2 | Model 3 | Model 1 | Model 2 | Model 3 |
| Constant | -2.66*** | -2.76*** | -3.24*** | -3.83*** | -2.23*** | -2.76*** |
| | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| Merton –KMV PD | 1.96*** | | 1.49*** | 2.09*** | | 1.80*** |
| | (0.00) | | (0.00) | (0.00) | | (0.00) |
| Profit < 0% | | 3.11*** | 2.86*** | | 2.71*** | 2.95*** |
| | | (0.00) | (0.00) | | (0.00) | (0.00) |
| Profit 0-6% | | 3.76*** | 3.37*** | | 3.21*** | 3.48*** |
| | | (0.00) | (0.00) | | (0.00) | (0.00) |
| Net value/ total liabilities | | -0.16 | -0.14 | | -0.29 | -0.21 |
| | | (0.40) | (0.49) | | (0.12) | (0.27) |
| Debt/ asset | | 0.04** | 0.03* | | 0.02 | -0.04 |
| | | (0.03) | (0.07) | | (0.91) | (0.80) |
| Real GDP growth | | 4.53 | 6.38 | | -19.42** | -25.54*** |
| | | (0.51) | (0.88) | | (0.03) | (0.01) |
| $\rho$ | 0.96*** | 0 | 0 | 0.97*** | 0 | 0 |
| | (0.01) | (1.00) | (1.00) | (0.01) | (1.00) | (1.00) |
| -2 Log Likelihood | 188.7 | 104.8 | 96.1 | 324.7 | 165.8 | 149.6 |
| AIC (smaller is better) | 194.7 | 118.8 | 112.1 | 330.7 | 179.8 | 165.6 |
| Area Under ROC curve | 0.881 | 0.928 | 0.956 | 0.785 | 0.915 | 0.987 |
| Number of observations used | 1,441 | 1,441 | 1,441 | 4,303 | 4,303 | 4,303 |
| Number of defaults | 27 | 27 | 27 | 32 | 32 | 32 |

*Table 2.VII: Default Prediction*

Type I and II errors when the threshold is the 9th decile of predicted probabilities

Panel A: Performance in Predicting Defaults by Sub-period (In-sample)

| | Model prediction | Period I: 1988-1995 | | Period II: 1996-2004 | |
|---|---|---|---|---|---|
| | | Actual defaults | Actual non-defaults | Actual defaults | Actual non-defaults |
| Model 3 (Hybrid) | Defaults | 25<br>92.59% | 119<br>8.42% | 32<br>100% | 398<br>9.32% |
| | Non-defaults | 2<br>7.41% | 1,295<br>91.58% | 0<br>0% | 3,873<br>90.68% |
| | Total | 27 | 1,414 | 32 | 4,271 |
| Model 2 (Statistical) | Defaults | 23<br>85.19% | 121<br>8.56% | 28<br>87.5% | 402<br>9.41% |
| | Non-defaults | 4<br>14.81% | 1,293<br>91.44% | 4<br>12.5% | 3,869<br>90.59% |
| | Total | 27 | 1,414 | 32 | 4,271 |
| Model 1 (Structural) | Defaults | 16<br>59.26% | 128<br>9.05% | 19<br>59.38% | 411<br>9.62% |
| | Non-defaults | 11<br>40.74% | 1,286<br>90.95% | 13<br>40.62% | 3,860<br>90.38% |
| | Total | 27 | 1,414 | 32 | 4,271 |

Panel B: Out of Sample Forecasts

| | Estimation period: 1988-1995 | | | Evaluation period: 1996-2004 | | |
|---|---|---|---|---|---|---|
| | Model 1 | Model 2 | Model 3 | Model 1 | Model 2 | Model 3 |
| # of observations | 1,441 | 1,441 | 1,441 | 4,303 | 4,303 | 4,303 |
| # of defaults | 27 | 27 | 27 | 32 | 32 | 32 |
| -2 Log likelihood | 188.7 | 104.8 | 96.1 | 331 | 180.4 | 187.7 |
| AUC | 0.881 | 0.928 | 0.956 | 0.788 | 0.902 | 0.959 |
| AIC (smaller is better) | 194.7 | 118.8 | 112.1 | 335 | 184.4 | 191.7 |
| Type I error | 40.74% | 14.81% | 7.41% | 40.6% | 18.75% | 0% |
| Type II error | 9.05% | 8.56% | 8.42% | 9.62% | 9.46% | 9.32% |

*Table 2.VIII: Probit Panel Regression of the Default Probability by Industry*

The dependent variable is the dummy variable which takes 1 if the firm defaults and 0 otherwise. Profitability < 0% is a dummy variable which takes 1 if the operating margin (EBITDA/Sales) is negative and 0 otherwise. 0% ≤ Profitability < 6% is a dummy variable which takes 1 if the operating margin ranges between 0% and 6%. The *p*-values are shown in parentheses below the coefficient. *, **, *** significant at the threshold of 10%, 5% and 1% respectively.

We did the same analysis with the mining industry (2 digits SIC codes [10-14]). The results are about the same. The coefficients of the PD variable are 3.14*** (Model 1) and 3.26*** (Model 3). Details are available from the authors.

| | Manufacturing | | | Other industries | | |
|---|---|---|---|---|---|---|
| | Model 1 | Model 2 | Model 3 | Model 1 | Model 2 | Model 3 |
| Constant | -4.99*** | -2.89*** | -3.32*** | -5.51*** | -2.30*** | -2.77*** |
| | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| Merton – KMV PD | 2.64*** | | 1.97*** | 3.01*** | | 1.52*** |
| | (0.00) | | (0.01) | (0.00) | | (0.00) |
| Profit < 0% | | 3.96*** | 3.97*** | | 2.54*** | 2.43*** |
| | | (0.00) | (0.00) | | (0.00) | (0.00) |
| Profit 0-6% | | 4.14*** | 4.45*** | | 2.80*** | 2.83*** |
| | | (0.00) | (0.00) | | (0.00) | (0.00) |
| Net value/ total liabilities | | -0.65 | -0.29 | | -0.36** | -0.27* |
| | | (0.18) | (0.45) | | (0.02) | (0.06) |
| Debt/ asset | | -0.55 | -0.68 | | -0.04* | 0.02 |
| | | (0.20) | (0.20) | | (0.07) | (0.19) |
| Real GDP growth | | -15.48 | -22.05 | | -5.18 | -6.50 |
| | | (0.26) | (0.13) | | (0.31) | (0.23) |
| $\rho$ | 0.76 | 0 | 0 | 0.85** | 0 | 0 |
| | (0.14) | (1.00) | (1.00) | (0.04) | (1.00) | (1.00) |
| -2 Log Likelihood | 182.7 | 72.2 | 57.8 | 334.7 | 198.7 | 182.6 |
| AIC (smaller is better) | 188.7 | 86.2 | 73.8 | 340.7 | 212.7 | 198.6 |
| AUC | 0.819 | 0.969 | 0.996 | 0.815 | 0.928 | 0.970 |
| Number of observations used | 2,415 | 2,415 | 2,415 | 3,329 | 3,329 | 3,329 |
| Number of defaults | 19 | 19 | 19 | 40 | 40 | 40 |

*Table 2.IX: Models' Performance by Industry*

| | | Manufacturing | | Other Industries | |
|---|---|---|---|---|---|
| | Model prediction | Actual defaults | Actual non-defaults | Actual defaults | Actual non-defaults |
| Model 3 (Hybrid) | Defaults | 19<br>100% | 222<br>9.26% | 38<br>95% | 294<br>8.94% |
| | Non-defaults | 0<br>0% | 2,174<br>90.74% | 2<br>5% | 2,995<br>91.06% |
| | Total | 19 | 2,396 | 40 | 3,289 |
| Model 2 (Accounting) | Defaults | 18<br>94.74% | 223<br>9.31% | 33<br>82.5% | 299<br>9.09% |
| | Non-defaults | 1<br>5.26% | 2,173<br>90.69% | 7<br>17.5% | 2,990<br>90.91% |
| | Total | 19 | 2,396 | 40 | 3,289 |
| Model 1 (Structural) | Defaults | 11<br>57.89% | 230<br>9.60% | 25<br>62.5% | 307<br>9.33% |
| | Non-defaults | 8<br>42.11% | 2,166<br>90.40% | 15<br>37.5% | 2,982<br>90.67% |
| | Total | 19 | 2,396 | 40 | 3,289 |

Type I and II errors when the threshold is the 9$^{th}$ decile of predicted probabilities.

*Table 2.X: Models' Performance Market Capitalization and Book-to-Market*

Panel A: By market capitalization

| | Small | | | Big | | |
|---|---|---|---|---|---|---|
| | Model 1 | Model 2 | Model 3 | Model 1 | Model 2 | Model 3 |
| # of observations | 2,872 | 2,872 | 2,872 | 2,872 | 2,872 | 2,872 |
| # of defaults | 45 | 45 | 45 | 14 | 14 | 14 |
| Merton –KMV PD | 2.93*** (0.00) | - | 1.42*** (0.00) | 2.59*** (0.00) | - | 1.53** (0.03) |
| AUC | 0.807 | 0.952 | 0.970 | 0.762 | 0.917 | 0.992 |
| Type I error | 40% | 11.11% | 8.89% | 50% | 14.29% | 0% |
| Type II error | 9.19% | 8.74% | 8.70% | 9.80% | 9.62% | 9.55% |

Panel B: By Book-to-Market

| | Low | | | High | | |
|---|---|---|---|---|---|---|
| # of observations | 2,872 | 2,872 | 2,872 | 2,872 | 2,872 | 2,872 |
| # of defaults | 38 | 38 | 38 | 21 | 21 | 21 |
| Merton –KMV PD | 2.615*** (0.00) | - | 1.49*** (0.01) | 4.79*** (0.01) | - | 1.28*** (0.01) |
| AUC | 0.814 | 0.953 | 0.989 | 0.871 | 0.9 | 0.934 |
| Type I error | 44.74% | 7.89% | 2.63% | 28.57% | 14.29% | 9.52% |
| Type II error | 9.39% | 8.89% | 8.82% | 9.54% | 9.43% | 9.4% |

Type I and II errors when the threshold is the 9$^{th}$ decile of predicted probabilities. The accounting and macroeconomic parameters are not reported.

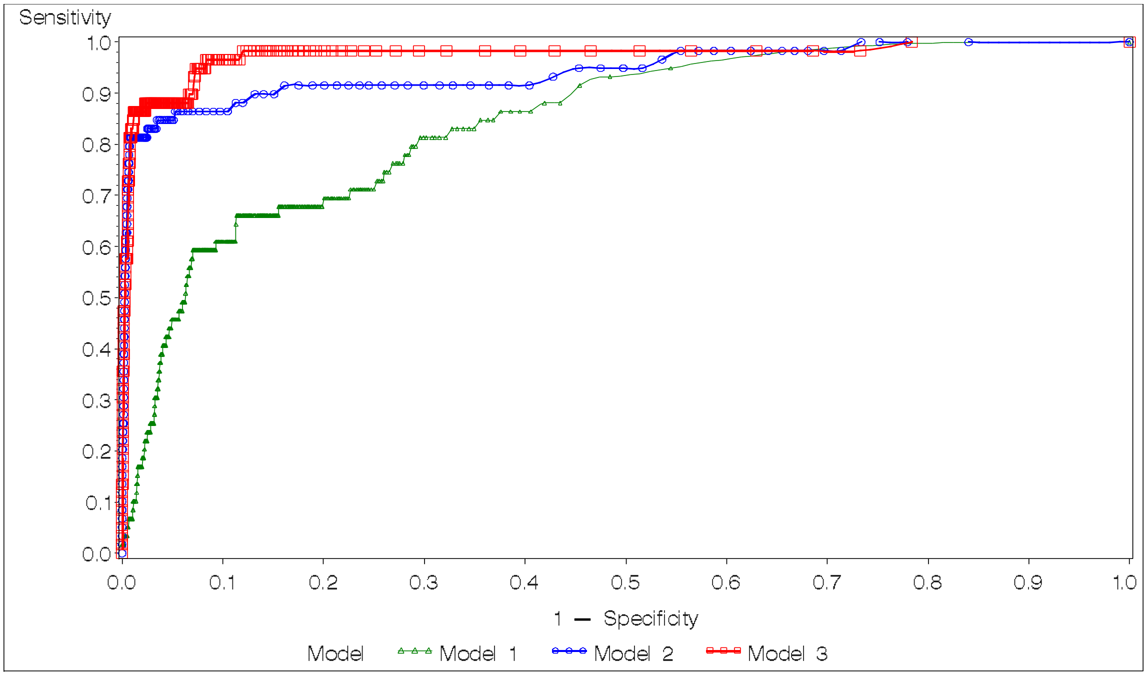
*Table2.XI: Jackknifed Probit Estimates of the Bankruptcy Models*

The reported estimates are the average coefficients when a default is withdrawn in each regression. The dependent variable is the dummy variable which takes 1 if the firm defaults and 0 otherwise. Profitability < 0% is a dummy variable which takes 1 if the operating margin (EBITDA/Sales) is negative and 0 otherwise. 0% ≤ Profitability < 6% is a dummy variable which takes 1 if the operating margin ranges between 0% and 6%. The *p*-values are shown in parentheses below the coefficient. *, **, *** significant at the threshold of 10%, 5% and 1% respectively. AUC = Area Under Curve (ROC Curve), AIC = Akaike Information Criteria.

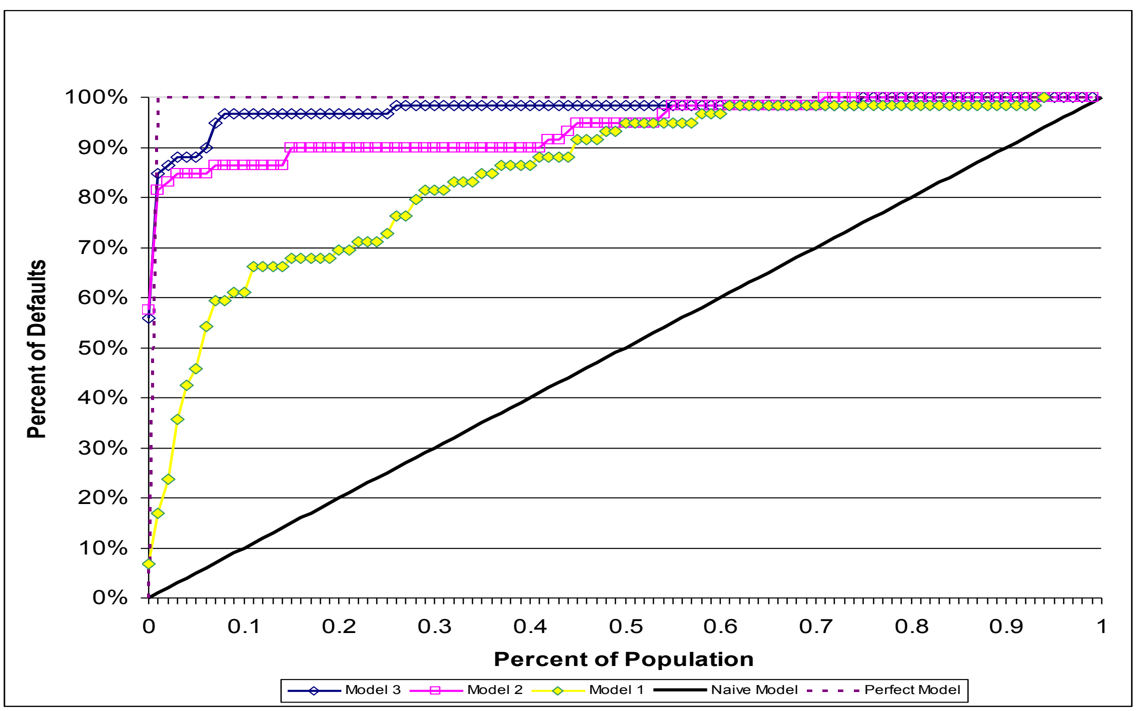
| Variable | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Constant | -2.79*** | -2.44*** | -2.84*** |
| | (0.00) | (0.00) | (0.00) |
| Merton–KMV PD | 1.72*** | | 1.34*** |
| | (0.00) | | (0.00) |
| Profit < 0% | | 2.64*** | 2.46*** |
| | | (0.00) | (0.00) |
| Profit 0-6% | | 2.89*** | 2.85*** |
| | | (0.00) | (0.00) |
| Net value/ total liabilities | | -0.22** | -0.16 |
| | | (0.04) | (0.12) |
| Debt/ asset | | 0.04** | 0.03* |
| | | (0.04) | (0.06) |
| Real GDP growth | | -7.93* | -9.03** |
| | | (0.06) | (0.03) |
| -2 Log Likelihood | 554.96 | 277.88 | 253.1 |
| AIC (smaller is better) | 558.96 | 289.88 | 267.2 |
| AUC | 0.814 | 0. 933 | 0.977 |
| N | 5,744 | 5,744 | 5,744 |

*Figure 2.1: CAP and ROC Curves*

Panel A: ROC Curves with Probit Panel Estimation*




Panel B: CAP Curves with Probit Panel Estimation



*For PD cut points (default/non-default model classification) varying from 0 to 1, each firm-year observation is classified as default (positive) if the model generated PD is above the cut point, and non defaulted otherwise. The outcomes of the model are classified as follow:

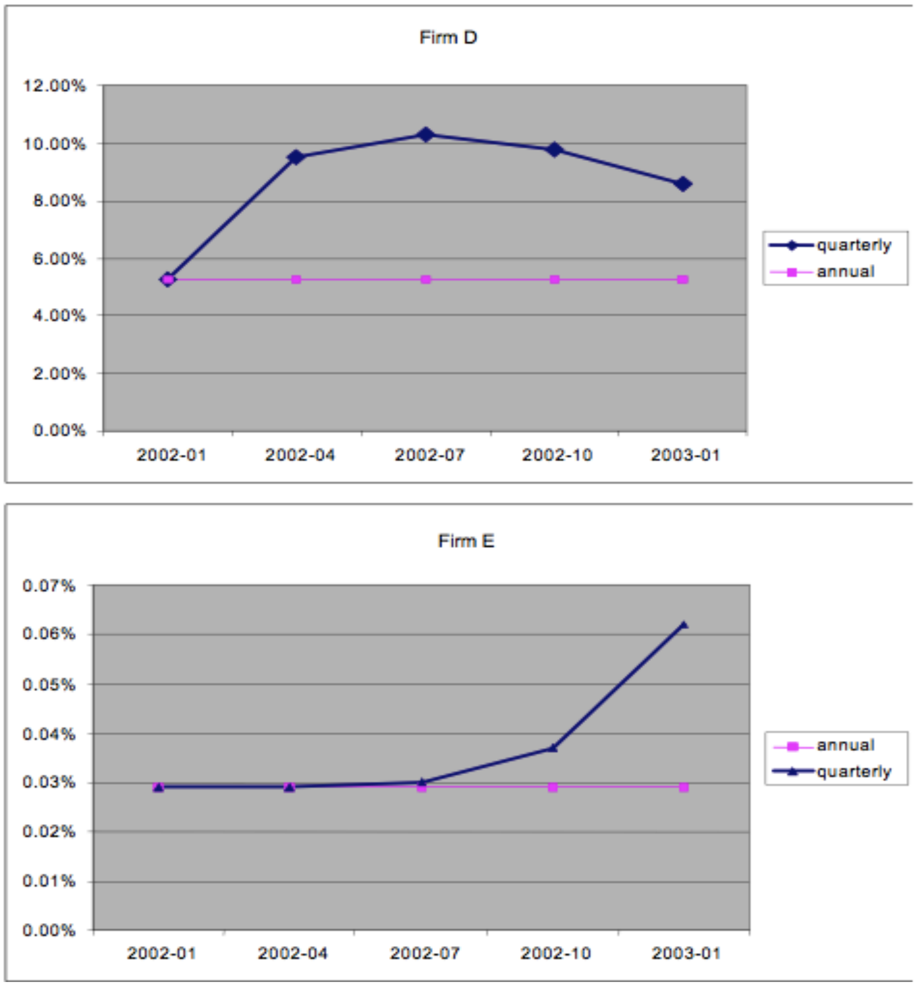
| Model outcome | Actual Condition | | |
|---|---|---|---|
| | Positive (Default) | Negative (Non-default) | Total |
| Positive (Default) | TP (True Positive) | FP (False Positive) | TP+FP (Total model's positive prediction) |
| Negative (Non-default) | FN (False Negative) | TN (True Negative) | FN+TN (Total model's negative prediction) |
| Total | TP+FN | FP+TN | TP+FN+ FP+TN |

Based on the above definition, the following measures are as follow:

Sensitivity = TP/(TP + FN) = (Number of true positive)/(Number of all positive)

Specificity = TN/(TN + FP) = (Number of true negative)/(Number of all negative)

*Figure 2.2: Quarterly Default Probabilities (1 year) of Non-defaulting Firms from Hybrid Model*



# Chapter 3

# On the Determinants of the Implied Default Barrier

## 3.1. Introduction

One of the most important assumptions in the structural models of credit risk is that the firm defaults when its value reaches a minimal threshold, which is often called the default barrier or the default boundary. All the structural models in the credit risk literature specify assumptions about the default barrier and calibrate the level of asset value below which the firm defaults. Most of the empirical tests of these models compare the credit risk premia generated by the structural models with those actually observed on the credit derivatives or debt contracts. Although these studies are numerous, little attention has been paid to the underlying assumptions regarding the level of the default barrier, which is the value of the assets below which the default is triggered. One exception is Davydenko (2007), who studies whether default is triggered by low market asset value or by liquidity shortages.

Structural models often rely on parameters that are not directly observable; one example is the default barrier where the dynamics and the location are not visible. Researchers must then specify the default barrier based on indirect information. While many structural models specify the analytic default barrier, the relation between the asset value at default and firm-specific characteristics was rarely explored empirically. Because a better understanding of the determinants of the default threshold could be valuable for modeling the decision to default and for default prediction, this paper seeks to identify the firm-specific factors and the macroeconomic variables that may influence the location of the default barrier.

Using a sample of public companies, we compare the default prediction obtained from a Merton model-based approach, where the default barrier is set as a given fraction of the firm's debt, with that generated by the Down-and-Out European Call option model (hereafter DOC option model) introduced by Brockman and Turtle (2003), where the firm defaults whenever its asset value reaches the estimated default barrier. Our results show that the estimated default barrier is significant on average for our sample of public firms. We also find that our implementation of the Merton-based approach and the DOC option model have similar in-sample fits in explaining default occurrence. However, the DOC option model provides superior out-of-sample forecasts of bankruptcies in our sample. For the subsample of the defaulted firms, the estimated asset values with the DOC option model are much closer to the model-implied default barrier than those of the surviving firms. On average, the estimated default barrier measures the value of the assets at the default time accurately. We focus on these estimates of the default threshold and perform a statistical analysis of the default barrier level on a set of firm characteristics. The results indicate that the DOC implied default barrier is affected not only by the level of leverage, but also by the liquidity of the firm and its debt cost, which underlines the importance of the liquidity shortage and external financing cost concerns. Further, the implied default barrier location is influenced by liquidation costs, renegotiation frictions, and equity holders' bargaining power, which supports the strategic default models.

The rest of the paper is organized as follows. Section 2 presents a brief literature review of structural models. Section 3 describes the methodology used to estimate the models' parameters and presents the data. Section 4 analyzes the results of the estimated barrier and compares the capacity to predict defaults of two models: the DOC option model and the Merton-KMV model. Section 5 discusses the choice of explanatory variables that affect the default barrier, together with the regression results. Section 6 analyses the simultaneous relation between default barrier and indebtedness. Section 7 concludes the paper.

## 3.2. Literature review

There are several structural models that propose default triggers. Most of them are first-passage-time approaches in the sense that they extend the seminal framework of Black and Scholes (1973) and Merton (1974) (hereafter BSM) by allowing default to occur whenever the value of a firm's assets crosses a pre-specified barrier, rather than only at the debt's maturity (Black and Cox, 1976). The default triggering barrier can be given either endogenously or exogenously. For the endogenous default trigger, the equity holders choose to default (or to reorganize) in order to maximize the value of their claims (e.g. Leland, 1994; Leland and Toft, 1996; Acharya and Carpenter, 2002). Exogenous default trigger models, in contrast, impose a pre-specified default barrier and extend the basic framework to include characteristics of bond markets, such as stochastic risk-free interest rates (Longstaff and Schwartz, 1995), stochastic default barrier (Hsu, Saá-Requejo, and Santa-Clara, 2002), and mean-reverting leverage (Collin-Dufresne and Goldstein, 2001).

The practical implementation of the structural models must specify assumptions regarding the level of the default barrier. Usually, in the exogenous structural models, the default barrier is expressed as a fraction of the face value of the debt (less than or equal to 1). It is therefore assumed in these models that the default barrier depends solely on the level of the face value of debt. For instance, Longstaff and Schwartz (1995) consider a default barrier that equals the total principal value of debt. Nonetheless, such a default barrier seems unrealistic because many firms continue to operate with a negative net worth. To deal with this concern, Huang and Huang (2003) suppose that the default barrier equals 60% of the face value of debt, while Leland (1994) calibrates the default barrier to match the observed recovery rates. This leads to a default barrier of 73% of the face value of the debt. Alternatively, the Merton-KMV model assumes that default occurs only at debt maturity and the default point is set to the short-term debt plus half of the long-term debt (Crosbie and Bohn, 2003; Vassalou and Xing, 2004).

The endogenous models pioneered by Black and Cox (1976), and extended by Leland and Toft (1996) and Acharya and Carpenter (2002) among others, offer a richer specification of the default barrier, because the equity holders/manager decide whether or not to default depending on the continuation value of the firm relative to current debt service payment. The default barrier corresponds to the cut-off point for the asset values below which it is more worthwhile for equity holders to default on the firm's debt. This setting makes the default barrier sensitive to other factors in addition to the principal value of the debt. For example, in Leland and Toft's (1996) model, the optimal barrier level decreases in debt maturity, asset volatility and the risk-free rate, whereas it increases in default costs and the book value of debt.

As mentioned by Davydenko (2007), this kind of model assumes the absence of either minimum cash-flow covenants or market frictions, which could limit the firm's ability to raise sufficient external financing. As a result, the firm will never fall into default for cash shortage reasons: If the firm faces a liquidity crisis, and the firm's value is above the default barrier, the equity holders will always be able to avoid default by raising new funds. Few models presented in the literature relax these assumptions (see Kim, Ramaswamy, and Sundaresan, 1993; Anderson and Sundaresan, 1996). Instead of setting the asset value as the default trigger, they assume that default occurs when the firm's cash-flow fails to meet the debt service payment. Given the unavailability or the limitations of external financing, the default becomes exogenous and happens only in the case of a cash crisis. Fan and Sundaresan (2000) combine the endogenous value-based and exogenous liquidity-based defaults by assuming an exogenous covenant on the minimum cash flow for the former, and costly external fund raising for the latter.

Another trend in the literature considers the possibility of debt contract renegotiation and deviation from the absolute priority rule, allowing strategic debt service. Indeed, Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) stipulate that in the presence of liquidation costs and the bargaining power of the equity holders, the firm creditors may accept a partial payment of the debt, which in turn may encourage opportunistic default by the equity holders. In addition, many firm-specific strategic factors were identified as having an effect on default and recovery decisions. Asquith, Gertner and Scharfstein (1994), Franks and Torous (1994), and Betker (1995) document that the complexity of debt structure, managerial share ownership, and asset tangibility influence the occurrence of the formal and informal reorganization and deviations from absolute rule. Therefore, we can expect the default barrier to depend both on strategic factors and on other firm-specific factors.

François and Morellec (2004) distinguish between default and liquidation of the firm, and explicitly account for the possibility of debt renegotiation under Chapter 11. In this setting, the firm is liquidated if its assets stay below the default barrier for a given period of time. Thus, the firm's equity is modeled as a Parisian down-and-out call option on the firm's assets. Moraux (2004) offers a rather different modeling by considering the cumulative time spent below the barrier (financial distress). Galai, Raviv and Wiener (2007) go further by considering not only the consecutive and the cumulative time spent in financial distress, but also the severity of this distress. Finally, Carey and Gordy (2007) develop a model where the default barrier is set mainly by private debtholders. They present evidence that the recovery rate increases sharply with the pre-bankruptcy share of private debt in all of the firm's debt[20].

To our knowledge, only Davydenko (2007) explicitly studies the value of assets at default, and investigates whether default is triggered by low asset values or by liquidity shortages. He uses a sample of low-grade US firms with observed market values of both debt and equity, which allows him to observe the asset value at default. He finds that the asset value at default varies largely in cross-section, and depends on balance sheet liquidity, asset volatility and tangibility. While on average a barrier of 72% of the face value of the debt correctly predicts the probability of default, the large cross-section variability regarding the default barrier of defaulted firms leads him to conclude that structural models based on a well-defined default trigger have a limited ability to predict defaults in cross-section.

In contrast with Davydenko (2007), we estimate the default barrier implied by Brockman and Turtle's (2003) model using a maximum likelihood estimation procedure for all the firms in our sample, and we do not limit our investigation to firms with directly observable asset value. We focus on the barrier level that is perceived by the market participants, because it is derived from the common equity price. Indeed, the default announcement could convey additional information about the defaulting firm's financial situation. Recovery rates may underestimate the asset value at which the firm

[20] Moody's reported recovery rates ( Cantor et. al., 2009 for instance) shows that bank loans average LGD are higher than bonds' LGD.

defaults, due to the presence of bankruptcy costs and departures from the absolute priority rule. In other words, we focus on the set of information prior to default, namely the equity market prices.

## 3.3. Estimation of the implied default barrier

### 3.3.1. Estimation method

The Brockman and Turtle (2003) model is an extension of the basic BSM framework, where the firm's equities are viewed as a down-and-out call option and default is triggered when the value of the assets crosses the barrier level such that bondholders are able to receive the remaining value of the firm before it deteriorates further. In this setup, the default barrier can be seen as a debt covenant. One goal is to estimate the barrier level implied by the traded equity prices. The methodology relies on calibrating the barrier level such that the Down-and-Out European call option price formula matches the observed equity prices. To be able to do so, one needs to know the value of assets, the instantaneous drift and volatility of the assets' return, and the face value of debt. Because the actual value of the firm's assets is not observable, one can approximate the total value of the assets by summing the market value of equity and the book value of debt. Brockman and Turtle (2003) use the time series of the quarterly market value obtained over a ten-year period to estimate the historical asset volatility.

They find an average barrier level significantly higher than zero, and the barrier to assets ratio is 69.2%, much larger than the average leverage ratio of their sample, which is equal to 44.7%. This result holds for the different industry sectors and for the nine first leverage deciles. These findings are counter-intuitive because many firms continue to operate with negative net worth, and it seems unrealistic that debt holders could get back more than their debt.

Wong and Choi (2009) point out this discrepancy and show that, in the down-and-out call framework, approximating the asset value by the market capitalization plus the book value of debt leads to a biased implied default barrier that is larger than the book value of corporate liabilities, regardless of the empirical data used. This underscores the necessity of using an alternative estimation procedure to measure the firm's asset value rather than a proxy.

The literature provides several ways of calibrating the firm's asset value, Vt, and the standard-deviation of the asset volatility, $\sigma_V$. In the framework of BSM, the first method, which is referred to here as the variance restriction method, makes use of Ito's lemma to obtain a system of two equations linking the unknown asset values and the asset volatility to the observed equity values and volatility (Jones, Mason and Rosenfeld, 1984; Ronn and Verma, 1986). However, several drawbacks of this method were identified. Indeed, Crosbie and Bohn (2003) point out that the equation relating the equity volatility to asset volatility holds only instantaneously. Furthermore, Duan (1994) criticizes the implicit assumption in the variance restriction method of constant equity volatility and its independence from the corporate asset value and time. He also points out the lack of statistical inference for the estimates of Vt and $\sigma_V$ with the variance restriction method.

Duan (1994) developed a transformed data maximum likelihood estimation method in order to estimate Vt and $\sigma_V$ from equity prices, which views the observed equity times series as a transformed data set where the theoretical equity pricing formula is used as a transformation. We will revisit this estimation method in greater detail below. In addition to the statistical inference provided by the maximum likelihood estimation, Ericsson and Reneby (2005) compare the three described estimation methods, and find that the transformed data maximum likelihood estimation method is superior. Wong and Choi (2004) also use this method in the down-and-out call option framework.

KMV developed an iterative method based on the variance restriction method, described in Crosbie and Bohn (2003). For the standard call approach, Duan, Gauthier and Simonato (2004) show that the KMV method estimates are identical to the maximum likelihood estimates for the Black-Scholes-Merton model. However, when more complex structural model involving unknown capital structure parameter is considered, such as in the DOC option model, the KMV method is unable to estimate the additional parameter involved, because only two equations are used. This contrasts with the transformed MLE method, which is able to estimate the capital structure parameter, namely the default barrier in the DOC option model. These features lead us to retain the MLE estimation as our preferred methodology for estimating the models' parameters.

### 3.3.2. Down-and-Out Call Option

As mentioned earlier, the DOC option model hinges on viewing the firm's equity as a DOC option on the firm's assets. We assume a geometric Brownian motion for the asset value, that is:

$$dlnV_t = (\mu - \sigma^2/2)dt + \sigma dW_t$$

where $V_t$ is the market value of the firm's assets at time t, $\sigma$ is the asset value volatility, $\mu$ is the expected return on assets and $W_t$ is a Wiener process. The down-and-out call option price is given by:

$$\begin{aligned} E_{DOC} &= VN(a) - Xe^{-rT}N(a-\sigma\sqrt{T}) - V(H/V)^{2\eta}N(b) + Xe^{-rT}(H/V)^{2\eta-2}N(b-\sigma\sqrt{T}) \\ &+ R(H/V)^{2\eta-1}N(c) + R(V/H)N(c-2\eta\sigma\sqrt{T}) \end{aligned} \tag{3.1}$$

where T is the time to maturity of the option, H is the default barrier, R is the rebate of the barrier option, that is the payment made to the equity holders if the value of the firm's assets breaches the barrier. *N(.)* is the cumulative distribution function for the standard normal distribution, and

$$a = \begin{cases} \dfrac{\ln(V/X) + (r+\sigma^2/2)T}{\sigma\sqrt{T}} & \text{if } X \ge H, \\ \dfrac{\ln(V/H) + (r+\sigma^2/2)T}{\sigma\sqrt{T}} & \text{if } X < H, \end{cases}$$

$$b = \begin{cases} \dfrac{\ln(H^2/VX) + (r+\sigma^2/2)T}{\sigma\sqrt{T}} & \text{if } X \ge H, \\ \dfrac{\ln(H/V) + (r+\sigma^2/2)T}{\sigma\sqrt{T}} & \text{if } X < H, \end{cases}$$

$$\frac{\ln(H/V) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \quad \text{and} \quad \eta = \frac{r}{\sigma^2} + \frac{1}{2}$$

where r is the risk-free rate and X is the nominal value of debt.

It is interesting to note that the DOC framework includes the standard call option framework as a special case. Indeed if we set the barrier H to zero in equation (3.1) we obtain the pricing formula of a European call option. Moreover, in our setting we assume that the Absolute Priority Rule holds, that is, the equity holders receive nothing if the firm defaults. Thus, the last two terms in equation (3.1) become null, and equation (3.1) is reduced to:

$$\begin{aligned} E_{DOC} &= VN(a) - Xe^{-rT}N(a - \sigma\sqrt{T}) - V(H/V)^{2\eta}N(b) + Xe^{-rT}(H/V)^{2\eta-2}N(b - \sigma\sqrt{T}) \\ &= \beta(V, \sigma, H) \end{aligned} \tag{3.2}$$

The function relating the equity price to the asset value, $\beta(V, \sigma, H)$, is invertible for any given asset volatility. Thus, we can invert it and express V as a function of $E_{DOC}$, $\sigma$ and H, that is $V^t = \beta^{-1}(E^t_{DOC}, \sigma, H)$.

We apply the Wong and Choi (2004) likelihood function in the DOC framework.[21] In the first specification, Vt is the asset value at time t, and $f(\ln V^t \mid \ln V^{t-1}; \mu, \sigma, H)$ is the conditional density function of $\ln V^t$, because the asset value should remain above the barrier between two successive observation dates (respectively t and t-1), the density function should account for this feature. The corresponding density function is given by:

$$f(\ln V^t \mid \ln V^{t-1}; \mu, \sigma, H) = \varphi(\ln(V^t / V^{t-1})) - e^{2\eta(\ln H - \ln V^{t-1})}\varphi(\ln(V^t V^{t-1}) - 2\ln H) \tag{3.3}$$

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[21] See Duan, Gauthier and Simonato (2004) for an alternative likelihood function to estimate the Brockman and Turtle model with the maximum likelihood method. See Gharghori et al. (2006) for a comparison of an option-based model with an accounting-based one.

where $\phi(x) = \frac{1}{\sigma\sqrt{2\pi\Delta t}}\exp\left(-\frac{(x-(\mu-\sigma^2/2)\Delta t)^2}{2\sigma^2\Delta t}\right)$ and $\Delta t$ is the time interval between two successive observation dates.

If the asset value was observable, the log-likelihood function would be:

$$L^V = \sum_{t=2}^{n} \ln f(\ln V^t \mid \ln V^{t-1}; \mu, \sigma, H).$$

However, because we do not observe Vt but rather $E_{DOC}^t$, we modify the model as follows:

$$\begin{aligned} L^E &= \sum_{t=2}^{T} \ln\left[ f(\ln \hat{V}^t \mid \ln \hat{V}^{t-1}; \mu, \sigma, H) \times \left(\frac{\partial \beta(V,\sigma)}{\partial \ln V}\right)^{-1}\right] \\ &= \sum_{t=2}^{T} \ln\left[ f(\ln \hat{V}^t \mid \ln \hat{V}^{t-1}; \mu, \sigma, H) \times \left(\hat{V}^t\,\frac{\partial \beta(V,\sigma)}{\partial V}\right)^{-1}\right] \end{aligned} \tag{3.4}$$

where $\hat{V}^t = \beta^{-1}(E_{DOC}^t, \sigma, H)$ and [22]

$$\begin{aligned} \frac{\partial \beta(V,\sigma)}{\partial V} &= N(a) + \frac{1}{\sigma\sqrt{T}} n(a)(1-\frac{X}{H}) + \frac{X}{V} e^{-rT} (\frac{H}{V})^{2\eta-2} N(b-\sigma\sqrt{T})(2-2\eta) \\ &\quad - (1-2\eta)(\frac{H}{V})^{2\eta} N(b) + \frac{1}{\sigma\sqrt{T}} n(b)(1-\frac{X}{H})(\frac{H}{V})^{2\eta} \qquad \text{if } H \geq X, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \beta(V,\sigma)}{\partial V} &= N(a) - (\frac{H}{V})^{2\eta} N(b)(1-2\eta) \\ &\quad + (2-2\eta)\frac{X}{V} e^{-rT} (\frac{H}{V})^{2\eta-2} N(b-\sigma\sqrt{T}) + \frac{1}{\sigma\sqrt{T}} n(b)(1-\frac{X}{H})(\frac{H}{V})^{2\eta} \qquad \text{if } H < X, \end{aligned}$$

We conduct simulations to check for the estimation's ability to retrieve the model parameters. We also estimate the Merton model, in order to compare the performance of

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[22] See Hao (2006).

the DOC option model in predicting default probabilities with the standard European call framework. In this setting the pricing equation becomes:

$$E_{SC} = VN(d) - Xe^{-rT}N(d - \sigma\sqrt{T}) \text{ where } d = \frac{\ln(V/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{3.5}$$

The corresponding transformed maximum log likelihood function as derived by Duan et al (2004) is given by:

$$L^E = -\frac{n}{2}\ln(2\pi\sigma^2\Delta t) - \frac{1}{2}\sum_{t=2}^{n}\left[\frac{\ln(\hat{V}^t/\hat{V}^{t-1}) - (\mu - \sigma^2/2)\Delta t}{\sigma\sqrt{\Delta t}}\right]^2 - \sum_{t=1}^{n}\ln\hat{V}^t - \sum_{k=1}^{n}\ln(\Phi(\hat{d})).$$

Here the $\hat{V}_t$ is obtained by inverting equation (3.5). Given its high non-linearity, the likelihood function in both cases is maximized using the Nelder-Mead Simplex Algorithm (Fminsearch in Matlab).

Once the models' parameters are estimated, the default probabilities are given by:

$$DP_{Barrier} = N\left(\frac{-(\ln(V_0/H) - (\mu - \sigma^2/2)\tau)}{\sigma\sqrt{\tau}}\right) + e^{-\frac{2(\mu-\sigma^2/2)\ln(V_0/H)}{\sigma^2}} N\left(\frac{-(\ln(V_0/H) + (\mu - \sigma^2/2)\tau)}{\sigma\sqrt{\tau}}\right)$$

for the DOC option model while for the standard call option this probability takes the following form:

$DP_{SC} = N(-d)$ where d is defined in (3.5).

### 3.3.3. Simulations

In order to assess the maximum likelihood function's ability to recover the asset drift, volatility and barrier level we use Monte-Carlo simulations. The performance of the estimation method is examined in this subsection. The procedure for the simulations is described below:

1. We begin by generating the time series of the asset value between the time 1 and $T$, where $\Delta t$ is the time interval between two successive observation dates. We refer to this time series by $\{V^0, V^1, .., V^t, ..., V^T\}$. Because the return on assets is assumed to be normal, the value of the assets follows a lognormal distribution. Thus, the value of $V^t$ is given by: $V^{t+1} = V^t \exp\left((\mu - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\varepsilon^t\right)$ where $\{\varepsilon^0, \varepsilon^1, .., \varepsilon^t, ..., \varepsilon^T\}$ is a sequence of independent and identically distributed standard normal random variables.

2. The second step is to compute the time series of the equity prices $\{E^0_{DOC}, E^1_{DOC}, .., E^t_{DOC}, ...E^T_{DOC}\}$ from the simulated asset values using equation (3.2).

3. Finally, we use the log-likelihood function given in equation (4) to estimate the asset drift ($\mu$), volatility ($\sigma$) and barrier level ($H$) from the obtained equity prices.

We conduct maximum likelihood estimation and compute the point estimates for each quantity. We repeat the simulation 1,000 times. We retain 200 daily observations of equity prices, that is $N$=200 and $\Delta t$ =1/250. We assume that the capital structure remains unchanged through the 200-day observation period, hence we do not take into account the survivorship consideration as did Duan, Gauthier, Simonato and Zaanoun (2003). Furthermore, we assume that the initial value of the assets is $V^0$=\$10,000 and the face value of the debt is $F$= \$6000. The true barrier level is set to $H$=5000, and the drift and volatility of the assets are set to $\mu = 0.1$ and $\sigma = 0.3$. The retained risk-free rate is $r$ = 5% and the maturity of the barrier option retained is $T$=20 years. To allow a better comparison with previous studies we express the barrier level as a fraction of the nominal value of total liabilities. That is:

$$H = \alpha F .$$

Note that there is no difference between the estimates of $H$ or $\alpha$. When the barrier level is nil, we obtain an $\alpha$ equal to zero. Moreover, a barrier level $H$ higher than the level of debt will lead to an $\alpha$ higher than 1, and when the barrier level is below the nominal debt, $\alpha$ will be less than 1. In this case, the true $\alpha$ is 0.833. Moreover, we assume that the debt of the firm is a zero coupon bond, and is rolled over for all the option maturity. We report the simulation results in Table 3.I.

<Insert Table 3.I here>

The average estimates of the three parameters are all close to their true value. To test whether the difference between the mean of the estimates and their corresponding true values are significant, we report in Table 3.I the *t-statistic* and the related *p-value* for each parameter. Despite the elevated standard deviation for the asset drift and the default barrier estimates, their means are not significantly different from the real value.

Bharath and Shumway (2004) find that the asset drift parameter is important in estimating the default probabilities. That is, in out-of-sample results, they find that distance to default (DD) computed with estimated continuously compounded return on assets, $\widehat{\mu}$, outperforms the DD measure where this parameter is set to the risk-free rate. Therefore, we decided to use real probabilities of default instead of risk-neutral ones.

### 3.3.4. Data

The study covers the public Canadian industrial firms listed in the Toronto Stock Exchange from January 1988 to December 2004. To be able to obtain default probabilities for the first year we needed market and accounting data on the previous year, because we needed to obtain one year of daily observations. Thus, in order to estimate the structural models we gathered data starting from January 1987.

Firms that went bankrupt or were in reorganization were identified using Financial Post Predecessors & Defunct, CanCorp Financials (Corporate Retriever), and Stock Guide. Between 1988 and 2004, 130 firms were identified as being in default: 112 were bankrupt and 18 were undergoing reorganization. After merging the accounting data with the daily market data, 77 firms remained in the intermediary database of defaults.

This attrition is mostly attributable to the fact that, for some firms, we had only incomplete market data and, for others, only one year of accounting data, rendering the data unusable for our study in both cases. In fact, application of the structural model requires at least 200 consecutive daily market prices coupled with available accounting data on the book value of debt for defaulted firms.

As in Vassalou and Xing (2004), we use the book value of debt[23] for the new fiscal year starting only four months after the end of the previous fiscal year. The goal is to ensure that we utilize only the data available to investors at the time of calculation. As a result, we needed at least two successive financial statements to obtain the 200 estimation observations required.

We examined the lags separating the default dates from the last financial statements of some defaulted firms in greater detail. Many firms do not publish financial statements during the years prior to their bankruptcy. We felt obliged to withdraw from the database defaults for which these lags exceeded 18 months. For the others, i.e., those that had defaulted between 12 and 18 months after their final financial statement, we moved the date of the default up to reconcile it with the last available year of financial statements. This filtering reduced the number of defaults retained in our sample to 60 companies.

The data on daily market capitalization of equities for both defaulting and surviving firms were obtained from DataStream. The accounting data for the non-default sample came from the Stock Guide database, while accounting data for the defaulted firms were gathered from various sources, including Stock Guide, CanCorp Financials, and the companies' financial statements from SEDAR. We end up with 4,916 observations (year-firm), representing 762 single firms, of which 56 are defaults. For further details on the data used, please refer to section 2.3. We should notice here that among these 56 defaults 11 are reorganizations.

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23 All the debt is considered here, with no regard to the maturity.

## 3.4. Analysis of the results

### 3.4.1. Estimated default barriers

For the estimation of both models considered here, we use a one-year window, which is equivalent to an average of 261 daily market value observations. Except for the market value of equities and the risk-free rate, which are the same for the two models, we should mention here some differences in the parameter choice, given that the two models have different definitions of variables. In the standard call option, the default point retained is the same as in Crosbie and Bohn (2002) and Vassalou and Xing (2004), namely the sum of current liability and 50% of the long-term debt. The time to maturity is adjusted in consequence, and is set to one year for the standard call option, because this amount of debt is supposed to mature one year later. The parameters of the DOC option model differ according to the underlying assumptions. Here the level of debt retained is the total liability because the option time to maturity is set to 20 years, which represents the life interval of the firm's equity. Brockman and Turtle use a 10-year interval. However, Reisz and Perlich (2004) try different time horizons between 5 and 20 years and find that this choice does not affect the default barrier. Moreover, Brockman and Turtle (2003) contend that varying the option maturity from 3 to 100 years has a minor effect on the barrier level estimates. In our case we assume a time to maturity of 20 years for the European DOC option.

<Insert Table 3.II here>

The barrier estimates are presented in Table 3.II. The estimated barrier to implied assets for the pooled sample in Panel B is around 29% and has a standard-deviation of 27%, which is significant at all the usual confidence levels. The median of this ratio is 25%. Thus, the first finding of this study is that the Canadian public firm's equities can be seen as a down-and-out call option on their assets, because, on average, the implied default barrier is not nil. A closer look at the barrier estimates shows that the percentage of observations with barriers greater than zero attains 77%. Moreover, the average leverage ratio in our sample is 54% of the book value of assets, as shown in Panel A of

Table 3.II. Compared with 29.38% for the implied barrier/estimated asset value, it seems that the default barrier is far below the face value of debt.

Our estimates of average barrier and median are in line with the results of Reisz and Perlich (2004), where the average barrier to implied asset value is 30.53% (median of 27.58%). This result contrasts with the average barrier of 69.2% found by Brockman and Turtle (2003). As mentioned earlier and as reported by Wong and Choi (2004) and Reisz and Perlich (2004), this discrepancy comes primarily from using the sum of the market value of equity plus the book value of debt as proxies for the market value of assets. This approximation overstates the default barrier estimate.

When we compare the ratio of the default barrier to the implied asset value between defaulting and surviving firms, we observe an obvious difference between the two sub-samples. Indeed, while for non-defaulting firms the median is as low as 25% of the asset value, the median for defaulting firms is 76% of the implied asset value, and the third quartile reaches 91%. Therefore, we can conclude that for defaulting firms the barrier level is much closer to the estimated asset value.

Regarding the asset drift estimate in Panel C of Table 3.II, we observe a difference between surviving and bankrupt firms. While the average asset drift for the former attains 5%, it drops to -18% for the defaulted group. This seems to corroborate Bharath and Shumway (2004), who found that the asset drift parameter contains valuable information about the default likelihood of a firm. This also confirms the use of physical probabilities instead of risk-neutral ones in order to compare the models' performance in predicting defaults. In Panels E and F of Table 3.II, we report the average implied barrier to market value estimates by year and by debt load respectively. This ratio varies substantially from year to year, going from a minimum of 19% in 1996 to a maximum of 43% in 1998. Further, the average barrier increases in debt load.

### 3.4.2. Comparison of models' capacity to predict defaults

In this section we compare the estimates from the DOC option model and the Merton-KMV model. We also compare these models' capacity to predict default occurrence in our sample of public Canadian firms. The aim here is to see whether the DOC option is able to predict the defaults more accurately than the Merton-KMV. The descriptive statistics of the estimated default probabilities with both models are reported in Table 3.III. The average default probability for non-defaulting firms obtained from the DOC option model is 10.88% versus 61.05% for defaulting firms, while for the Merton-KMV model those average probabilities are respectively 13.66% and 48.51%.

<Insert Table 3.III here>

In Table 3.IV we compare the predicted default probability, default barrier to asset value as well as firm characteristics such that the firm size, liquidity volatility and debt cost between liquidated and reorganized defaulted firms. The default barrier to estimated asset value is slightly higher for bankrupted firms, with 0.65 and 0.63 respectively, while the default probabilities is higher for reorganized firms which emerged from default as a going concern (69.01% compared to 73.59%). Moreover, the bankrupted firms have lower leverage, more liquidity, higher debt costs, higher assets volatility and are slightly larger than reorganized firms. However, none of the differences between these variables is significant at the 5%. Thus we cannot conclude that there is a difference in default barrier, default probability and firm's characteristics between bankrupted and reorganized firms. We notice here that the default barrier is estimated using market prices of at least one year prior to the date of default. The dissimilarity in default probabilities between defaulted and surviving firms is pronounced, but there are no noticeable differences between bankrupted and reorganized firms.

<Insert Table 3.IV here>

As a first comparison of the models' performance in predicting defaults, we report in Table 3.V the number and the percentage of default and non-default observations in each decile of default probabilities, where the defaults are grouped into deciles of the estimated default probabilities with the DOC option and Merton-KMV models, and the 10th decile represents the highest default probabilities. The advantage of this classification is that it is not affected by the calibration technique. That is, the overall level of the estimated default probabilities has no effect on the number of hits and false alarms in each decile. We notice that for the 10th decile the DOC option model captures 68% of the defaults while the Merton-KMV model predicts only 55% of the defaults in the whole sample. This difference in the number of hits in the 10th decile between the two models is not outweighed by a proportional increase in the number of misclassified observations for the DOC option model, because we observe the same number of observations in the 10th decile for the two models. The same figure appears when we consider the first quintile, where 75% of defaults are captured with the DOC option model compared with 66% for that of Merton-KMV. The DOC option default probability quintiles appear to classify default risk across firms much more effectively than those of Merton-KMV. This finding is in line with those of Reisz and Perlich (2004) and Hao (2006) where the DOC approach attains higher accuracy than the standard call approach.

<Insert Table 3.V here>

In order to better compare the ability of these models to forecast bankruptcy one year in advance, we perform two probit regressions where the dependent variable is a dummy equal to 1 if the firm went bankrupt in a given year, and 0 otherwise. The only independent variable is the estimated default probability using the DOC option model for the first regression, and the Merton-KMV default probability for the second. The fit and associated statistics for these regressions are reported in Table 3.VI.

<Insert Table 3.VI here>

The estimated coefficients when the whole period is used as the estimation period in Panel A, are 1.91 for the first model and 1.65 for the second model, respectively, and both are statistically significant at all the usual confidence levels, with p-values below 0.001. The maximum rescaled R square of Nagelkerke (1991) is a generalization of the coefficient of determination to a more general linear model. It shows that the default probabilities from the DOC option model have more explanatory power than their Merton-KMV counterpart in explaining default occurrence. However, the percentage of concordant observed values with the models’ predictions are similar (75.4 % for the DOC option model compared with 75.2% for that of KMV-Merton).

The Receiver Operating Characteristic curves (ROC) are used extensively for binary response models. Furthermore, the ROC curve accounts not only for Type I errors but also for Type II errors. Indeed, the ROC curve reports the percentage of 'hits', (i.e., defaults correctly classified) as a function of the 'false alarm' (i.e., non-defaults erroneously classified as defaults), for every cutoff point between 0 and 1. A better model would have a ROC curve closer to the upper left corner, while a perfectly random classification of observations would have the main diagonal as an ROC curve. We report the ROC curves for the two competing models in Figure 1. It appears from this figure that the DOC option model ROC curve dominates that of Merton-KMV for most of the cut-off points.

The area under the ROC curve summarizes the model's ranking ability, and ranges from 0.5 to 1, 1 being the best achievable value, corresponding to a perfect model. We retain the AUC measure as our primary statistic to compare the two models. Moreover, DeLong, DeLong and Clarke-Pearson (1988) offer a nonparametric test for the difference of AUC for correlated ROC curves. This statistic follows a Chi-square distribution with 1 degree of freedom.

We report the AUC in Table 3.VI, as well as the Chi-square statistic for the difference in the AUC. The DOC option model achieves a higher AUC than the KMV-Merton model, 0.86 versus 0.824. However, the statistical test of no difference between the two AUC is not rejected. Indeed, the Chi-square statistic attains 1.2, corresponding to a p-value of 0.273. Thus, we cannot conclude that the DOC option model is superior in explaining default incidence.

We now assess the ability of the two models to forecast defaults in out-of-sample estimations. For the out-of-sample validation, we split the full data set into a training sample, which is used to estimate the coefficient on the structural models' default probabilities in the probit model. These estimates are then used to compute the scores for the remaining unused data (i.e., out-of sample data). The out-of-sample validation allows evaluation of the ability of these scores to predict future defaults. Sobehart, Keenan and Stein (2000), suggest that quantitative models of credit risk should be developed and validated using an out-of-sample test, in order to avoid embedding undesirable sample dependency. In Panels B and C of Table 3.VI, we report the out-of sample estimation results. In Panel B, the training sample goes from 1988 to 1995, for a total of 1310 observations containing 23 defaults, while the out-of sample contains 3606 observations, of which 33 are defaults. The probit estimation on the first subsample shows a higher AUC for the Merton-KMV, 0.883 versus 0.845. However, the difference between these two performance measures is not significant at all the usual confidence levels. The opposite figure appears in the out-of-sample validation, the AUC measure in out-of-sample is in favor of the DOC option model. In fact, the Chi-square statistic value is 3.224, which is significant at the 10% confidence level. Thus, it seems that the DOC option model achieves better bankruptcy prediction in out-of-sample.

In Panel C of Table 3.VI, we check for the robustness of the previous result to the choice of the cutoff year for the separation of the training and the out-of-sample data. We perform another out-of-sample test. The training sample expands from 1988 to 1996, for a total of 1567 observations including 26 bankruptcies and reorganizations. The remaining sample contains 3349 firm-year observations, of which 30 are defaults. Here again, while the in-sample estimation shows no statistically significant difference in the AUC between the two models, the out-of-sample validation shows that the area under the curve for DOC option is significantly higher than that of the Merton-KMV. Indeed, the Chi-square statistic, testing for no difference between the areas under the two ROC curves, is 6.907 and rejects the null with a p-value below 1%.

Hence, even if the DOC option and Merton-KMV models are equivalently accurate in predicting bankruptcy occurrence in in-sample probit estimation, the former seems to achieve better predictive power in out-of-sample tests. We should notice here that the implemented Merton-KMV model here is not the model actually used to estimate the distance to default by the Moody's-KMV corporation. The proprietary model includes further steps and adjustments beside those described in Crosbie and Bohn (2003) and applied in this paper.

## 3.5. Barrier determinants

### 3.5.1. Independent variables

The theoretical financial literature identifies several firm-specific factors that can explain both the decision to default and the output of the reorganization process. These factors can therefore explain the barrier level. Roughly, we can group them in two broad categories: Strategic and non-strategic factors. In the following subsections we discuss these factors and justify the choice of the proxies.

#### 3.5.1.1. Non-strategic factors

Most of the exogenous default models, including the basic Merton (1974) and Longstaff and Schwartz (1995) models, specify the barrier level as a fraction of the debt. It is therefore natural that our first barrier determinant is the leverage of the firm. We expect a positive relation between the firm's leverage and our implied barrier measure. We measure the leverage as the ratio of total liabilities to book asset value.

In opposition to value-based models, where default depends on the value of the assets, we find cash-based models where the default is assumed to happen whenever the firm's cash flows are insufficient to cover its debt payments.[24] Cash-based models include Ross (2005), Anderson and Sundaresan (1996) and Kim, Ramaswamy, and Sundaresan (1993). These models assume that the firm has no access to external financing, implying that default can occur due to a cash shortage, even if the company has a positive net worth. However, this assumption is restrictive because external financing could be accessible at a given cost, depending on the financial soundness of the firm and its debt capacity. The existence of financing costs raises the issue of liquidity management. Indeed, firms may accumulate a cash cushion to avoid external financing during downturns. Asvanunt, et al. (2007), Acharya et al (2006) and Anderson and Carverhill (2007) account for liquidity management and financing costs. If a cash shortage can cause default, or at least accelerate default occurrence, we could expect an adjustment of

[24] Davydenko (2007) compares the value-based and cash-based models and their different assumptions

the implied barrier level to the cash holding of the firm. More cash in the firm's assets should be associated with a lower barrier to the estimated asset market value. We measure the liquidity as the ratio of cash and equivalents to the book value of assets. The same reasoning applies to external financial constraints, if the firm can contract new debt at low cost in order to avoid cash shortage defaults, its implied default barrier should account for this effect, and we should observe a lower barrier level when the debt costs are low. Hence, we expect a positive relation between the debt cost, measured by the ratio of interest expenses to total liabilities, and the implied barrier.

While credit risk models are scale free, we include the size, as measured by the logarithm of assets, to account for information availability. Indeed, Yu (2005) finds that accounting transparency is associated with lower credit spreads. Because large firms typically have lower information asymmetries than smaller firms, the latter may have higher uncertainty regarding the barrier location. We could reasonably hypothesize that market participants presume a higher barrier level when this uncertainty is greater, therefore we expect a negative relation between the firm's size and the default barrier.[25] Finally, we control for the asset volatility because it is related to firm risk. The firm risk is measured by the estimated asset volatility. We also control for the state of the economy as measured by the growth rate of the real GDP of the Canadian economy.

#### 3.5.1.2. Strategic factors

Strategic factors are specific to the endogenous default models. They fall into three categories: 1) Costs of liquidation, 2) Relative bargaining power and 3) Renegotiation friction. Betker (1995), Franks and Torous (1994), among others, find that these factors have an effect on the occurrence and the outcome of reorganizations. In addition, models put forth by Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Hart and Moore (1998) and Fan and Sundaresan (2000) allow for strategic defaults. In

[25] The size of the firm can be seen as a strategic factor. Indeed, large/mature firms are often associated with high bargaining power in debt renegotiation with debt holders, whereas small/young firms are considered weak firms in renegotiation. See for instance Hackbarth et al (2007). Moreover, Houston and James (1996), Johnson (1997), Krishnaswami et. al. (1999) and Denis and Mihov (2003) report evidence that the proportion of public debt in total debt increases with the size and the age of the firm, while Carey and Gordy (2007) observe a higher recovery rate for firms with more bank debt. This could be another way in which size affects the default barrier.

contrast with liquidity default, when the firm defaults due to insufficient cash flows, strategic default happens when equity holders decide to forgo debt payment even if they have the necessary funds. Indeed, if deadweight costs associated with liquidation of the firm's assets are high, it could be beneficial for the debt holders to concede some of their debt in order to allow the firm to survive. Therefore, equity holders may be interested in defaulting opportunistically to benefit from such debt cutback. As the likelihood of strategic default is higher when the liquidation costs and equity holders' bargaining powers are more pronounced, we can anticipate a positive relation between these variables and the level of default barrier. Moreover, Davydenko and Strebulaev (2007) find that the credit spread is positively sensitive to liquidation costs and bargaining power, and negatively related to renegotiation friction.

As a proxy for the liquidation costs we use the percentage of fixed assets. Fixed assets are measured by the total value of capital assets including land, buildings, computers, factories, office equipment, leasehold improvements, and assets under capital leases, net of accumulated depreciation and amortization. Therefore, fixed assets are the physical assets of the firm which are easiest to sell in case of liquidation. As a result we can expect a negative relation between the proportion of fixed assets and the default barrier. As an additional measure of the asset specificity we use the R&D expenditures to the book value of assets. Indeed the research expenditures could be a good proxy for the asset specificity of the firm. We anticipate a positive relationship between these asset specificity measures and the level of the default barrier. Indeed, more specific assets are generally harder to liquidate in case of bankruptcy and imply higher liquidation costs. To account for the equity holders' bargaining power we use the percentage of votes attached to the voting shares of a company held by the directors and other individuals or companies that own more than 10% of all voting rights. Here we choose to retain the percentage of votes instead of shareholding because we believe that it better reflects the control held by the manager and major equity holders over the firm's assets.

Finally, renegotiation frictions could prevent debt renegotiation, but also reduce recovery rates ex-post. Hart and Moore (1998) and Fan and Sundaresan (2000), argue that renegotiation frictions could prevent strategic defaults, but they also render the liquidation costs harder to avoid in liquidity default, and thus decrease recovery rates. Our proxy for renegotiation friction is the portion of current liabilities relative to total liabilities. Indeed, Berglöf and von Thadden (1994) point out that for financially distressed firms, short-term creditors rarely forgive debt, while concessions often are made by subordinated long-term claimholders. Thus, if the strategic default effect prevails, as higher short-term debt indicates more renegotiation friction, it could prevent strategic defaults and therefore lower the default barrier level. However, when liquidity default risk effect is more pronounced, short-term debtholders may prevent debt renegotiation and force bankruptcy with the associated liquidation costs, which have the potential to increase the ex ante default threshold. Therefore, the overall effect of the renegotiation friction is ambiguous, as measured by the current to total debt ratio on the default barrier location.

However, as short-term liabilities are paid first, they have de facto higher seniority relative to long-term unsecured debt. Davydenko and Strebulaev (2007) observe that a larger proportion of current debt relative to long-term debt increases the liquidity shortage risk, because more cash flows are used for day-to-day debt service. Hence, we conjecture that more short-term debt could push the default barrier upward. We use the ratio of short-term liabilities to total liabilities as a proxy of renegotiation frictions.

Because the short term debt proxy for renegotiation friction can be contaminated by liquidity default risk, we also use the proportion of the outstanding public debt to the book value of total debt as an alternative renegotiation friction proxy. In fact, Davydenko and Strebulaev (2007) find a negative relationship between credit spread and the proportion of public debt. Therefore, the public debt seems to have the potential to deter strategic default of equity holders, because it is more difficult for firms with multiple dispersed creditors to renegotiate their debt, as argued by Hege and Mella-Barral (2004), Gertner and Scharfstein (1991) and Berglöf and von Thadden (1994), among others. Moreover, Carey and Gordy (2007) allege that private debt holders (banks) endogenously set the asset value threshold below which firms declare bankruptcy, and find evidence of a strongly increasing recovery rate in the share of private debt. This could be another possible explanation for the negative relation between the proportion of public debt and the default barrier.

In order to compute the outstanding amount of public debt for companies in our sample, we start by scanning the new Canadian bond issues lists of FISD and the SDC Platinum databases to compile a list of companies in our dataset that are active on the bond market. For each identified issuer, we manually collect information on outstanding public debt for each fiscal year from the long-term debt section of printed Moody's/Mergent international manuals. Moreover, because the FISD and SDC Platinium databases are not exhaustive for Canadian issuers, we also look for the remaining Canadian firms in the Moody's/Mergent international manuals to check if they have public debt in their capital structure. We end up with 104 unique bond issuers out of 575 single firms in our dataset, for a total of 867 firm-year observations between 1988 and 2004. The remaining firms are assumed to have only private debt in their capital structure.

### 3.5.2. Regression analysis results

#### 3.5.2.1. Descriptive statistics

Our dependent variable is the ratio of the implied barrier estimated from market price to the estimated asset value. Here we choose to standardize the implied barrier by the estimated asset value instead of the book value of debt or the book value of assets because these accounting measures can diverge substantially from their corresponding market values. We believe that the estimated market prices give a better measure of the value of the assets under management.

For our regression analysis we drop observations with nil implied barrier estimates, which reduce our initial sample of 4,916 to 3,609 firm-year observations.[26] In fact, including the nil estimated barriers observation could blur the relationship between the

[26]We compared the characteristics, such as the size and leverage, of the whole sample (4,916 observations) with the restricted sample (3,609 observations). We find no significant difference. We also performed the regression analysis on the entire sample; the results found are similar to the restricted sample.

default barrier and firm characteristics. After dropping observations with insufficient data on independent variables we end up with 3,232 firm-year observations for 575 single firms, covering 17 years from 1988 to 2004. The average number of years by firm is hence 5.6 years.

<Insert Table 3.VII here >

Our default sample of 127 firm-year observations comes from our default database, and includes 50 observations of defaults or reorganizations. The descriptive statistics for both dependent and independent variables are reported in Table 3.VII. The average default barrier attains 40.1% of the estimated asset value, while the average leverage is 48.2% of the book value of assets. Thus on average the ratio of barrier to estimated assets is below the leverage ratio in our sample. For regression analysis we discuss in the following subsections the regression results for strategic and non-strategic factors.

#### 3.5.2.2. Non-strategic factors

The regression analysis results of the implied barrier on the non-strategic factors are presented in Table 3.VIII. The objective here is to test whether the implied default barrier estimated from equity price, viewed as a down-and-out call option, is adjusted by market participants to account for the possibility of cash shortages and impossibility of contracting new debts.

<Insert Table 3.VIII here>

Regression (1) in Table 3.VIII shows the result of regressions of the default barrier on the non-strategic factors. As expected, the leverage ratio is positively associated with the implied default barrier. The coefficients on the leverage ratio are positive and significant at the 1% level in all the regressions. The question that we seek to answer in this study is whether indebtedness is the only driver of the ex-ante perceived default barrier. Our regression results demonstrate clearly that this is not the case. The liquidity measure is negatively related to the implied barrier. These coefficients are also highly significant in all regressions. Thus, the implied default barrier associated with firms with more cash holdings accounts for the fact that they may eventually default at lower asset value, because they can handle debt payments and avoid default due to liquidity constraints. This result supports the underlying assumptions of cash-based models, such as Ross (2005), Anderson and Sundaresan (1996) and Kim, Ramaswamy, and Sundaresan (1993). The debt cost also has a significant positive impact on the location of the default threshold. Financing frictions seem to be a major determinant of the value at which the firm is expected to default. A firm's higher ability to contract new debt at a low cost decreases its default threshold. This result, in combination with the liquidity concern results, shows that credit risk models with endogenous cash management in the presence of external financing costs seem to better describe the reality of the firm, despite their complexity.

The negative relation of the ex-ante default barrier with volatility is concordant with endogenous barrier models like that of Leland and Toft (1996). Higher volatility makes the option to wait more valuable, and decreases the level of default barrier. The size of the firm has the expected sign. Larger firms benefit from the lower perceived ex ante default barrier level, probably for informational reasons. Large firms have greater visibility and are followed more closely by analysts. This helps reduce the uncertainty regarding the asset level below which firms default. Consequently, the default barrier for large firms is lower than that of smaller companies. Finally, the GDP growth rate is positively and significantly related to the default barrier. This result may seem surprising, because one could expect lower barrier levels in economic expansion. However, a possible explanation lies in the expectations of investors regarding future economic conditions. If firms set the default barrier in accordance with these expectations instead of with actual economic conditions, and given the cyclical aspect of the real GDP growth rate, a positive relationship between GDP growth and barrier level may result.

To check the robustness of our results, we perform a panel regression with random and fixed effect using the same specifications as in regressions (2) and (3) of Table 3.VIII. Our results are robust to the inclusion of both firm-specific constants and random error terms. Moreover, the R square of the model achieves 29.9% for the random effect regression and 26.8% with fixed effect panel regression.

We also test whether our results for the liquidity and debt cost are driven by the presence of outliers in our data. To detect the presence of such outliers we applied the method of Hadi (1992, 1994) for outlier detection in multivariate data to these variables. This leads to the exclusion of 62 observations. Columns (4) and (5) in Table 3.VIII report the random and fixed effect panel regressions results for the remaining observations. Here again our results are not altered either in terms of the coefficient estimates sign or their statistical significance. The debt cost becomes even significant at the 1% level in the fixed effect panel regression.

Moreover, a closer look at the liquidity variable shows that 78 observations have a ratio of cash and equivalents to total assets above 50%. Among them, only two observations come from our defaulted firms' data set. These observations may represent firms in asset liquidation process. To ensure that our results are not driven by observations where the assets are liquidated or reorganized, we drop them from the sample in regression (6), and find that our results are not driven by observations where firms undertake asset liquidation. As an additional check of the liquidity effect on the barrier level, we try another measure of asset liquidity, namely the current ratio. The current ratio is measured as current assets to current liabilities, and is a proxy for the ability of the firm to meet its short-term obligations with its short-term assets. In unreported results, this alternative measure of liquidity also has a negative and significant coefficient estimate, both in random and fixed effect panel regressions.

On the debt cost side, we observe that a fairly high proportion (around 14%) of firms in our sample do not use long-term debt, and rely solely on accounts payable to suppliers. Because suppliers generally allow a 90–day grace period for payment, these firms have zero or low interest expenses. We test whether the positive relation between debt cost and the location of the default barrier are driven by these observations. Regression (7) in Table 3.VIII shows the contrary. The coefficient on the debt cost variable is higher for the remaining sample and is significant at all the usual confidence levels.

The overall results show the importance of liquidity shortages and costs of external finance as drivers of the default barrier level location. It seems clear from the regression analysis that market participants adjust, through equity prices, the level of assets at which the firm is expected to default for liquidity shortage concerns and for the difficulty to raise new debt financing. This result holds, even after controlling for the leverage, asset volatility, firm size and economic conditions, and is robust to the presence of potential outliers.

#### 3.5.2.3. Strategic factors

We now turn to the results of the regression analysis of the implied default barrier on strategic factors. We report these results in Table 3.IX. All of the regressions include the non-strategic factors. We do not report them in Table 3.IX for the sake of brevity. All the non-strategic factors are significant and have the same signs as in Table 3.VIII. It should be noted here that regressions are restricted to observations where we were able to find data on the voting rights of the directors and major shareholders. The final sample contains 3085 observations for 509 single firms. Our estimates in Table 3.VIII are robust to the exclusion of firms without data on voting rights.

Liquidation costs are an important strategic factor in endogenous default models. Firms' creditors should be more willing to forgive a part of their debt when the asset values for going concerns are much higher than their liquidation value, and when the liquidation costs are high. This gives equity holders more incentives for strategic default in order to benefit from these debt concessions. If equity market participants are aware of such strategic default effects, the implied barrier should increase with default costs. Regression (1) of Table 3.IX supports this hypothesis. As expected, the coefficient on the fixed effect is negative and significant at the 1% confidence level in regression (1) of Table 3.IX. This liquidation cost effect is also supported by Davydenko and Strebulaev (2007), who find that the proportion of non-fixed assets is positively correlated with credit spread.

<Insert Table 3.IX here>

As an alternative liquidation cost proxy, we use the ratio of research and development expenses to asset book value. Regression (3) of Table 3.IX shows, however. that R&D is not relevant and does not have the expected sign, with a t-statistic of –0.33. We obtain the same figure after including random effects to account for the panel pattern in our data. Given that a large number of firms in our data set do not undertake R&D programs, in unreported results we used a dummy variable set to 1 if the R&D expenses are non-nil, and 0 otherwise. We found that the R&D dummy is positive as expected but it is not significant, with a *t-statistic* of 1.44. It seems from these results that the proportion of fixed assets better approximate liquidation costs in our data.

Regarding the renegotiation friction, we use two alternative proxies; the short-term debt to total debt ratio and the public debt to total debt ratio. Regressions (5) to (6) show that the short term debt coefficient is positive and statistically significant at all the usual confidence levels. This supports the higher liquidity default risk in the presence of more short-term debt rather than the strategic default explanation. Indeed, the effect of the liquidity risk seems to dominate the strategic default effect on the ex ante default barrier in our data. Further, short-term debt is a noisy measure of renegotiation friction because it is related to liquidity default risk.

In order to better isolate the effect of renegotiation friction on the estimated default barrier, we use the public debt variable in specifications (1) to (4) of Table 3.IX. In all four regressions, the estimated default barrier decreases significantly with the proportion of public debt in the firm's capital structure. The coefficient on the renegotiation friction proxy is negative and significant at the 1% level in these regressions. This result underlines the role of renegotiation frictions in discouraging potential opportunistic shareholders from defaulting despite the liquidation costs that renegotiation could avoid.

Finally, our proxy for the bargaining power of CEOs and major shareholders is positively and significantly related to the implied default barrier. This result indicates that more shareholder bargaining power implies a higher default barrier. This is consistent with strategic default effect: higher bargaining power of equity holders encourages the occurrence of strategic defaults, as equity holders could gain more in renegotiation. Moreover, once the firm defaults, higher shareholder bargaining power implies greater deviation from the absolute priority rule. This result is consistent with

Betker (1995), who finds that deviations from APR in Chapter 11 increase sharply with CEO shareholding. In addition, Davydenko and Strebulaev (2007) find that CEO shareholding increases the credit spread.

The regression results gives support to the strategic default effect on the implied default threshold[27]. This evidence is in line with endogenous default models, in the spirit of Leland (1994) and Fan and Sundaresan (2000), for instance, where the shareholders deliberately choose to default in order to benefit from debt cutback.

## 3.6. Robustness tests

It is intuitive that more leverage implies a higher implied default barrier level, and we find evidence of this relationship in the non-strategic factor regressions. However, one could argue that this level of default threshold could also influence borrowing decisions. Therefore, we can reasonably suspect a simultaneous relationship between default barrier and indebtedness, where the optimal leverage ratio would be the result of equilibrium.

To be able to account for the endogenous relationship linking leverage and implied default barrier, we need to specify the rest of the leverage determinants. The financial literature identifies several determinants of capital structure choice. Definitely, the first motive for companies to contract debt is to benefit from the debt tax shield. We therefore include the actual tax rate, defined as the ratio of the tax payment to the earnings before tax as an explanatory variable for the debt equation. As firms also benefit from a non-debt tax shield, we add the depreciation and amortization scaled by the book value of total assets as an additional independent variable. We expect a positive sign for both tax rate and depreciation and amortization variables.

Moreover, Titman and Wessels (1988) argue that the firm's collateral value increases its ability to contract more debt. Our proxy for collateral value is the book-to-market ratio. Indeed, we conjecture that value companies, those with high book-to-market ratio, have

[27] Although the R squared is slightly increased by adding non-strategic factors, the coefficients on them are statistically significant.

more productive assets in place than do low book-to-market firms, whose value is primarily driven by less pledgeable growth options. Thus, we expect a positive sign for the coefficient on the book-to-market ratio. In the same spirit, we added the ratio of R&D expenses to the book value of assets, to account for the fact that firms operating in technology intensive sectors have more specific assets. Because these kinds of assets are less valuable in case of liquidation, we could expect a negative relation with leverage. The R&D expenses can be interpreted as a measure of the firm's uniqueness. As an additional proxy for firm uniqueness we use selling, general and administrative expenses scaled by net sales; the same logic makes us anticipate a negative sign. Finally, the profits generated by the firm's operations decrease its need for external financing and should be associated with less debt. Our measure for profitability is the EBITDA divided by net sales; here again we expect a negative relation with leverage.

<Insert Table 3.X here>

We report the results of the three-stage least-square estimation in Table 3.X, the dependent variables being the estimated default barrier to estimated asset value and the leverage ratio (book value of debt to book value of assets). We notice that all the independent variables in the leverage equation have the expected signs and are significant at the 1% confidence level. Moreover, in regard to the default barrier equation, the previous results hold for both strategic and non-strategic factors. The endogeneity between the leverage and default barrier does not bias our regression results.

<Insert Table 3.XI here>

Furthermore, in the DOC framework, we have to assume a lifespan of the firms. In other words, to model the firm's equity as a DOC option, we have to set the option maturity which represent the lifetime of the company. In the previous analysis, we assumed a lifespan of 20 years. To ensure that our findings are not affected by the choice of this input parameter, we estimated the default barrier assuming lifespans of 5 and 10 years. The results in Table 3.XI show that the average estimated barrier is not particularly sensitive to the choice of the firm's lifespan assumption. Indeed, changing the option maturity from 20 to 5 years moves the mean of estimated barrier from 0.29 to 0.26. Thus, the economic importance of the default barrier estimates is robust to the alternative maturity choice. Moreover, the correlation of the default barriers estimated with 20 years maturity and those estimated with 10 years maturity attains 0.99, whereas the correlation between the 20 years and the 5 years default barriers achieve 0.96. Finally, the estimates of the implied barrier on the non strategic and strategic factors keep the same signs and significance when the assumed firm's lifespans are changed. We can therefore conclude that our findings are robust to the choice of the maturity parameter.

## 3.7. Conclusion

In the structural models of credit risk, default is often assumed to happen when the market value of assets falls below a given barrier. The financial theory stipulates different assumptions regarding the default barrier, ranging from Merton (1974), whose threshold is simply the debt value at maturity, to more sophisticated settings where the default barrier is determined endogenously by stakeholders, as in Leland and Toft (1996). However, due to the unobservability of the firm's asset value, these assumptions were not directly tested, but rather the overall model performance is assessed in predicting either defaults or credit spreads. In this paper, we use the maximum likelihood estimation method of Duan (1994) in order to infer the implied default barrier of the DOC option model from equity prices. We use a sample of public Canadian firms to compare the KMV-Merton model with that of DOC option model in terms of default prediction accuracy. We find that our implementation of the KMV-Merton and the DOC option models perform equally well for in-sample fitting. However, the DOC option default probability estimate attains higher accuracy in out-of-sample default forecasting. Moreover, the implied barrier for defaulting firms is close to their estimated asset market value at default. We also use regression for the implied default barrier against firm-specific and macro-economic factors, and find that not only does the capital structure influence the default barrier location, but firm-specific factors also do. Further, the ex ante implied default barrier is adjusted to both non-strategic factors and strategic factors. It is sensitive to asset liquidity, debt cost and liquidation costs, renegotiation friction and equity holders' bargaining power. Thus, it seems that the market adjusts the implied default threshold, through equity prices, to account for firm-specific determinants that go beyond the level of debt. Our results give new insights for modeling the decision to default and for default predictions.

## References

Acharya, V.V., and J.N. Carpenter, 2002. Corporate Bond Valuation and Hedging with Stochastic Interest Rates and Endogenous Bankruptcy, *Review of Financial Studies* 15, 1355–1383.

Acharya, V.V., J.Z. Huang, M.G. Subrahmanyam, and R.K. Sundaram, 2006, When Does Strategic Debt-service Matter? *Economic Theory* 29, 363–378.

Acharya, V.V., S.A. Davydenko, and I.A. Strebulaev, 2007, Cash Holdings and Credit Spreads, Working paper, London Business School.

Anderson, R.W., and A. Carverhill, 2007, Liquidity and Capital Structure, Centre for Economic Policy Research Working paper No. 6044.

Anderson, R.W., and S.M. Sundaresan, 1996. The Design and Valuation of Debt Contracts, *Review of Financial Studies* 9, 37–68.

Asquith, P., R. Gertner, and D. Scharfstein, 1994, Anatomy of Financial Distress: An Examination of Junk Bond Issuers, *Quarterly Journal of Economics* 109, 625–658.

Asvanunt, A., M. Broadie, and S. Sundaresan, 2007, Managing Corporate Liquidity: Welfare and Pricing Implications, Working paper, Columbia University.

Berglöf, E., and E.L. von Thadden, 1994, Short-term Versus Long-Term Interests: Capital Structure with Multiple Investors, *Quarterly Journal of Economics* 109, 1055–1084.

Betker, B.L., 1995, Management's Incentives, Equity's Bargaining Power, and Deviations from Absolute Priority in Chapter 11 Bankruptcies, *Journal of Business* 68, 161–183.

Bharath, S.T and Shumway, T. (2004). Forecasting Default with the Merton-KMV Model. Working paper, University of Michigan.

Black, F. and M. Scholes, 1973. The Pricing of Options and Corporate Liabilities, *Journal of Political Economy* 81, 637-654.

Black, F., and J.C. Cox, 1976. Valuing Corporate Securities - Some Effects of Bond Indenture Provisions. *Journal of Finance* 31(2), 351-367

Brockman,.P. and H.J. Turtle, 2003. A Barrier Option Framework for Corporate Security Valuation. *Journal of Financial Economics* 67(3), 511–529.

Cantor R., Emery K., Matos A., Ou S. and Tenneant J., February 2009, Corporate Default and Recovery Rates, 1920-2008, Global and Regional Corporate Default Studies.

Carey, M., and M. Gordy, 2007, The Bank as Grim Reaper, Debt Composition and Recoveries on Defaulted Debt, Board of Governors of the Federal Reserve System Working Paper.

Collin-Dufresne, P., and R.S. Goldstein, 2001, Do Credit Spreads Reflect Stationary Leverage Ratios? *Journal of Finance* 56, 1929–1957.

Crosbie, P. and J. Bohn, 2003. Modeling Default Risk. Mimeo, Moody's/KMV.

Davydenko, S. A. and I. A. Strebulaev, 2007, Strategic Actions and Credit Spreads: An Empirical Investigation, *Journal of Finance* 62 (6), 2633-2671.

Davydenko, S.A., 2007. When Do Firms Default? A Study of the Default Barrier. EFA 2005 Moscow Meetings. Paper Available at SSRN: http://ssrn.com/abstract=672343.

DeLong E.R., D.M. DeLong, and D.L. Clarke-Pearson (1988), Comparing the Areas Under Two or More Correlated Receiver Operating Characteristic Curves: A Nonparametric Approach, *Biometrics*, 44, 837-845.

Denis, D.J., and V.T. Mihov. 2003. The Choice Among Bank Debt, Non-bank Private Debt, and Public Debt: Evidence from New Corporate Borrowings. *Journal of Financial Economics* 70:3–28.

Duan, J.C., 1994. “Maximum Likelihood Estimation Using Price Data of the Derivative Contract”, Mathematical Finance, 4(2), 155-167. Correction: Duan, J.C., 2000, “Maximum Likelihood Estimation Using Price Data of the Derivative Contract” (*Mathematical Finance* 1994, 4(2), 155-167), *Mathematical Finance*, 10 (4), 461-462.

Duan, J.,G. Gauthier, and J. Simonato, 2004. On the Equivalence of the KMV and Maximum Likelihood Methods for Structural Credit Risk Models. Working Paper, University of Toronto.

Duan, J.,G. Gauthier, J.G. Simonato, and S. Zaanoun, 2003. Estimating Merton's Model by Maximum Likelihood with Survivorship Consideration. Working Paper, HEC Montréal.

Ericsson, J. and Reneby, J., 2005. Estimating Structural Bond Pricing Models, *Journal of Business* 78, 707–735.

Fan, H., and S.M. Sundaresan, 2000, Debt Valuation, Renegotiation, and Optimal Dividend Policy, *Review of Financial Studies* 13, 1057-1099.

François, P., and E. Morellec, 2004, Capital Structure and Asset Prices: Some Effects of Bankruptcy Procedures, *Journal of Business* 77, 387-411.

Franks, J R., and W.N. Torous, 1994, A Comparison of Financial Recontracting in Distressed Exchanges and Chapter 11 Reorganizations, *Journal of Financial Economics* 35, 349–370.

Galai, D., A. Raviv, and Z. Wiener, 2007, Liquidation Triggers and The Valuation of Equity and Debt, *Journal of Banking & Finance* 31 (12), 3604-3620.

Gertner, R., and D. Scharfstein, 1991, A Theory of Workouts and the Effects of Reorganization Law, *Journal of Finance* 46, 1189-1222

Gharghori, P., H. Chan, and R. Faff, 2006, Investigating the Performance of Alternative Default-Risk Models: Option-Based Versus Accounting-Based Approaches, *Australian Journal of Management* 31 (2), 207-234.

Hackbarth, D., Hennessy, C.A. and H.E. Leland, 2007, Can the Tradeoff Theory Explain Debt Structure?, *The Review of Financial Studies*, 20 (5), 1389-1428.

Hadi, A.S., 1992. Identifying Multiple Outliers in Multivariate Data, *Journal of the Royal Statistical Society*. Series B, 54, 761-771.

Hadi, A.S., 1994. A Modification of a Method for the Detection of Outliers in Multivariate Samples, *Journal of the Royal Statistical Society*, Series B, 56 (2).

Hao, H., 2006. Is the Structural Approach More Accurate than the Statistical Approach in Bankruptcy Prediction? Working Paper, Queen's School of Business.

Hart, O.D., and J. Moore, 1998, Default and Renegotiation: A Dynamic Model of Debt, *Quarterly Journal of Economics* 113, 1–41.

Hege, U., and P. Mella-Barral, 2005, Repeated Dilution of Diffusely Held Debt, *Journal of Business* 78 (3), 737-786.

Houston, J., and C. James. 1996. Bank Information Monopolies and the Mix of Private and Public Debt Claims, *Journal of Finance* 51, 1863–1889.

Hsu, J.C., J. Saá-Requejo, and P. Santa-Clara, 2002, Bond Pricing with Default Risk, Working paper, UCLA.

Huang, J., and M. Huang, 2003. How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk? Working paper, Pennsylvania State University.

Jones, P.E., S.P. Mason, and E. Rosenfeld, 1984, Contingent Claims Analysis of Corporate Capital Structures: An Empirical Analysis, *Journal of Finance* 39, 611–625.

Johnson, S. 1997. An Empirical Analysis of the Determinants of Corporate Debt Ownership Structure. *Journal of Financial and Quantitative Analysis* 32, 47–69.

Kim, I.J., K. Ramaswamy, and S. Sundaresan, 1993, Does Default Risk In Coupons Affect the Valuation of Corporate Bonds? A Contingent Claims Model, *Financial Management* 22, 117-131.

Krishnaswami, S., P. Spindt, and V. Subramaniam. 1999. Information Asymmetry, Monitoring, and the Placement Structure of Corporate Debt. *Journal of Financial Economics* 51, 407–34.

Leland, H.E., and K.B. Toft, 1996. Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads. *Journal of Finance* 51 (3), 987-1019.

Leland, H.E., 1994. Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. *Journal of Finance* 49 (4), 1213–52.

Longstaff, F.A., and E.S. Schwartz, 1995. A Simple Approach to Valuing Risky Fixed and Floating Rate Debt, *Journal of Finance* 50, 789–819.

LoPucki, L.M., and W.C. Whitford, 1990, Bargaining Over Equity's Share in the Bankruptcy Reorganization of Large, Publicly Held Companies, *University of Pennsylvania Law Review*, 139, 125-96.

Mella-Barral, P., 1999, The Dynamics of Default and Debt Reorganization, *Review of Financial Studies* 12, 535–578.

Mella-Barral, P., and W.R.M. Perraudin, 1997, Strategic Debt Service, *Journal of Finance* 52, 531–556.

Moraux, F., 2004, Valuing Corporate Liabilities when the Default Threshold is not an Absorbing Barrier, University of Rennes Working Paper.

Nagelkerke, N.J.D., 1991. A Note on a General Definition of the Coefficient of Determination. *Biometrika* 78 (3), 691-692.

Reisz A.S. and C. Perlich, 2007, A Market-based Framework for Bankruptcy Prediction. *Journal of Financial Stability* 3 (2), 85-131.

Ronn, E.I., and A.K. Verma, 1986. Pricing Risk-adjusted Deposit Insurance - An Option Based Model. *Journal of Finance* 41 (4), 871-895

Ross, S.A., 2005, Capital Structure and the Cost of Capital, *Journal of Applied Finance* 15, 5–23.

Sobehart, J., S. Keenan, and R. Stein, 2000, Benchmarking Quantitative Default Risk Models: A Validation Methodology, Moody's Investors Service.

Vassalou, M. and Y. Xing, 2004. Default Risk in Equity Returns. *Journal of Finance* 59 (2), 831–68.

Wong, H., and T. Choi, 2009. Estimating Default Barriers from Market Information. *Quantitative Finance* 9 (2), 187-196.

Yu, F. (2005). Accounting Transparency and the Term Structure of Credit Spreads. *Journal of Financial Economics* 75 (1), 53-84.

*Table 3.I: Monte Carlo Study of the MLE Estimation*

The true values used in the Monte Carlo simulation for $\mu$, $\sigma$ and $\alpha$ are respectively 0.1, 0.3 and 0.833. Mean, Median, Standard-deviation, Min and Max are the sample statistics of the estimates from 1,000 simulations. The values used in the simulations are as follows: V0=10,000, F=6,000, $\alpha$=0.833 (H=5000), r = 0.05, $\Delta t$=1/250, and N =200 is the number of daily observations. We use the Wong and Choi (2009) maximum likelihood function as given in equation (3.4).

| | $\hat{\mu}$ True=0.1 | $\hat{\sigma}$ True=0.3 | $\hat{\alpha}$ True=0.833 |
|---|---|---|---|
| Mean | 0.103 | 0.301 | 0.824 |
| Median | 0.095 | 0.296 | 0.849 |
| Std | 0.321 | 0.068 | 0.350 |
| Min | -0.866 | 0.109 | 0 |
| Max | 1.144 | 0.548 | 1.840 |
| t-stat | 0.34 | 0.42 | -0.86 |
| p-value | 0.73 | 0.68 | 0.39 |

*Table 3.II: Estimated Barrier, Asset Drift and Volatility*

This table presents the MLE estimates of the default barrier. The results refer to firm-year observations. Panel A presents the leverage ratio for the sample. The ratio is the book value of total liabilities divided by the book value of total assets. Panel B presents the barrier as a fraction of the last estimated asset market value. Panels C and D present the estimated asset drift and volatility respectively with the DOC option model. Panels E and F show the average implied barrier to assets by year and debt load respectively.

| | N[28]. | Mean | Std | Min | Q1 | Median | Q3 | Max |
|---|---|---|---|---|---|---|---|---|
| Panel A: Leverage ratio (total liabilities/ total assets) | | | | | | | | |
| Overall sample | 4916 | 0.54 | 0.70 | 0 | 0.30 | 0.49 | 0.66 | 19.5 |
| Non-defaulted | 4860 | 0.53 | 0.69 | 0 | 0.30 | 0.49 | 0.66 | 19.5 |
| Defaulted | 56 | 1.12 | 1.16 | 0.26 | 0.62 | 0.85 | 1.23 | 8.36 |
| Panel B: Barrier to implied asset value | | | | | | | | |
| Overall sample | 4916 | 0.29 | 0.27 | 0 | 0 | 0.25 | 0.5 | 0.99 |
| Non-defaulted | 4860 | 0.29 | 0.27 | 0 | 0 | 0.25 | 0.5 | 0.99 |
| Defaulted | 56 | 0.65 | 0.31 | 0 | 0.47 | 0.76 | 0.91 | 0.99 |
| Panel C: Estimated asset drift* | | | | | | | | |
| Overall sample | 4916 | 0.05 | 0.31 | -0.43 | -0.13 | 0.02 | 0.19 | 1.02 |
| Non-defaulted | 4860 | 0.05 | 0.31 | -0.43 | -0.13 | 0.03 | 0.2 | 1.02 |
| Defaulted | 56 | -0.18 | 0.28 | -0.43 | -0.32 | -0.2 | 0 | 0.9 |
| Panel D: Estimated asset volatility | | | | | | | | |
| Overall sample | 4916 | 0.53 | 0.40 | 0.00 | 0.24 | 0.41 | 0.70 | 3.98 |
| Non-defaulted | 4860 | 0.53 | 0.40 | 0.00 | 0.24 | 0.41 | 0.70 | 3.98 |
| Defaulted | 56 | 0.57 | 0.36 | 0.05 | 0.26 | 0.58 | 0.82 | 1.77 |

*To avoid the effect of outliers, the asset drift estimates were limited to the interval between the 1st and the 99th percentiles.

[28] Firm-year observations.

*Table 3.II (Continued)*

| | ♯ Obs. | Average | Std | Student t-statistic | t-statistic p value |
|---|---|---|---|---|---|
| Panel E: Average implied barrier/ MVA estimate by year | | | | | |
| 1988 | 100 | 0.38 | 0.27 | 14.07 | < 0.001 |
| 1989 | 113 | 0.29 | 0.26 | 11.95 | < 0.001 |
| 1990 | 138 | 0.42 | 0.29 | 16.84 | < 0.001 |
| 1991 | 155 | 0.35 | 0.27 | 16.38 | < 0.001 |
| 1992 | 155 | 0.33 | 0.26 | 15.96 | < 0.001 |
| 1993 | 173 | 0.21 | 0.24 | 11.83 | < 0.001 |
| 1994 | 190 | 0.23 | 0.23 | 13.60 | < 0.001 |
| 1995 | 230 | 0.30 | 0.26 | 17.04 | < 0.001 |
| 1996 | 257 | 0.19 | 0.22 | 14.03 | < 0.001 |
| 1997 | 304 | 0.26 | 0.29 | 15.69 | < 0.001 |
| 1998 | 370 | 0.43 | 0.31 | 25.99 | < 0.001 |
| 1999 | 420 | 0.36 | 0.28 | 26.16 | < 0.001 |
| 2000 | 443 | 0.31 | 0.29 | 22.74 | < 0.001 |
| 2001 | 497 | 0.25 | 0.23 | 23.88 | < 0.001 |
| 2002 | 534 | 0.28 | 0.25 | 25.54 | < 0.001 |
| 2003 | 567 | 0.25 | 0.24 | 24.18 | < 0.001 |
| 2004 | 270 | 0.21 | 0.24 | 14.38 | < 0.001 |
| Panel F: Average barrier/implied MVA estimate by debt load | | | | | |
| Debt proportion $\leq 0.1$ | 334 | 0.21 | 0.24 | 22.08 | < 0.001 |
| $0.1 <$ Debt proportion $\leq 0.2$ | 399 | 0.24 | 0.25 | 19.17 | < 0.001 |
| $0.2 <$ Debt proportion $\leq 0.3$ | 472 | 0.28 | 0.28 | 21.72 | < 0.001 |
| $0.3 <$ Debt proportion $\leq 0.4$ | 611 | 0.28 | 0.26 | 26.62 | < 0.001 |
| $0.4 <$ Debt proportion $\leq 0.5$ | 691 | 0.32 | 0.27 | 31.15 | < 0.001 |
| $0.5 <$ Debt proportion $\leq 0.6$ | 719 | 0.31 | 0.27 | 30.79 | < 0.001 |
| $0.6 <$ Debt proportion $\leq 0.7$ | 743 | 0.31 | 0.27 | 31.29 | < 0.001 |
| $0.7 <$ Debt proportion $\leq 0.8$ | 406 | 0.31 | 0.27 | 23.13 | < 0.001 |
| $0.8 <$ Debt proportion $\leq 0.9$ | 199 | 0.34 | 0.29 | 16.53 | < 0.001 |
| Debt proportion $> 0.9$ | 342 | 0.33 | 0.31 | 19.68 | < 0.001 |

*Table 3.III: Comparison of PDs between the Merton-KMV and the DOC Models*

This table presents the default probability estimates obtained from the Merton-KMV model and the DOC option model for a 1-year horizon. The probabilities presented here are real probabilities and not risk neutral.

| | # Obs. | Mean | Std | Min | Q1 | Median | Q3 | Max |
|---|---|---|---|---|---|---|---|---|
| *Panel A: DOC option-Wong and Choi* | | | | | | | | |
| Overall sample | 4916 | 10.88% | 21.79% | 0% | 0% | 0% | 8.36% | 100% |
| Non-defaulted | 4860 | 10.30% | 20.82% | 0% | 0% | 0% | 7.77% | 99.5% |
| Defaulted | 56 | 61.05% | 39.49% | 0% | 16% | 81.37% | 96.93% | 100% |
| *Panel B: Merton-KMV* | | | | | | | | |
| Overall sample | 4916 | 13.66% | 22.36% | 0% | 0% | 0.6% | 19.7% | 99.9% |
| Non-defaulted | 4860 | 13.26% | 22.36% | 0% | 0% | 0.53% | 19.15% | 99.5% |
| Defaulted | 56 | 48.51% | 32.01% | 0% | 15.96% | 58% | 74.06% | 99.99% |

*Table 3.IV: Comparison between Bankruptcies and Reorganizations*

*Leverage is the book value of total liabilities divided by the book value of total assets. Liquidity is the cash and cash equivalents divided by the book value of total assets. Debt Cost* is the ratio of interest expenses to the book value of total liabilities. *Asset volatility* is the estimated asset volatility from the DOC option model. *Size* is the logarithm of the total book assets in millions of dollars. The *p-values* of the *t-test* of the difference between the two groups are reported in brackets.

| | Mean | | |
|---|---|---|---|
| Variable | Bankruptcies | Reorganizations | Difference |
| *Default barrier* | 0.6563 | 0.6286 | 0.0278 |
| | | | (0.7935) |
| *Default probability* | 0.6901 | 0.7359 | -0.046 |
| | | | (0.7114) |
| *Leverage* | 1.005 | 1.5974 | -0.592 |
| | | | (0.1315) |
| *Liquidity* | 0.1039 | 0.0366 | 0.0673 |
| | | | (0.4722) |
| *Debt Cost* | 0.0516 | 0.0246 | 0.027 |
| | | | (0.4283) |
| *Volatility* | 0.6031 | 0.4531 | 0.1501 |
| | | | (0.2158) |
| *Size* | 10.675 | 9.9806 | 0.6947 |
| | | | (0.2888) |
| *N* | 45 | 11 | |

*Table 3.V: Default probabilities by decile*

Performance of the DOC option and Merton-KMV models in predicting bankruptcy probabilities within one year. We report the number and percentage of defaults after classifying observations into deciles based on the estimated default probabilities. The 10th decile is the largest one and the 1st decile is the smallest. The DOC option and Merton-KMV models are estimated using the Maximum Likelihood method. We use the Wong and Choi maximum likelihood function to estimate the DOC option model's parameters.

| | DOC option model | | | | | | Merton-KMV model | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Decile | N | Defaults | Cumulative sum | Non-defaults | Cumulative sum | Average DP | N | Defaults | Cumulative sum | Non-defaults | Cumulative sum | Average DP |
| 10 | **491** | 38 | 38 | 453 | 453 | **68.06%** | **491** | 31 | 31 | 460 | 446 | **68.34%** |
| | | 68% | 68% | 9% | 9% | | | 55% | 55% | 9% | 9% | |
| 9 | **492** | 4 | 42 | 488 | 941 | **29.99%** | **492** | 6 | 37 | 486 | 918 | **38.64%** |
| | | 7% | 75% | 10% | 19% | | | 11% | 66% | 10% | 19% | |
| 8 | **492** | 2 | 44 | 490 | 1431 | **9.18%** | **492** | 7 | 44 | 485 | 1388 | **20.10%** |
| | | 4% | 79% | 10% | 29% | | | 13% | 79% | 10% | 29% | |
| 7 | **491** | 5 | 49 | 486 | 1917 | **1.57%** | **491** | 3 | 47 | 488 | 1863 | **7.57%** |
| | | 9% | 88% | 10% | 39% | | | 5% | 84% | 10% | 39% | |
| 6 | **492** | 2 | 51 | 490 | 2407 | **0.09%** | **492** | 5 | 52 | 487 | 2337 | **1.80%** |
| | | 4% | 91% | 10% | 50% | | | 9% | 93% | 10% | 50% | |
| 5 | **492** | 1 | 52 | 491 | 2898 | **0.00%** | **492** | 2 | 54 | 490 | 2812 | **0.23%** |
| | | 2% | 93% | 10% | 60% | | | 4% | 96% | 10% | 60% | |
| 4 | **491** | 1 | 53 | 490 | 3388 | **0.00%** | **491** | 0 | 54 | 491 | 3289 | **0.01%** |
| | | 2% | 95% | 10% | 70% | | | 0% | 96% | 10% | 70% | |
| 3 | **492** | 2 | 55 | 490 | 3878 | **0.00%** | **492** | 0 | 54 | 492 | 3766 | **0.00%** |
| | | 4% | 98% | 10% | 80% | | | 0% | 96% | 10% | 80% | |
| 2 | **412** | 0 | 55 | 412 | 4290 | **0.00%** | **492** | 0 | 54 | 492 | 4244 | **0.00%** |
| | | 0% | 98% | 8% | 88% | | | 0% | 96% | 10% | 90% | |
| 1 | **571** | 1 | 56 | 570 | 4860 | **0.00%** | **491** | 2 | 56 | 489 | 4720 | **0.00%** |
| | | 2% | 100% | 12% | 100% | | | 4% | 100% | 10% | 100% | |
| Total | 4916 | 56 | 56 | 4860 | 4860 | 10.88% | 491 | 56 | 56 | 4860 | 4860 | 13.66% |

*Table 3.VI: Comparative Performance in Predicting Bankruptcy*

This table reports the comparative performance in predicting bankruptcy one year ahead. The estimated default probability from structural models is used as a regressor in probit regression including an intercept. In sample estimation we use the full data set to estimate probabilities, while out-of-sample uses the coefficient estimated from the in-sample estimation to evaluate the predictive performance of the resulting probabilities on the remaining unused out-of-sample data. For each probit regression considered, we report the maximum rescaled $R^2$ of Nagelkerke (1991), and the percentage of bankruptcy concordant with the model, and the area under the receiver operating characteristic curve (AUC) measures of the ability of the model to correctly rank observations. In the last column we add the nonparametric test of difference between areas under correlated ROC curves of Delong et al (1988).

| *Model* | DOC option | Merton-KMV | $\chi^2_{(1)}$ Statistic (*p-value*) |
|---|---|---|---|
| *Panel A: In-sample estimation: estimation period 1988-2004* | | | |
| # obs. | 4916 | 4916 | |
| # defaults | 56 | 56 | |
| Default probability | 1.99 | 1.65 | |
| | (0.001) | (0.001) | |
| Max rescaled $R^2$ | 0.256 | 0.152 | |
| Percent concordant | 75.4 | 75.2 | |
| Area under ROC | 0.860 | 0.824 | 1.200 |
| | | | (0.273) |
| *Panel B: Out-of-sample validation; estimation period 1988-1995; evaluation period 1996-2004* | | | |
| *In sample* | | | |
| # obs. | 1310 | 1310 | |
| # defaults | 23 | 23 | |
| Max rescaled $R^2$ | 0.292 | 0.22 | |
| Percent concordant | 71.7 | 86.8 | |
| Area under ROC | 0.845 | 0.883 | 0.614 |
| | | | (0.433) |
| *Out-of-sample* | | | |
| # obs. | 3606 | 3606 | |
| # Defaults | 33 | 33 | |
| Max rescaled $R^2$ | 0.283 | 0.107 | |
| Percent concordant | 73.5 | 62.2 | |
| Area under ROC | 0.871 | 0.787 | 3.224 |
| | | | (0.072) |
| *Panel C: Out-of-sample validation; estimation period 1988-1996; evaluation period 1997-2004* | | | |
| *In sample* | | | |
| # obs. | 1567 | 1567 | |
| # defaults | 26 | 26 | |
| Max rescaled $R^2$ | 0.303 | 0.23 | |
| Percent concordant | 71.7 | 83.4 | |
| Area under ROC | 0.837 | 0.884 | 0.722 |
| | | | (0.395) |
| *Out-of-sample* | | | |
| # obs. | 3349 | 3349 | |
| # defaults | 30 | 30 | |
| Max rescaled $R^2$ | 0.274 | 0.095 | |
| Percent concordant | 74.1 | 61.4 | |
| Area under ROC | 0.883 | 0.780 | 6.907 |
| | | | (0.009) |

*Table 3.VII: Descriptive statistics of dependent and independent variables*

*Default* barrier is the implied default barrier divided by the last estimated asset value. *Leverage* is the book value of total liabilities divided by the book value of total assets. *Liquidity* is the cash and cash equivalents divided by the book value of total assets. *Debt Cost* is the ratio of interest expenses to the book value of total liabilities. *Asset volatility* is the estimated asset volatility from the DOC option model. *Size* is the logarithm of the total book assets in millions of dollars. *GDP Growth* is the real GDP growth rate. *Fixed assets* is the total value of capital assets including land, buildings, computers, factories, office equipment, leasehold improvements and assets under capital leases, net of accumulated depreciation and amortization to the book value of total assets. *R&D* is the ratio of research and development expenses to total assets. *Short-term debt* is the ratio of current liabilities to total liabilities. *Voting* is the percentage of votes attached to the voting shares of a company held by the directors and other individuals or companies that own more than 10% of all voting rights. *Public Debt* is the ratio of the amount of outstanding public debt to the book value of total debt.

| Variable | N | Mean | Median | Std. Dev. | Minimum | Maximum |
|---|---|---|---|---|---|---|
| *Default barrier* | 3232 | 0.401 | 0.383 | 0.242 | 0 | 0.992 |
| *Leverage* | 3232 | 0.482 | 0.488 | 0.293 | 0.002 | 8.361 |
| *Liquidity* | 3232 | 0.093 | 0.034 | 0.137 | 0 | 0.988 |
| *Debt Cost* | 3232 | 0.034 | 0.031 | 0.031 | 0 | 0.284 |
| *Volatility* | 3232 | 0.458 | 0.359 | 0.334 | 0.007 | 3.983 |
| *Size* | 3232 | 11.67 | 11.38 | 2.18 | 4.63 | 17.54 |
| *GDP Growth* | 17 | 0.031 | 0.031 | 0.018 | -0.021 | 0.055 |
| *Fixed Assets* | 3232 | 0.426 | 0.401 | 0.278 | 0 | 1 |
| *R&D* | 3232 | 0.029 | 0.000 | 0.09 | 0 | 1.855 |
| *Short-term debt* | 3232 | 0.565 | 0.553 | 0.284 | 0 | 1 |
| *Voting* | 3085 | 0.304 | 0.242 | 0.257 | 0 | 0.961 |
| *Public Debt* | 3232 | 0.15 | 0 | 0.311 | 0 | 1 |

*Table 3.VIII: Regression analysis of the implied default barrier on non-strategic factors*

This table reports the results of regression analysis of the implied default barrier on non-strategic variables. The dependent variable is the implied default barrier divided by the last estimated asset value. The sample consists of all firm-year observations with non-nil estimated barriers. *Leverage* is the book value of total liabilities divided by the book value of total assets. *Asset volatility* is the estimated asset volatility from the DOC option model. *Liquidity* is the cash and cash equivalents divided by the book value of total assets. *Debt Cost* is the ratio of interest expenses to the book value of total liabilities. *Size* is the logarithm of the total book assets in millions of dollars. *GDP growth* is the annual growth rate of the real GDP. Values of *t-statistics* are reported in parentheses. Coefficients marked ***, ** and * are significant at the 1%, 5% and 10% significance levels, respectively. *Hausman test* is the Hausman test for random effect following a chi square distribution with *k* degrees of freedom, where *k* is the number of independent variables. The corresponding p value is reported below.

| | All | | | Without outliers | | | |
|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| *Leverage* | 0.106*** | 0.103*** | 0.07*** | 0.102*** | 0.069*** | 0.069*** | 0.08*** |
| | (8.16) | (6.84) | (3.71) | (6.69) | (3.50) | (3.49) | (3.91) |
| *Liquidity* | -0.112*** | -0.142*** | -0.188*** | -0.135*** | -0.202*** | -0.242*** | -0.189*** |
| | (-3.97) | (-4.47) | (-5.16) | (-3.66) | (-4.69) | (-5.03) | (-3.76) |
| *Debt Cost* | 0.277** | 0.237* | 0.3** | 0.399** | 0.472*** | 0.316** | 0.444*** |
| | (2.26) | (1.81) | (2.06) | (2.48) | (2.57) | (2.12) | (2.75) |
| *Volatility* | -0.438*** | -0.522*** | -0.611*** | -0.522*** | -0.613*** | -0.612*** | -0.595*** |
| | (-34.88) | (-37.98) | (-38.94) | (-37.64) | (-38.41) | (-38.47) | (-35.44) |
| *Size* | -0.044*** | -0.05*** | -0.036*** | -0.051*** | -0.036*** | -0.036*** | -0.032*** |
| | (-22.45) | (-17.16) | (-6.71) | (-17.24) | (-6.49) | (-6.43) | (-5.20) |
| *GDP Growth* | 1.44*** | 1.38*** | 1.3*** | 1.466*** | 1.38*** | 1.36*** | 1.25*** |
| | (7.06) | (7.36) | (6.75) | (7.34) | (7.10) | (6.99) | (6.01) |
| *Const.* | 1.02*** | 1.15*** | 1.04*** | 1.14*** | 1.03*** | 1.04*** | 0.967*** |
| | (35.43) | (29.59) | (15.76) | (29.37) | (15.09) | (15.07) | (12.48) |
| *Fixed effect* | No | No | Yes | No | Yes | Yes | Yes |
| *Random effect* | No | Yes | No | Yes | No | No | No |
| $R^2$ | 0.299 | 0.299 | 0.268 | 0.298 | 0.269 | 0.269 | 0.257 |
| *Observations* | 3232 | 3232 | 3232 | 3170 | 3170 | 3154 | 2716 |
| *Single firms* | 575 | 575 | 575 | 574 | 574 | 571 | 508 |
| *Hausman test* | - | - | 147.3 | - | 141.91 | 144.74 | 106.74 |
| *p-value* | - | - | 0.00 | - | 0.00 | 0.00 | 0.00 |

*Table 3.IX: Regression analysis of the implied default barrier on strategic factors*

This table reports the results of regression analysis of implied default barrier on strategic variables. The dependent variable is the implied default barrier divided by the estimated asset value. The sample consists of all firm-year observations with non-nil estimated barriers. *Fixed assets* is the total value of capital assets including land, buildings, computers, factories, office equipment, leasehold improvements and assets under capital leases, net of accumulated depreciation and amortization to the book value of total assets. The *R&D* is the research and development expenses to total assets. *Short-term debt* is the ratio of current liabilities to total liabilities. *Voting* is the percentage of votes attached to the voting shares of a company held by the directors and other individuals or companies that own more than 10% of all voting rights. *Public Debt* is the ratio of the amount of outstanding public debt to the book value of total debt. *Leverage, Liquidity, Debt cost, Volatility, Size, GDP growth,* and the constant are included in all specifications. Values of *t-statistics* and *z-statistic* are reported in parentheses. Coefficients marked ***, ** and * are significant at the 1%, 5% and 10% significance level, respectively.

| | | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|---|
| Liquidation costs | *Fixed assets* | -0.044*** | -0.035* | | | | |
| | | (-3.09) | (-1.91) | | | | |
| | *R&D* | | | -0.137 | 0.001 | -0.053 | 0.001 |
| | | | | (-0.33) | (0.02) | (-1.27) | (0.02) |
| Renegotiation frictions | *Short- term debt* | | | | | 0.088*** | .082*** |
| | | | | | | (6.16) | (4.95) |
| | *Public debt* | -0.048*** | -0.064*** | -0.057*** | -0.068*** | | |
| | | (-3.44) | (-3.58) | (-4.10) | (-3.84) | | |
| Bargaining power | *Voting* | 0.079*** | 0.08*** | 0.086*** | 0.086*** | 0.076*** | 0.079*** |
| | | (7.05) | (3.86) | (6.00) | (4.15) | (5.22) | (3.78) |
| *Random effect* | | No | Yes | No | Yes | No | Yes |
| $R^2$ | | 0.30 | 0.30 | 0.31 | 0.33 | 0.30 | 0.30 |
| *Observations* | | 3085 | 3085 | 3085 | 3085 | 3085 | 3085 |
| *Single firms* | | 509 | 509 | 509 | 509 | 509 | 509 |

*Table 3.X: Three Stage Least Square Estimation of Barrier and Leverage Equations*

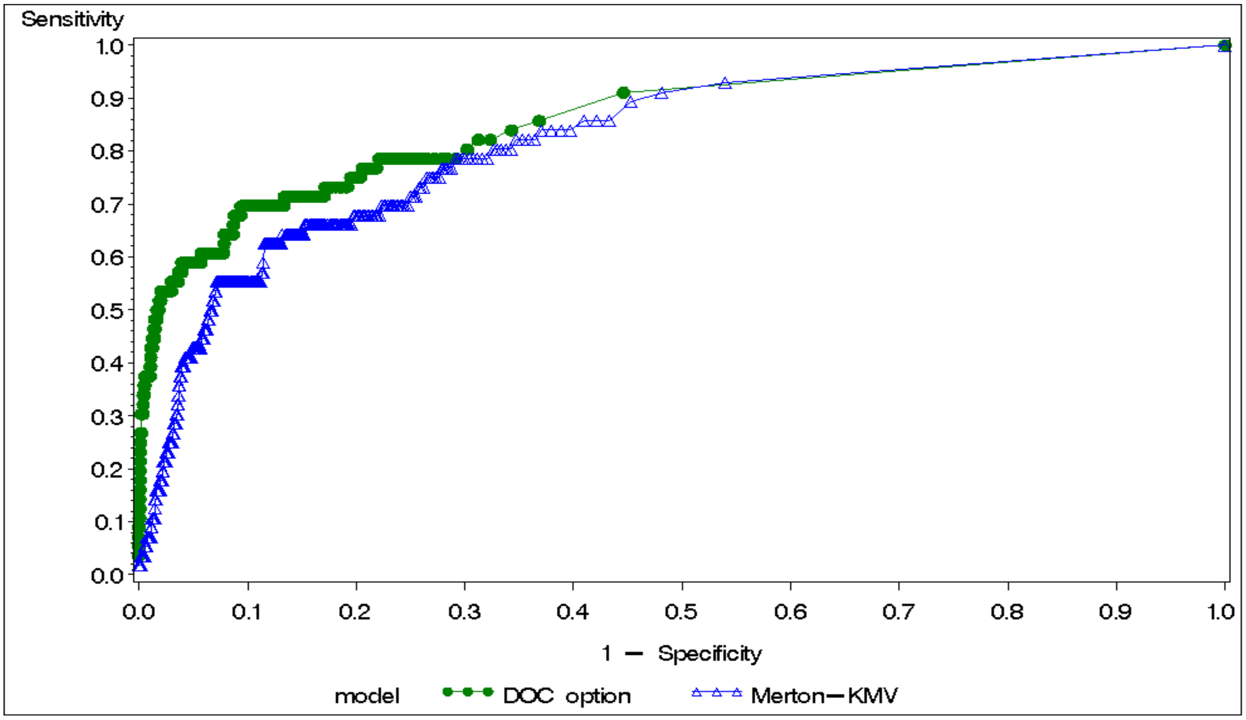
This table reports the results of the three stage least square regressions for panel data with fixed assets where the endogenous variables are the implied default barrier divided by the estimated asset value and the leverage ratio, defined as the book value of total liabilities divided by the book value of total assets. The sample covers all firm-year observations with non-nil estimated barriers and sufficient data. *Asset volatility* is the estimated asset volatility from the DOC option model. *Liquidity* is the cash and cash equivalents divided by the book value of total assets. *Debt Cost* is the interest expenses to the book value of total liabilities. *GDP growth* is the annual growth rate of the real GDP. *R&D* is the research and development expenses scaled by the book value of assets. Profitability is the EBITDA to net sales ratio. *Tax rate* is the tax payment of the year divided by the earnings before taxes. *Dep & Amt* is the depreciation and amortization scaled by the book value of total assets at the end of the year. *Selling & Adm* is the selling and corporate expenses divided by the net sales. Values of *z-statistics* are reported in parentheses. Coefficients marked ***, ** and * are significant at the 1%, 5% and 10% significance level, respectively.

| | Barrier equation | Leverage equation |
|---|---|---|
| *Constant* | 0.426*** (8.86) | 0.250*** (17.59) |
| *Default barrier* | | 0.26*** (8.86) |
| *Leverage* | 0.191** (2.53) | |
| *Liquidity* | -0.187*** (-4.35) | |
| *Debt Cost* | 0.823** (5.21) | |
| *Volatility* | -0.334*** (-27.01) | |
| *GDP Growth* | 1.252*** (6.51) | |
| *R&D* | 0.132*** (2.86) | -0.235*** (-4.99) |
| *Fixed assets* | -0.075*** (-4.94) | |
| *Public debt* | -0.06*** (-4.03) | |
| *Voting* | 0.07*** (4.80) | |
| *Profitability* | | -0.22** (-2.18) |
| *Book-to-Market* | | 0.028*** (16.42) |
| *Tax rate* | | 0.099*** (5.37) |
| *Dep & Amt* | | 1.158*** (10.01) |
| *Selling & Adm* | | -0.062*** (-2.50) |
| *N* | 3085 | 3085 |
| $R^2$ | 0.21 | 0.08 |
| *Chi2 stat* | 1160.33*** | 549.75*** |

*Table 3.XI: Implied Default Barrier for Various Option Lives*

| | N. | Mean | Std | Min | Q1 | Median | Q3 | Max |
|---|---|---|---|---|---|---|---|---|
| 5 years | 4916 | 0.26 | 0.26 | 0 | 0 | 0.19 | 0.42 | 0.99 |
| 10 years | 4916 | 0.27 | 0.26 | 0 | 0 | 0.22 | 0.46 | 0.99 |
| 20 years | 4916 | 0.29 | 0.27 | 0 | 0 | 0.25 | 0.5 | 0.99 |

*Figure 3.1: DOC option and Merton-KMV model's ROC Curve.*




For PD cut points (default/non-default model classification) varying from 0 to 1, each firm-year observation is classified as default (positive) if the model generated PD is above the cut point, and non defaulted otherwise. The outcomes of the model are classified as follow:

| Model outcome | Actual Condition | | |
|---|---|---|---|
| | Positive (Default) | Negative (Non-default) | Total |
| Positive (Default) | TP (True Positive) | FP (False Positive) | TP+FP (Total model's positive prediction) |
| Negative (Non-default) | FN (False Negative) | TN (True Negative) | FN+TN (Total model's negative prediction) |
| Total | TP+FN | FP+TN | TP+FN+ FP+TN |

Based on the above definition, the following measures are as follow:

Sensitivity = TP/(TP + FN) = (Number of true positive)/(Number of all positive)

Specificity = TN/(TN + FP) = (Number of true negative)/(Number of all negative)

# Conclusion générale

À travers cette thèse nous avons cherché à apporter des éléments de réponse à certaines problématiques en matière de risque de crédit.

Dans le premier chapitre, nous avons examiné si l'évaluation continue des probabilités de défaut obtenues à partir des modèles structurels des entreprises canadiennes cotées en bourse permet d'améliorer la prédiction des défauts en comparaison avec les modèles statistiques. Pour ce faire, nous utilisons un modèle hybride incluant à la fois les probabilités de défaut structurelles et les variables financières et économiques. Les résultats indiquent que les probabilités de défaut structurelles contribuent de façon significative à prédire l'occurrence des défauts. Les autres variables ne perdent pas pour autant leur pouvoir explicatif. Nous pouvons conclure dès lors que les modèles structurels peuvent être considérés comme complémentaires aux données comptables et macroéconomiques, plutôt que des substituts.

Dans le deuxième chapitre, nous nous intéressons à une composante centrale des modèles structurels de risque de crédit : la majorité de ces modèles supposent que la firme fait défaut lorsque la valeur de ses actifs baisse suffisamment pour toucher une barrière de défaut. En utilisant la méthode de maximum de vraisemblance, nous estimons les barrières de défaut implicite à partir des prix des actions. Les actions sont considérées comme des options barrières dans ce cadre d'analyse. Nous trouvons que les barrières de défaut sont strictement positives pour les entreprises canadiennes. Nous trouvons aussi que ces barrières sont sensibles à la liquidité des actifs et au coût de la dette en plus du niveau d'endettement. Les barrières de défaut sont aussi sensibles aux facteurs de défauts stratégiques, tel que les coûts de liquidation des actifs, les obstacles à la renégociation et le pouvoir de négociation des actionnaires. Les résultats indiquent donc que le marché ajuste les barrières de défaut implicites dans les prix des actions pour tenir compte d'autres facteurs pouvant influencer les défauts corporatifs en plus du niveau d'endettement. Ces résultats permettent une meilleure compréhension des décisions de défaut et donnent un certain support au modèle de défaut stratégique.

Finalement, le dernier chapitre vise à établir une revue des principaux modèles structurels. Les hypothèses et la logique qui sous-tendent ces modèles sont discutées et comparés tout au long du chapitre. Les applications et les résultats empiriques relatifs à ces modèles sont aussi examinés et répertoriés. L'objectif étant de fournir une vue d'ensemble sur les contributions et les applications sans cesse croissantes en matière de modèles structurels de risque de crédit.