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# Three Essays on Corporate Credit Spreads

By

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# Résumé

Une importante question de recherche dans la littérature récente sur le risque de crédit aborde la proportion de l'écart de rendement entre les obligations privées et gouvernementales (prime de crédit) pouvant être attribuée au risque de défaut (proportion de la prime de défaut). Des études antérieures ont démontré que la proportion de la prime de crédit qui peut être expliquée par le risque de défaut est faible (Elton, Gruber, Agrawal et Mann, 2001 et Huang et Huang, 2003). Ce phénomène est connu sous le terme de puzzle des primes de crédit (Chen, Collin-Dufresne et Golstein, 2005; Driessen, 2005; Amato et Remolona, 2003). Cette thèse se compose de trois essais.

Dans le premier essai, nous réexaminons la proportion de la prime de défaut en considérant différentes problématiques associées au calcul des probabilités de transition et de défaut obtenues à partir des données historiques des transitions de notation de crédit. Un résultat important de notre recherche est que la proportion attribuable au risque de défaut dans les primes de crédit est sensible à la structure par terme des probabilités de défaut de chaque classe de risque. De plus, cette proportion peut devenir importante lorsque des analyses de sensibilité sont réalisées en fonction des taux de recouvrement, des cycles de défaut et de l'information considérée dans les données historiques des transitions de notation de crédit.

Dans le second essai, nous incorporons la corrélation entre les taux d'intérêts sans risque et la probabilité de défaut ainsi que le risque de changement de régime des facteurs macroéconomiques et des probabilités de défaut afin d'expliquer la prime de crédit. Notre approche permet une prime de risque liée aux facteurs macroéconomiques à travers la modélisation du facteur d'actualisation en fonction des facteurs macroéconomiques (Evans, 2003) et avec un changement de régime (Bansal et Zhou, 2002). Ainsi, nous contribuons à la littérature en examinant si le risque de changement de régime des

facteurs macroéconomiques peut expliquer la prime de crédit. Pour atteindre cet objectif, nous généralisons le modèle de Bansal et Zhou (2002) pour évaluer les obligations corporatives en dérivant une formule analytique des probabilités cumulées de défaut et des obligations gouvernementales et corporatives. Le modèle est calibré aux données de la consommation, de l'inflation, des structures par terme des taux d'intérêts sans risque et des probabilités cumulées de défaut de la période 1987-1996. Les résultats démontrent que les probabilités de défaut et les primes de défaut changent de régime. En moyenne, la prime de défaut représente 41% de la prime de crédit des obligations industrielles cotées Baa et ayant une maturité de 10 ans. La proportion de défaut moyenne est de 48.5% dans le régime de défauts élevés et de 35.5% dans le régime de faibles défauts. De plus, la proportion de défaut est variable et peut atteindre 72.5% dans le cycle de défauts élevés et 25% dans le cycle de faibles défauts.

Dans le troisième essai, nous modélisons la dynamique des primes de crédit. La majorité des modèles du risque de crédit supposent que les primes de crédit suivent un seul régime avec un effet de niveau dans la volatilité. Ces modèles semblent inappropriés puisque la dynamique des primes de crédit dépend du cycle de défaut et des conditions macroéconomiques. D'ailleurs, plusieurs recherches ont démontré que les conditions macroéconomiques ont un impact considérable sur les probabilités de défaut et les pertes en cas de défaut (voir Allen et Saunders, 2003 pour une revue). Dans cet essai, nous proposons un modèle pour la volatilité des primes de crédit qui tient compte de l'effet de niveau, du changement de régime et de l'effet GARCH.

Les résultats empiriques du troisième essai montrent que l'effet de niveau et l'effet GARCH sont essentiels pour la modélisation des primes de crédit. L'effet de niveau est cohérent avec le processus racine-carrée fortement utilisé dans les modèles à forme réduite, à l'exception des obligations très risquées (cotées Ba et moins) où l'élasticité de niveau est plus élevée. La volatilité conditionnelle des obligations très risquées est plus persistante. Les bonnes et les mauvaises nouvelles ont le même effet sur la volatilité des primes de crédit à l'exception des obligations cotées A où une prime de crédit non anticipée positive (mauvaise nouvelle) a pour conséquence d'augmenter la prime de crédit, par contre une prime de crédit non anticipée négative n'a aucun effet sur la volatilité de la prime de crédit. Nous trouvons aussi qu'il y a deux régimes dans la

dynamique de la prime de crédit. Le premier régime est caractérisé par une volatilité élevée et l'autre par une volatilité faible. Durant le régime de volatilité élevée, la prime de crédit moyenne est plus faible (élevée) pour les obligations les moins (plus) risquées. Les primes de crédit sont plus stationnaires dans le régime de volatilité élevée. De même, durant ce régime l'effet de niveau est plus important. Enfin, les cycles de volatilité élevée des obligations très risquées sont liés aux cycles économiques et de défaut. Pour les obligations moins risquées (cotées Baa et plus), les cycles de primes de crédit ne sont pas nécessairement liés aux cycles économiques et de défaut.

**Mots clés:** Prime de crédit, risque de défaut, prime de risque, puzzle de primes de crédit, changement de régime, volatilité stochastique, modèle de consommation.

# Abstract

An important research question examined in the recent credit risk literature focuses on the proportion of corporate yield spreads which can be attributed to default risk. Past studies have verified that only a small fraction of corporate spreads can be explained by default risk (Elton, Gruber, Agrawal and Mann, 2001 and Huang and Huang, 2003). This question is now associated to the credit spread level puzzle (Chen, Collin-Dufresne and Golstein, 2005; Driessen, 2005; see Amato and Remolona, 2003 for a review).

In the first essay, we re-examine the proportion of the corporate yield spread attributed to default risk in the light of the different issues associated with the computation of transition and default probabilities obtained from historical rating transition data. One main finding of our research is that the estimated default-risk proportion of corporate yield spreads is highly sensitive to the term structure of default probabilities estimated for each rating class. Moreover, this proportion can become a large fraction of the yield spread when sensitivity analyses are made with respect to recovery rates, default cycles in the economy, and information considered in the historical rating transition data.

In the second essay, we introduce both a correlation between risk free rates and default intensity and a regime shift risk to explain corporate yield spreads. Our approach considers a risk premium related to macroeconomic factors by using a pricing kernel with discrete regime shifts as in Bansal and Zhou (2002) and observed macroeconomic factors as in Evans (2003). Therefore, we contribute to the literature by assessing the ability of observed macroeconomic factors and the possibility of changes in regimes to explain the default risk proportion in yield spreads. For this purpose, we extend the Markov Switching risk-free term structure model of Bansal and Zhou (2002) to the corporate bond setting and develop recursive formulas for default probabilities, risk-free and risky zero-coupon bond prices. The model is calibrated with consumption, inflation, risk-free yield and default data over the 1987-1996 period. Empirical results show that default

probabilities and default spreads exhibit a regime shift. On average the default spread explains 41% of the observed corporate yield spread for industrial Baa during the 1987-1996 period for ten year zero-coupon bonds. The average default spread proportion is 48.5% in the high default regime and 35.5% in the low default regime. More interesting, the default spread proportion can attain 72.4% in the high default regime and 25% in the low default regime.

In the third essay, we investigate the dynamics of corporate bond spreads. Most reduced form models used diffusion models with a level effect for default intensity and a constant recovery rate. However, these basic models seem to be inappropriate since corporate bond spreads can exhibit a structural change in the level and the volatility in the high default cycle. Moreover, many studies showed that macroeconomic conditions affect observed loss given default as well as default probabilities (see Allen and Saunders, 2003 for a review). In this paper, we propose and estimate a comprehensive model for corporate credit spreads volatilities that includes, simultaneously, the level and the GARCH effects in addition to regime shifts.

Empirical results show that the level and the GARCH components are essential in modeling the conditional volatility of corporate spreads. The level elasticity parameter is found to be in accordance with the square-root specification used in reduced form models except for long term non-investment grade bonds where the elasticity level is higher than 0.5. The conditional volatility is more persistent for non-investment grade spreads. Negative and positive unexpected corporate spreads have the same effect on the conditional volatility except for A short term corporate spreads, for which positive shocks lead to an increase in conditional volatility while negative shocks have no influence on the conditional volatility.

Corporate spreads are found to exhibit regime shifts between a high volatility regime and a low volatility regime. The high volatility regime has lower (higher) average spread for investment (non-investment) grade bonds. In the high volatility regime, corporate spreads are more stationary than in the low volatility regime. For all corporate spreads, the elasticity parameter is higher in the high volatility regime. Non-investment grade spread regimes are related to economic and default cycles. In contrast, there is a

weak relation between investment grade credit spread regimes and economic and default cycles. Performance of different model specifications in forecasting volatility and matching unconditional moments are also compared.

**Keywords:** Corporate spread, Default risk, default risk premium, credit spread level puzzle, regime shift, stochastic volatility, consumption-based approach.

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## Chapter 2: Default Risk, Default Risk Premium and Corporate Yield Spreads



## Chapter 3: Regime Shift in Corporate Bond Spreads

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# Default risk in corporate yield spreads

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**Abstract**

An important research question examined in the recent credit risk literature focuses on the proportion of corporate yield spreads attributed to default risk. We reexamine this topic in the light of the different issues associated with the computation of transition and default probabilities obtained with historical data. One significant finding is that the estimated default-risk proportion of corporate yield spreads is highly sensitive to the term structure of the default probabilities. This proportion can become a large fraction of the yield spread when sensitivity analyses are made with respect to recovery rates, default cycles, and information considered in the historical rating transition data.

Keywords: Default risk, corporate yield spread, recovery rate, default probability, data filtration, default cycle.

JEL classification: G24, G32, G33.

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# 1 Introduction

An important research question studied in the recent credit risk literature examines the proportion of corporate yield spreads explained by default risk. This question is not only important for the pricing of bonds and credit derivatives but also for computing banks' optimal economic capital for credit risk (Crouhy, Galai, and Mark, 2000; Gordy, 2000). For example, in CreditMetrics (1997), all corporate yield spreads are assumed to be explained by default risk, so the implied capital charge for default risk can be too high if other factors enter in the composition of the spreads.

Elton, Gruber, Agrawal and Mann (Elton et al., 2001) have verified that only a small fraction of corporate yield spreads can be attributed to default risk or expected default loss. They got their result from a reduced form model and have shown that the expected default loss explains no more than 25% of corporate spot spreads. The remainder is attributed to a tax premium and a risk premium for systematic risk. More recently, Huang and Huang (2003) reached a similar conclusion with a structural model. They verified that, for investment-grade bonds (Baa and higher ratings), only 20% of the spread is explained by default risk.

One of the key inputs needed for such assessments is an estimate of the term structure of default probability, that is, the probability of defaulting for different time horizons. To get these quantities, databases on historical default frequencies from Moody's and Standard and Poor's can be used. For example, one can first come up with an estimate of transition probabilities between rating classes and then use them to compute the term structure of the default probability. This is the approach used in Elton et al. (2001). Although this method appears straightforward, obtaining probability estimates with such a procedure is not a trivial exercise. Many important issues arise in the process and the different choices made by the analyst might lead to different results.

A first choice to make when computing default and transition probabilities is the period over which the estimation is to be performed. As shown in Bangia et al. (2002), transition-matrix estimates are sensitive to the period in which they are computed. Business and credit cycles might have a serious impact on the estimated transition matrices and might lead to highly different estimates for the default-risk proportion.

A second issue calling for close attention concerns the statistical approach (Altman, 1998). Because defaults and rating transitions are rare events, the typical cohort approach used by Moody's and Standard and Poor's will produce transition probabilities matrices with many cells equal to zero. This does not mean that the probability of the cell is nil but that its estimate is nil. Such a characteristic could lead underestimates of the default-risk fraction in corporate yield spreads. Lando and Skodeberg (2002) have shown that a continuous-time analysis of rating transitions using generator matrices improves the estimates of rare transitions even when they are not observed in the data, a result that cannot be obtained with the discrete-time cohort approach of Carty and Fons (1993) and Carty (1997).

Finally, a third important issue arising in the computation of default and transition probabilities is the data filtering process which determines the information considered about issuers' movements in the database. For example, we must decide whether or not to consider issuers that are present at the beginning of the estimation period but leave for reasons other than default (withdrawn rating or right censoring). Another choice is whether or not to consider issuers that enter the database after the starting date of estimation. Again, these choices might have non-negligible impacts on the final estimates.

In this article, we revisit the topic of corporate rate spread decomposition in light of the above considerations. In a first step, we introduce a simple discrete-time model for estimating the proportion of the corporate rate spread attributable to default risk. This model is then implemented using estimates of transition probabilities from the cohort method and computed for different periods of the default cycle. The results show that matching the periods over which the probabilities are computed with the period over which the spreads are examined can raise the proportion of the corporate yield spreads attributable to default risk. For example, for A rated bonds, this proportion jumps from 9% to 23% (Table 3b, panel B, 1987-1991). For Baa bonds, this proportion jumps from 32% to 65% (Table 3b, panel C, 1987-1991). As we shall see in more detail, the 1987-1991 sub-period corresponds to a high default cycle in the bond market. The above results are confirmed by a the continuous-time analysis of transition ratings in line with Lando and Skodeberg (2002) (Table 3b, MDC columns). The analysis also highlights the importance of using the proper recovery rates when estimating the proportion of the yield spread attributed to default risk. For Baa bonds, for example, the above proportion of 65% corresponds to a recovery rate of 49%. This proportion jumps to 74% when the recovery rate is cut to 40%, a more plausible rate in a high default risk period (Table 5).

In a second step, we look to see how sensitive our results are to the modelling approach taken and to the information considered in the database. For this purpose, we introduce an alternative continuous-time model. We also use this model to compute the approximate confidence intervals, in the spirit of Christensen, Hansen, and Lando (2004). Three data filtering processes for different types of information are examined: the first includes issuers that enter after the starting date of estimation (entry firms hereafter) as well as withdrawn-rating observations; the second one excludes entry firm observations; and the third excludes both entry firms observations and withdrawn-rating observations. Our results —when excluding withdrawn-rating and entry firm observations— show that the average spread proportion attributed to default risk for a maturity of ten years climbs to 71% for Baa rated bonds during the 1987-1991 period, with a 95% approximate interval of 56% and 86% when the recovery rate is 49% (Table 8b).

The rest of the paper is organized as follows. In Section 2, we describe how the empirical bond-spread curves are estimated. In Section 3, we present the discrete-time model used to estimate the default proportion of the corporate yield spread for different rating categories and maturities. This section also presents the results on the default-risk proportion obtained with this model and examines their sensitivity to assumptions about default cycles, estimation methodology of probabilities and recovery rates. The sensitivity of our modelling approach is then examined in Section 4 with the use of a continuous-time model. This section also presents results including inference obtained by Monte Carlo simulations and a detailed sensitivity analysis about the information considered in the databases. Section 5 concludes.

## 2 Empirical bond-spread curves

### 2.1 *Database description*

The data come from the Lehman Brothers Fixed Income Database (Warga, 1998). We choose this data to enable comparisons with other articles in this literature using the same information. Moreover, the database covers two default cycles, an issue that will become important in the analysis. The data contains information on monthly prices (quote and matrix), accrued interest, coupons, ratings, callability and returns on all investment-grade corporate and government bonds for the period from January 1987 to December 1996. All bonds with matrix prices and options were eliminated; bonds not included in Lehman Brothers' bond indexes and bonds with an odd frequency of coupon payments were also dropped.[1] Appendix A.1, provides details on the treatment of accrued interest.

As in Elton et al. (2001), month-end estimates of the yield-spread curves on zero-coupon bonds for each rating class are needed to implement the models. These yield-spread curves are obtained by first estimating the parameters associated with the Nelson and Siegel curve fitting approach. Appendix A.2 provides a brief summary of this approach. All bonds with a pricing error higher than $5 are dropped. We then repeat this estimation and data removal procedure until all bonds with a pricing error larger than $5 have been eliminated. Using this procedure, 776 bonds were eliminated (one Aa, 90 A and 695 Baa) out of a total of 33,401 bonds found in the industrial sector, which is the focus of this study.

### 2.2 *Empirical analysis of bond-spread curves*

For each of the 120 months of this study, we first estimate the forward rate curve associated with government and private bonds.[2] The industrial corporate bonds are then grouped in three categories: Aa, A, and Baa. For each category, we estimate the corporate forward rate curves. Our results are coherent,

[1] We did, however, keep three categories from the list of eliminations in Elton *et al.*(2001), because we lacked the information needed to identify them: government flower bonds, inflation-indexed government bonds, and bonds with floating rate debt. As we shall see, this makes no difference in the results.

[2] Recent studies (see, for example, Hull *et al.* 2005) argued that the Treasury rate is not the appropriate risk-free-rate. We shall return to this issue in Section 4.3.

in that all of our estimated empirical bond-spread curves are positive. Moreover, the bond-spread curves between a high rating class and a lower rating class are also positive. Throughout this article, we shall use the expression “spot rate” to indicate the yield to maturity on a zero-coupon bond. Figure 1 shows the empirical spreads on industrial bonds of six years maturity for the ratings Aa, A, and Baa. We observe that the spreads are higher during the January 1987-December 1991 sub-period.

(Figure 1 here)

Table 1 reports our results for corporate yield spreads for two to ten years of maturity. The results are very close to those presented in Table 1 of Elton et al. (2001) for the industrial sector. The small discrepancies might be explained by differences in data sets and estimation algorithms. In panel A, the results cover the entire 10-year period, while in panels B and C they refer to two sub-periods of five years. It is important to observe that the average spreads are higher in the 1987-1991 sub-period than in the 1992-1996 sub-period. This difference will matter in the next sections where we explain the proportions of the corporate yield spreads associated with default risk. The two sub-periods represent two different default cycles. The 1987-1991 sub-period is usually associated with a high default cycle while the sub-period 1992-1996 tends to coincide with a low default cycle. The 1987-1991 period contains a macroeconomic recession and the US loan crisis.

Figure 2 presents default rates extracted from Moody’s database (Moody’s, 2005). We observe that the distribution of these rates over time has a shape similar to that of the empirical spreads for six years maturity in Figure 1. This suggests that the link between the default-risk proportion and the default risk should vary with the default cycles (see also Manning, 2004, for a similar conclusion).

(Figure 2 here)

Another interesting graph shown in Figure 3 examines how the average recovery rate varies with time. These recovery rates were obtained from Moody’s database (Moody’s, 2005) and are defined as the ratio of the defaulted bond's market price, observed 30-days after its default date, to its face value (par). The average recovery rates vary significantly with the default cycles defined above. For example, the average recovery rate for all bonds during the 1987-1991 sub-period is equal to 40.8% while that of the 1992-1996 sub-period is equal to 45.5%. Those for senior unsecured bonds vary between 45.5% and 50.66% for the same sub-periods. It is also documented in Moody's (2005) that the recovery rates are even lower for industrial bonds. Altman et al. (2003) present an interesting review of the literature on the link between recovery rate and default probability.

(Figure 3 here)

Table 2 compares the average root mean squared errors of the difference between theoretical bond prices computed using the Nielson-Siegel model and the actual bond prices for treasuries and industrial corporate bonds. Again our results are similar to those of Elton et al. (2001).

# 3 Discrete-time model

## 3.1 *Model*

The corporate yield spread is defined as the difference between the yield curves of the risky zero-coupon bond and the risk-free, zero-coupon bond. Therefore, to characterize corporate yield spreads, one need only to model the values of a risk-free and a corporate zero-coupon bond. The model developed here, unlike that of Elton et al. (2001), avoids specifying a coupon rate that might absorb effects unrelated to default risk. The model we propose thus focuses on zero-coupon bonds and assumes that a corporate yield spread might be totally explained by the recovery rate and the possibility of default. The model will be used to measure how much the observed corporate yield spread is explained by these two components.

The model is built on the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$. In this section, time is measured in discrete periods. Let $f(t, T)$ denote the risk-free, continuously compounded forward rate (annualized) at time $t$ for a loan that starts at period $T$ and ends one period later. The discount factor for the period $t$

to $T$ is $\beta(t,T) = \exp\left(-\sum_{s=t}^{T-1} f(s,s)\,\Delta_t\right)$ where $\Delta_t$ is the length (in years) of one period of time. In the following, it is assumed that:

**(i)** There exists at least one martingale measure $Q$ under which the discounted value of any risk-free, zero-coupon bond is a martingale.

**(ii)** Under the martingale measure $Q$, the default time $\tau$ of the corporate bond is independent of the risk-free, forward interest rates $f(t,T)$, $t \leq T$.

**(iii)** In case of default, a fraction $\rho_\tau$ of the market value of an equivalent risky bond is recovered at the default time. This fraction is a deterministic function of the default time.[3]

**(iv)** Under the martingale measure $Q$, the probability $q_s$ that the default will arise in $s$ periods from now will remain constant through time, that is, for $s = 1, 2, 3, \ldots$,

$$\begin{aligned} q_s &= Q\left[\tau = s \,|\tau > 0\right] \\ &= Q_t\left[\tau = t + s \,|\tau > t\right] \text{ for any } t, \end{aligned} \tag{1}$$

where $Q_t$ denotes the conditional probability indicated by the information (the $\sigma-$field $\mathcal{F}_t$) available at period $t$.

**(v)** Investors are risk neutral with respect to default risk.

Assumption (i) is needed to price a bond at its expected discounted payoff. Assumption (ii) is used to simplify the bond-value computations. We did not consider using the $Q-$ forward measure to relax this hypothesis since there are as many forward measures as there are zero-coupon bonds. Assumption (iii) differs from the ones found in different studies. The recovery of a fraction of the face value of the bond is not suitable for zero-coupon bonds because it would allow disproportionate cash flows with respect to the

[3] It is important to emphasize that the recovery is at the default time, as in Duffie and Singleton (1999). See Madan and Unal (1998) and Jarrow and Turnbull (1995) for alternative models.

bond price in case of early default. This, in turn, might give rise to negative spreads. When working with coupon-paying bonds, however, the recovery of a fraction of the face value can be used as a proxy for the recovery of a fraction of the market value because the latter is not too far from its face value. Assumption (iv) is not crucial to the model's application, and the model we propose can easily be extended to non-homogeneous default probabilities. This assumption is required only in the estimation stage of the model since there would otherwise be too many parameters to estimate. Finally, Assumption (v) implies that the law of the default time $\tau$ is the same under the objective measure $P$ and the martingale measure $Q$. This assumption justifies using databases containing information about default probabilities under the objective measure $P$ to estimate the parameters of the default distribution under the martingale measure $Q$ which is required in the model. Since, in practice, risk-averse investors usually require a default-risk premium, the impact of this hypothesis tends to underestimate the proportion of the spread that may be explained by default risk. Dionne et al. (2005) find that the default-risk premium proportion is low (4%)

**Theorem 1** *Under Assumptions (i), (ii), (iii), (iv) and (v) the time $t$ value $\widetilde{P}(t,T)$ of a corporate zero-coupon bond with a face value of one dollar and a time to maturity of $(T-t)$ periods is expressed as*

$$\widetilde{P}(t,T) = P(t,T)\, p_{t,T}, \tag{2}$$

*where $P(t,T)$ is the time $t$ value of an equivalent risk-free, zero-coupon bond, $p_{T,T}=1$,*

$$p_{t,T} = \left\{ \sum_{u=1}^{T-t} \rho_{u+t} p_{t+u,T} q_u + \left(1 - \sum_{u=1}^{T-t} q_u\right) \right\},\ t \in \{1,2,...,T-1\}, \tag{3}$$

*and $q_s$ is the probability, under the risk-neutral measure $Q$, that the default will occur in exactly $s$ periods from now.*

The result is established by using induction on the time to maturity. A complete proof may be found in Appendix B.1. The default-spread curve at time $t$ is therefore

$$S(t,T) = \frac{\ln P(t,T)}{\Delta_t (T-t)} - \frac{\ln \widetilde{P}(t,T)}{\Delta_t (T-t)} = -\frac{\ln p_{t,T}}{\Delta_t (T-t)}. \tag{4}$$

## 3.2 *Parameter estimation*

To compute the corporate yield spreads implied by Equation (4), one needs estimates of the recovery rates $\rho_t$ and the probabilities $q_s$, $s = 1, 2, 3, ...$, i.e. the term structure of default probabilities. As argued in the introduction, the statistical approach adopted for the estimation of the default probabilities can influence the estimated proportion of the corporate spreads attributable to default risk.

A first approach, which imposes little structure on the data, requires forming a cohort at a point in time and counting the defaults after one period, two periods, and so on. The drawback of such an approach stems from the large standard errors associated with the estimates. Generating accurate estimates needs the observation of many defaults, an unlikely possibility when working with investment grade bonds. For such a case, many estimated probabilities would simply be zero. This approach would also make it difficult to include the information provided by new firms entering the database.

Another approach found in the literature uses estimates of periodic transition matrices available from Moody's or Standard and Poor's via the cohort method of Carty and Fons (1993) and Carty (1997). The transitions from one credit rating class to another are counted and estimates of transition probabilities are calculated using the number of bonds in the cohort at the start of the period. Probabilities of defaulting for more than one period can then be conveniently computed from this transition matrix using simple matrix multiplications. This convenience comes at the cost of imposing a Markovian structure on the data and it is not clear if such a structure holds. As with the preceding approach, there are also several drawbacks associated with such estimates of default probabilities. Defaults and rating transitions are rare events and these transition matrices contain many cells with estimated probabilities equal to zero. This might lead to an underestimation of the default-spread. Again, as with the preceding approach, if one tries to build confidence intervals around these estimates, the results turn out to be unsatisfactory.

Recently, Lando and Skodeberg (2002) have suggested estimating a Markov-process generator rather than the one-year transition matrix. As with the cohort approach, this method also adds a Markovian structure not needed by the model we propose. Lando and Skodeberg (2002) have shown that this continuous-time analysis of rating transitions using generator matrices improves the estimates of rare transitions even when they are not observed in the data, a result that cannot be obtained with the discrete-time analysis of Carty and Fons (1993) and Carty (1997). So a continuous-time analysis of defaults permits estimates of default probabilities even for cells that have no defaults. This is possible because the approach draws on the information contained in the transition from one class to another to infer better estimates of the default probabilities. Finally, as shown in Christensen, Hansen, and Lando (2004), inference in such a framework is informative and can be conveniently computed.[4]

The next section will examine the proportion of the corporate spreads which can be attributed to default risk using the last two approaches described above for the estimation of the term structure of default probabilities. The recovery rate $\rho_\tau = \rho$ will be assumed constant through time. In Elton et al. (2001), $\rho$ is about 60% for Aa and A bonds and about 50% for Baa bonds. This section also illustrates how sensitive corporate default spreads are to the value of the recovery rate.

### 3.3 *Empirical results with discrete-time model*

Table 3a summarizes the different sources of information about the transition matrices used in this section. Table 3b summarizes our results on default spread proportions generated with the discrete-time model. The default spread proportions are measured as the fractions of the estimated default spreads over the observed credit spreads. In the Elton et al. (2001) panel, we estimate their model using transition matrices drawn from five different sources.[5] We repeat the same sensitivity analysis for the default-spread proportions, using our model estimated with zero-coupon bonds and recovery rates as a fraction of the market value at the default time (Dionne et al., 2005 panel). The results are similar to those in the Elton (2001) panel, the latter being obtained with a coupon rate set so that a 10-year bond with that coupon would be

[4] Other recent references on the estimation of migration matrices and the resulting inferences issues are Jafry and Schuermann (2004) and Hanson and Schuermann (in press).

[5] Table A1 in Appendix reproduces the default spreads analysis of Elton *et al.* (2001) using our data and their transition matrix. We see that the two data sets produce equivalent results.

selling close to par for all periods.

The first column of Table 3b in both panels presents the default-spread proportions obtained with a transition matrix calculated from Standard and Poor's data (SPEG). This matrix was estimated with the cohort approach over the 1981-1995 period (Altman, 1995). Notice that Elton et al. (2001) used this matrix to measure default risk over the entire 1987-1996 period.[6] Using this transition matrix over the 1987-1991 and 1992-1996 sub-periods produces larger default-spread proportions in the latter case simply because the empirical corporate yield spreads are smaller during those years (see Table 1). Moreover, because this transition matrix underestimates the defaults during the high default cycle period of 1987-1991, it also underestimates the default-spread proportions for that period.

Column MDEG repeats the analysis using a matrix estimated over the 1970-1992 period with Moody's data. This matrix is also used in Elton et al. (2001). We observe that the corresponding default-spread proportions are, in general, lower than for the Standard and Poor's matrix, most likely because the two matrices were estimated over two different periods. The results presented in Table 1 indicate clearly that the estimated corporate yield spreads are very different in the two sub-periods. It might thus be more appropriate to use different transition matrices estimated with data corresponding to these two different sub-periods.

The MDB1 column reports the estimates obtained with transition matrices computed over the corresponding sub-periods with Moody's database. The matrices are one-year transition-matrix estimates obtained with the cohort approach described in, for example, Christensen et al. (2004) equation (1). It should be noticed that splitting the sample in high and low default periods might amplify (or shrink) the proportions in the

[6] Elton et al. (2001) were aware of this fact: "... our transition matrix estimates are not calculated over exactly the same period for which we estimate the spreads. However there are three factors that make us believe that we have not underestimated default spreads." (p. 263) We must emphasize that Huang and Huang (2003) also used a single transition matrix estimated over the 1970-1998 period (Keenan et al., 1999) and a single recovery rate of 51.3% for all ratings (Altman and Kishore, 1996) to calibrate their model. Our discussion is more related to Elton et al. (2001) contribution because we are concerned with a reduced form model.

high (low) default sub-period. For example, in a high default sub-period, we assume that a ten-year bond is priced with probabilities from the high default period even if this period is not expected to last for ten years. The reverse effect might also be obtained for the low default period. Low default probabilities are used to price a ten-year bond even if the low default cycle is not expected to last for ten years.

The fourth column (MDB2) also reports estimates made with cohort transition matrices computed with Moody's database on the corresponding sub-periods but now considering different information than in the MDB1 case. The data considered here is obtained using the filtering approach suggested in Christensen et al. (2004) [7]. The specific details regarding this approach are presented in Appendix A3.

The results in columns MDB1 and MDB2 are similar. It is interesting to observe that the default-risk proportion increases from 31.57% with the MDEG procedure to more than 60% with both MDB1 and MDB2 procedures for Baa bonds during the 1987-1991 sub-period. We also observe that the default risk proportion is significantly lower in the 1992-1996 sub-period which is associated with a much lower default cycle. The main difference with previous studies is that appropriate transition matrices are matched with the different default cycles.

The fifth column (MDC) uses the same data and filtering technique as for the MDB2 column and applies the estimation method of Lando et al. (2002) using continuous-time credit migration. The estimated generators are presented in Table A2. The default-spread proportions do not differ significantly with those computed for Aa bonds using the discrete cohort method (MDB2) for a ten-year period (Table 3b) but are slightly lower for A and Baa bonds. Some important differences appear also when we consider shorter periods and higher credit ratings. Table 3c shows that the average default-spread proportions are higher with the MDC method of estimation when we look at industrial Aa and A ratings for two-year periods. These results are confirmed in Table 4a. We observe that, for these rating categories, the implied default probabilities are much higher in the 1987-1991 sub-period with the continuous-time transition matrix ap-

[7] With one exception that does not yield a significant difference. They used the senior unsecured bond rating to construct the issuer rating while we used the estimated senior unsecured issuer rating given by Moody's database.

proach (MDC_1). The main reason is that traditional methods of estimation yield zero estimates of short term default probabilities for higher rating classes whatever the default cycle.

It is important to note that the implied default probabilities in one year are much higher than those reported in Table 3 of Christensen et al. (2004) for the same 1987-1991 sub-period. For the Aa rating, we obtain a default probability of 0.0116 while they obtained a default probability of 0.00159 (in percentage), which is 10 times lower. For A and Baa, we respectively obtain 0.027 and 0.29, while they obtained 0.006 and 0.05. The differences are related to the data filtering process.[8] In their paper, they went back to initial entry of the firm even when this happened before 1987 to facilitate the comparison with the extended model in the second part of their article. When this backward looking method is not applied to the data, the default probabilities are comparable (details are available upon request).

Finally, Table 5 presents a sensitivity analysis of default-spread proportions with respect to recovery rates. We observe that the results are very sensitive to the different values chosen. For example, with a recovery rate of 40% (which is the rate often observed in high default cycles for industrial bonds), the default-spread proportion can increase up to about 74% in the industrial Baa rating class of corporate bonds when a discrete-time analysis of the data is done for the 1987-1991 sub-period. With the continuous-time analysis, it goes up to 70%. A sensitivity analysis with respect to coupon rates (not reported here) also shows that using bonds paying coupons does affect the results, which supports our approach of using zero-coupon bonds.

# 4 Continuous-time model

## 4.1 *Model*

To test the robustness of the results derived from our theoretical model, we examine here an alternative model specification where default can occur on a continuum of dates. This departs from the analysis of the previous section which relies on estimates of default probabilities for different discrete horizons.

[8] We thank David Lando for providing this explanation.

In this section, time is no longer expressed in terms of the number of periods but as a continuous function. As shown in Theorem 2, the model presented here depends on a recovery rate $\rho$ and the intensity $\{\lambda_t : t \geq 0\}$ associated with the distribution of $\tau$, the default time. The risk-free discount factor for the time interval $(t, T]$ is $\beta(t,T) = \exp\left(-\int_t^T f(s,s)\ ds\right)$ where $f(t,T)$ denotes the instantaneous continuously compounded risk-free forward rate. In the following, it is assumed that:

**(i)** There exists at least one martingale measure $Q$ under which the discounted value of any risk-free, zero-coupon bond is a martingale.

**(ii)** Under the martingale measure $Q$, the default time $\tau$ of the corporate bond is independent of the risk-free, forward interest rates $f(t,T)$, $t \leq T$.

**(iii)** In case of default, a constant fraction $\rho$ of the market value of an equivalent risky bond is recovered at the default time.

**(iv*)** Under the martingale measure $Q$, the intensity process $\{\lambda_t : t \geq 0\}$ of the default time $\tau$ is deterministic but may vary through time.

**(v)** Investors are risk neutral with respect to default risk.

The first three assumptions are identical to those used in the discrete-time model. Assumption (iv*) can be relaxed easily, since any intensity process that satisfies the requirements of the Duffie-and-Singleton (1999) model is acceptable. However, the expression given in Theorem 2 will be modified. Finally, assumption (v), which implies that the distribution of $\tau$ will remain the same under the empirical probability measure $P$ and the martingale measure $Q$, is again required for the use of databases containing information about default probabilities.

**Theorem 2** *Under the Assumptions (i), (ii), (iii), (iv*) and (v), the time $t$ value of a corporate zero-coupon* *bond is*

$$\widetilde{P}(t,T) = P(t,T)\exp\left(-(1-\rho)\int_t^T \lambda_s ds\right). \tag{5}$$

This result is a particular case of the Duffie-and-Singleton model (1999) and the proof is in Section B.2. In this particular case, the corporate yield spread curve at time $t$ is given by

$$S(t,T) = \frac{\ln P(t,T)}{T-t} - \frac{\ln \widetilde{P}(t,T)}{T-t} = \frac{1-\rho}{T-t}\int_t^T \lambda_s ds. \tag{6}$$

For practical reasons such as the estimation of the model, we need to impose some additional structure on the distribution of $\tau$. This is summarized in the following additional hypothesis:

**(iv**)** Under the martingale measure $Q$, the default time $\tau$ is driven by a time-homogeneous Markov process $X$ modelling the credit rating migrations of the firm. This Markov process $X$ is characterized by the generator matrix $\mathbf{\Lambda}$ and we assume that $\mathbf{\Lambda}$ is diagonable.

In this context, the intensity is

$$\lambda_t = \frac{\sum_{k=1}^m a_k d_k \exp(d_k t)}{1 - \sum_{k=1}^m a_k \exp(d_k t)} \tag{7}$$

where the constants $d_1, ...d_m$ are the eigen values associated with the generator matrix $\mathbf{\Lambda}$ and the constants $a_1, ..., a_m$ are functions of the components of the eigen vectors of $\mathbf{\Lambda}$ and are described explicitly in the proof found in Appendix B.3.

## 4.2 *Parameter estimation*

To perform comparisons similar to those made for the discrete-time model, we must have available estimates of the generators associated with the different transition matrices used in the analysis. However, as shown in Israel et al. (2001), the existence of such a generator for a given transition probability matrix is not guaranteed. We therefore borrow the procedure suggested in Israel et al. (2001) to verify the existence of an underlying generator for the transition matrices used in the previous sections. The results show that such a generator does not exist for the transition matrices from Standard and Poor's and Moody's used in Tables 3 and 4 (SPEG and MDEG) nor for the one computed from Moody's database (MDB).

As proposed in Israel et al. (2001), a possible solution to this problem is to obtain a generator that will produce a transition matrix close to the original transition matrix. We therefore follow their procedure to obtain the estimated generators. Using these estimates, we next compute the intensities with equation (7). The spreads are then computed using the following discrete approximation of equation (6):

$$\frac{1-\rho}{T-t}\int_t^T \lambda_s ds \cong \frac{1-\rho}{n}\sum_{j=1}^{n}\hat{\lambda}_{j\Delta t} \tag{8}$$

with $\Delta t = (T-t)/n = 10^{-6}$ and $\widehat{\lambda}_j$ being the estimated default intensity process.

### 4.3 *Empirical results with continuous-time model*

Table 6 presents the results associated with the continuous-time model and compares them to those obtained with our discrete-time model. As shown in this table, the results are robust to the model specification. We should mention that these results and those obtained with the discrete-time model should be interpreted in light of the risk neutrality assumption made to justify using databases with information about default probabilities under the objective measure. Relaxing this hypothesis would require, as reported in some studies (Duffie et al., 2003, and Jarrow and Purnanandam, 2003), scaling up the martingale intensity when we consider the jump risk premium. This scaling would yield even higher proportions of estimated default spread in credit spread. Unfortunately, the correct scaling factors are unknown and model dependent. However, the effect that an uniform scaling would produce can be assessed by noticing that the spread is a linear function of the intensity in the continuous-time model.[9]

We now report the sensitivity of our results to the data filtering process and the information considered when computing the transition matrix. Such an analysis is important for financial institutions that are building their own internal rating system for Basel II and for the regulators who will have to monitor these

[9] See Driessens (2005) for the analysis of different default risk premia. According to his results, the jump risk premium is not statistically different from zero.

systems. Approximate confidence intervals obtained by simulation are also reported in these tables. These intervals are approximate because the simulation procedure uses the estimated generator as if it were the true generator. The statistical uncertainty associated with this estimate is not taken into account. Hence, the intervals presented in this table are smaller than genuine confidence intervals. Appendix C gives the details of the simulation procedure and Table 7 reports the distribution of issuers by rating at the starting date of the simulation period.

Tables 8a, b, and c report results for the default spread proportions while Tables 9a, b, and c reports those for the estimated default probabilities. The main difference between Tables 8a, b, and c is in the treatment of the data. Table 8a uses censored data (withdrawn rating) and issuers that enter after the starting date (entry firms) of estimation. In Table 8b, we excluded withdrawn-rating and entry firm data. In Table 8c, we excluded only entry firm data. The same treatments are applied for Tables 9a, b, and c. As the results show, important differences are observed. Tables 8b and 9b report higher default proportions and higher default probabilities. As already mentioned, these tables exclude withdrawn-rating and entry firms and are more in the spirit of the standard cohort analysis of Moody's while Tables 8c and 9c include withdrawals. Moody's also produces statistics including withdrawals (right censoring). We observe, from Table 10, that the number of defaults is the same in panels b and c while the numbers of issuers and rating observations are higher in panel c. Inclusion of the withdrawals reduces default probabilities and default risk proportions in yield spreads. The same conclusion is obtained when entry firms are added (Tables 8a and 9a). Default-risk proportions and implied default probabilities are even lower. Two alternative explanations are possible: either the treatment of firms that enter and leave is not free of bias or the censored and entry issuers represent different default risks. Additional research is needed to discriminate between the two explanations.

Finally, Table 11 examines possible explanations for the low default-risk proportion in yield spreads for the 1992-1996 period. As recently argued in Hull, Predescu and White (2005), Treasury rates might not be proper benchmarks. Regulation on this market might artificially raise the demand for these bonds and reduce their yields.

Since, in theory, Credit Default Swaps (CDS) spreads should be close to the credit spread of the liquid bonds issued by the reference entity over the risk-free rate, Hull et al. (2005) used the credit default swap data to estimate the benchmark risk-free rate used by participants in credit markets. Their main result is that the benchmark five-year, risk-free rate is on average about 83% of the way from the Treasury rate to the swap rate. We use this result to approximate the benchmark risk-free rate for all maturities during the 1992-1996 period and to assess whether the low default-risk proportions we got for that period can be attributed to our Treasury benchmark. The results are reported in Table 11. Although the estimated proportions are higher, they are far below those anticipated.

Some authors have suggested liquidity risk as a possible explanation of yield spreads (see, for example, Longstaff, 2003). As argued by Duffie and Singleton (2003), liquidity risk should not be an important factor when using quote prices since that are not affected by trading size. Another benchmark could also be the Aaa rating to eliminate tax and some of the possible residual liquidity explanations. The results in Table 11 show that this does not substantially affect the results.

# 5 Conclusion

We have revisited the estimation of default-risk proportions in corporate yield spreads. Past studies have found that only a small proportion of the spreads can be attributed to default risk. We find here that such results do not hold for all periods of the default cycle. It has been documented that the 1987-1991 period corresponds to a high default cycle, while the 1992-1996 period corresponds to a low default cycle. When the transition matrices are appropriate for the default cycle under examination, substantial differences in the results are observed. The estimated proportions can reach 71% of the estimated spread for maturities of ten years for Baa bonds during the 1987-1991 period, with an approximate confidence interval of 56% to 86% when the recovery rate is 49% (Table 8b). This conclusion is important for financial institutions planning to use internal rating systems and for the regulators that will have to monitor these systems.

We have also shown that the results can vary substantially when changing the recovery rate assumption. When the recovery rate is cut to 40%, a rate that seems more appropriate for industrial bonds in high default cycles, the above proportion for Baa bonds goes up to 83%, with approximate confidence intervals of 67% to 98% (computation details are available upon request). The default-spread proportions are multiplied by 1.5 on average when the recovery rate drops from 60% to 40% (Table 5). The default-risk proportion is linear on recovery rate because, in our model, default intensity is not stochastic.

Our study could be extended in several directions by relaxing some of the restrictive assumptions that have been used. First, the assumption of risk neutrality could be relaxed. The computation of risk-neutral probabilities different from the default probabilities under the objective measure could then be obtained. Building confidence intervals around such estimates might produce results that would leave a tight fit for taxes once liquidity premia are taken into account. This would produce results consistent with the vast and successful literature on derivative securities in which the inclusion of taxes has been found to be of little help.

Finally, it should be noticed that we have observed substantial increases in the estimated proportion in the high default cycle only. The results in the low default cycle maintain that a small proportion of the spread is attributable to the default risk. As a result an important question remains: Why are the proportions of yield spread associated to default risk so low in low credit-risk cycles? This puzzle is discussed in recent contributions from the literature. It is now labelled the "credit spread level puzzle" (Chen, Collin-Dufresne, and Goldstein, 2005). It has been suggested that macroeconomic or Fama-French factors could explain the common variations of credit spreads (Elton and Gruber, 2001; Driessen, 2005; Chen et al., 2005). Time variation in risk premia is another possible explanation (Dionne et al., 2005), as well as non-Markov effects associated with hidden exited states (Christensen et al., 2004).

# Appendices

# A Parameters' estimation

## A.1 *Treatment of accrued interest*

We show that accrued interest is calculated one day after the transaction date and not on the transaction date after February 1991. As illustration, we provide three examples. First, when the date of the next coupon payment is the same as the transaction date and if it is not an odd coupon payment, the accrued interest in the Warga (1998) database is equal to zero before February 1991 and equal to the accrued interest of one day after February 1991. Second, when the date of the next coupon payment is the day following the transaction date and if it is not an odd coupon payment, the accrued interest in the database is equal to the coupon amount minus the accrued interest of one day before February 1991 and equal to zero after February 1991. Finally, for the other bonds, the accrued interest in the database is equal to the theoretical accrued interest (defined below) before February 1991 and equal to the theoretical accrued interest plus the accrued interest of one day after February 1991.

In order to get a similar accrued interest for the entire period, we calculated the theoretical accrued interest which is the same as that in the database for the first period (01-1987 to 02-1991) and we corrected the accrued interest for the second period (03-1991 to 12-1996). The theoretical accrued interest (AI) is given by this formula:

$$\mathrm{AI} = \frac{nC}{N}$$

where $n$ is the number of interest-bearing days, $N$ is the number of days between two successive coupons[10] and $C$ is the semiannual coupon amount.

In the database there are two methods of calculating the parameters $n$ and $N$. For government bonds it is the actual/actual method. For corporate bonds it is the 30/360 method. Let $(d_1, m_1, y_1), (d_2, m_2, y_2)$ and $(d_3, m_3, y_3)$ represent, respectively, the date from which accrued interest is calculated, the settlement date, and the relevant interest payment date with $d_i$ the day's number from 1 to 31, $m_i$ the month number from 1 and 12 and $y_i$ the year number. The parameters $n$ and $N$ are given by the next formula for the actual/actual method:

$$\begin{array}{ll} n & = (d_2, m_2, y_2) - (d_1, m_1, y_1) \\ N & = (d_3, m_3, y_3) - (d_1, m_1, y_1) \end{array}$$

and by the next formula for the 30/360 method:

$$\begin{array}{l} n = \left(\tilde{d}_2 - \tilde{d}_1\right) + 30\,(m_2 - m_1) + 360\,(y_2 - y_1) \\ N = 180 \end{array}$$

with

$$\tilde{d}_1 = \begin{cases} 30 & : \quad m_1 \neq 2 \text{ and } d_1 = 31 \\ 30 & : \quad m_1 = 2 \text{ and } d_1 \geq 28 \\ d_1 & : \quad \text{otherwise} \end{cases}$$

and

$$\tilde{d}_2 = \begin{cases} 30 & : \quad m_2 \neq 2 \text{ and } d_2 = 31 \\ 30 & : \quad m_1 = m_2 = 2,\ d_2 \geq 28 \text{ and } d_1 \geq 28 \\ d_2 & : \quad \text{otherwise} \end{cases} .$$

## A.2 *The Nelson-Siegel (1987) model*

The empirical corporate yield spreads are obtained using Nelson and Siegel's approach which parameterize the instantaneous forward rate as

$$f_{NS}(t, T) = a_t + b_t \exp\left(-\frac{T-t}{\eta_t}\right) + c_t \frac{T-t}{\eta_t} \exp\left(-\frac{T-t}{\eta_t}\right)$$

[10] Or the emission date and the first coupon date if the transaction date falls before the first coupon payment date.

where $a_t$, $b_t$, $c_t$ and $\eta_t$ are constants to be estimated for every month and rating class. The time $t$ value of a zero-coupon bond paying one dollar at maturity can then be written as

$$P_{NS}(t,T) = \exp\left\{-\alpha_t (T-t) - \beta_t \eta_t \left(1 - \exp\left(-\frac{T-t}{\eta_t}\right)\right) - \gamma_t (T-t) \exp\left(-\frac{T-t}{\eta_t}\right)\right\},$$

where $\alpha_t = a_t$, $\beta_t = b_t + c_t$ and $\gamma_t = -c_t$.

For a given date $t$, we observe $n$ bond prices $V_{obs}(t,T_1), ..., V_{obs}(t,T_n)$. Using Nelson-Siegel parametrization and the fact that a coupon bond[11] can be expressed as a portfolio of zero-coupon bonds, we can write the $i$th bond price as:[12]

$$V_{NS}(t,T_i) = C_i \sum_{k=1}^{K_i} P_{NS}(t,T_{i,k}) + P_{NS}(t,T_{i,K_i}) \tag{9}$$

where $C_i$ is the coupon rate of the $i^{th}$ bond, $T_i$ is its maturity date, $K_i$ is its number of remaining coupons and $T_{i,1}, ..., T_{i,K_i}$ are the coupon dates with $T_{i,K_i} = T_i$. Note that $V_{NS}(t,T_i)$ is a function of the known quantities $C_i$, $K_i$, $T_{i,1}, ..., T_{i,K_i} = T_i$ and of the unknown quantities $\alpha_t$, $\beta_t$, $\gamma_t$ and $\eta_t$. For each date $t$ in our sample, we choose $\alpha_t$, $\beta_t$, $\gamma_t$ and $\eta_t$ by minimizing the objective function

$$g(\alpha_t, \beta_t, \gamma_t, \eta_t) = \sum_{i=1}^{n} \left(V_{NS}(t,T_i) - V_{obs}(t,T_i)\right)^2$$

using the Levenberg-Marquardt algorithm (Marquardt, 1963). To minimize the chances of converging to a local rather than a global minimum, a grid search of $20^4 = 160\ 000$ points is performed to find a suitable starting point for the numerical minimization.

## A.3 *Data description for transition matrix estimation*

The rating transition histories used to estimate the generator are taken from Moody's Corporate Bond Default Database (January, 09, 2002). We consider only issuers domiciled in the United States and having at least one senior unsecured estimated rating. We started with 5,719 issuers (in all industry groups) with 46,305 registered debt issues and 23,666 ratings observations. For each issuer we checked the number of default dates in the Master Default Table (Moody's, January, 09, 2002). We obtained 1,041 default dates for 943 issuers in the period 1970-2001. Some issuers (91) had more than one default date. In the rating transition histories, there are 728 withdrawn ratings that are not the last observation of the issuer. Theses irrelevant withdrawals were eliminated and so we obtained 22,938 ratings observations. Table A3 compares our data set with that of Christensen et al. (2004).

The most important and difficult task is to get a proper definition of default. In order to compare our results with recent studies, we treat default dates as do Christensen et al. (2004). First, all the non withdrawn-rating observations up to the date of default have typically been unchanged. However, the ratings that occur within a week before the default date were eliminated.

Rating changes observed after the date of default were eliminated unless the new rating reached the B3 level or higher and the new ratings were related to debt issued after the date of default. In theses cases we treated theses ratings as related to a new issuer. It is important to emphasize that the first rating date of the new issuer is the latest date between the date of the first issue after default and the first date we observe an issuer rating higher than or equal to B3.

The same treatment is applied for the case of two and three default dates. Finally, few issuers have a registered default date before the first rating observation in the Senior Unsecured Estimated Rating Table (Moody's, January, 09, 2002). In theses cases, we considered that there was no default. With this procedure we got 5821 issuers with 965 default dates.

We aggregated all rating notches and so we got the nine usual ratings Aaa, Aa, A, Baa, Ba, B, Caa-C, Default and NR (Not Rated) with 15,564 rating observations.

[11] Our data set contains mainly coupon bonds.

[12] Note that equation (9) implicitely assumes that the recovery rate is at the default time. See Jarrow and Turnbull (1995) for a similar equation.

# B Proofs

## B.1 *Proof of theorem* 1

In this section, we determine the value $\widetilde{P}(t,T)$ of a corporate zero-coupon bond in the discrete-time setting described in Section 3. The value $\widetilde{P}(t,T)$ of the corporate bond is expressed as the expectation, under the martingale measure $Q$, of its discounted cash flows :

$$\begin{aligned}
&\widetilde{P}(t,T)\\
&= \mathrm{E}_t^Q\left[\sum_{s=t+1}^{T-1}\beta(t,s)\,\rho_s\widetilde{P}(s,T)\,\mathbf{1}_{\tau=s}+\beta(t,T)\left(\mathbf{1}_{\tau>T}+\rho_T\mathbf{1}_{\tau=T}\right)\right]\\
&= \sum_{s=t+1}^{T}\rho_s\mathrm{E}_t^Q\left[\beta(t,s)\,\widetilde{P}(s,T)\,\mathbf{1}_{\tau=s}\right]+\underbrace{\mathrm{E}_t^Q\left[\beta(t,T)\right]}_{=P(t,T)}\;\underbrace{\mathrm{E}_t^Q\left[\mathbf{1}_{\tau>T}\right]}_{Q_t[\tau>T|\tau>t]}\\
&= \sum_{s=t+1}^{T}\rho_s\mathrm{E}_t^Q\left[\beta(t,s)\,\widetilde{P}(s,T)\,\mathbf{1}_{\tau=s}\right]+P(t,T)\;\left(1-\sum_{u=1}^{T-t}q_u\right).
\end{aligned}$$

where the second equality hinges on the independence between the default time $\tau$ and the risk-free forward rate $f$.

The remainder of the proof is based on the induction on the time to maturity. Indeed, if there is only one period before maturity, then the value of the corporate zero-coupon bond is

$$\begin{aligned}
\widetilde{P}(T-1,T) &= \rho_T\mathrm{E}_{T-1}^Q\left[\beta(T-1,T)\,\mathbf{1}_{\tau=T}\right]+P(T-1,T)\;\;(1-q_1)\\
&= \rho_T P(T-1,T)\,q_1+P(T-1,T)\;\;(1-q_1)\\
&= P(T-1,T)\underbrace{\{1-(1-\rho_T)\,q_1\}}_{=p_{T-1,T}},
\end{aligned}$$

and for $t<T-1$, the value of the corporate zero-coupon bond is

$$\begin{aligned}
\widetilde{P}(t,T) &= \sum_{s=t+1}^{T}\rho_s\mathrm{E}_t^Q\left[\beta(t,s)\,\widetilde{P}(s,T)\,\mathbf{1}_{\tau=s}\right]+P(t,T)\;\left(1-\sum_{u=1}^{T-t}q_u\right)\\
&= \sum_{s=t+1}^{T}\rho_s\mathrm{E}_t^Q\left[\beta(t,s)\,P(s,T)\,p_{s,T}\mathbf{1}_{\tau=s}\right]+P(t,T)\;\left(1-\sum_{u=1}^{T-t}q_u\right)\\
&= \sum_{s=t+1}^{T}\rho_s p_{s,T}\mathrm{E}_t^Q\left[\beta(t,s)\,P(s,T)\right]\mathrm{E}_t^Q\left[\mathbf{1}_{\tau=s}\right]+P(t,T)\;\left(1-\sum_{u=1}^{T-t}q_u\right)\\
&= \sum_{s=t+1}^{T}\rho_s p_{s,T}P(t,T)\,q_{s-t}+P(t,T)\;\left(1-\sum_{u=1}^{T-t}q_u\right)\\
&= P(t,T)\underbrace{\left\{\sum_{u=1}^{T-t}\rho_{u+t}p_{t+u,T}q_u+\left(1-\sum_{u=1}^{T-t}q_u\right)\right\}}_{=p_{t,T}}
\end{aligned}$$

where the second equality comes from the use of the induction hypothesis and the third one is justified by the independence between the default time $\tau$ and the risk-free forward rate $f$. Q.E.D.

## B.2 *Proof of theorem* 2

In this section, the value of a corporate zero-coupon bond is determined in the continuous-time setting described in Section 4. Recall that, in case of default, the bondholder recovers, at time $\tau$, a fraction of the market value of an equivalent bond. The value of the corporate zero-coupon bond is expressed as the expectation, under the martingale measure $Q$, of its discounted payoff :

$$\begin{aligned}
\widetilde{P}(t,T) &= \mathrm{E}_t^Q\left[\beta(t,T)\,\mathbf{1}_{\tau>T} + \beta(t,\tau)\,\rho\widetilde{P}(\tau,T)\,\mathbf{1}_{\tau\leq T}\right] \\
&= \mathrm{E}_t^Q\left[\exp\left(-\int_t^T \left[f(s,s) + (1-\rho)\,\lambda_s\right] ds\right)\right] \text{(from Duffie and Singleton (1999))} \\
&= \underbrace{\mathrm{E}_t^Q\left[\exp\left(-\int_t^T f(s,s)\,ds\right)\right]}_{=P(t,T)} \exp\left(-(1-\rho)\int_t^T \lambda_s ds\right). \quad \text{Q.E.D.}
\end{aligned}$$

## B.3 *Proof for the form of $\lambda_t$ under assumption (iv**)*

If the generator matrix $\mathbf{\Lambda}$ is diagonable, then one can write $\mathbf{\Lambda} = \mathbf{PDP}^{-1}$ where the columns of the matrix $\mathbf{P}$ contain the eigen vectors of $\mathbf{\Lambda}$ and $\mathbf{D} = (d_i)$ is a diagonal matrix filled with the eigen values of $\mathbf{\Lambda}$. Let $\mathbf{Q}_t = (Q[X_t = j\,|X_0 = i])_{i,j=1,\ldots,m}$ denotes the transition matrix of the Markov process $X$. Then

$$\begin{aligned}
\mathbf{Q}_t = \exp(\mathbf{\Lambda}t) &= \sum_{k=1}^{\infty}\frac{(\mathbf{\Lambda}t)^k}{k!} = \sum_{k=1}^{\infty}\frac{\mathbf{PD}^k\mathbf{P}^{-1}t^k}{k!} = \mathbf{P}\exp(\mathbf{D}t)\,\mathbf{P}^{-1} \\
&= \left(\sum_{k=1}^{m} p_{ik}\exp(d_k t)\,p_{kj}^{-1}\right)_{i,j=1,\ldots,m}
\end{aligned}$$

where $p_{ij}$ are the components of $\mathbf{P}$, $p_{ij}^{-1}$ are the components of $\mathbf{P}^{-1}$, and the first equality is justified by the definition of the generator of a time-homogenous Markov process. Let $\tau_i$ be the default time of a firm initially rated $i$ and note that the default state corresponds to state $m$. The cumulative distribution of $\tau_i$ is $Q[\tau_i \leq t] = Q[X_t = m\,|X_0 = i]$. Therefore, the intensity associated with $\tau_i$ is

$$\lambda_{i,t} = \frac{\frac{\partial}{\partial t}Q[X_t = \text{ default}\,|X_0 = i]}{1 - Q[X_t = \text{default}\,|X_0 = i]} = \frac{\sum_{k=1}^{m} p_{ik}p_{km}^{-1}d_k\exp(d_k t)}{1 - \sum_{k=1}^{m} p_{ik}p_{km}^{-1}\exp(d_k t)}. \quad \text{Q.E.D.}$$

# C Approximate confidence interval computation

The first step of our simulation procedure computes a generator matrix for a given period using the maximum likelihood estimator given in Lando and Skodeberg (2002). Let the length of this period be $T$. The estimated generator is considered to be the *true* generator governing the data generating process. This does not take into account the statistical uncertainty associated with this estimate. Hence, this procedure produces approximate confidence intervals that will underestimate genuine confidence intervals that would account for this variability.

The second step uses the estimated generator obtained in the first step and the sample of issuers at the beginning of the period to simulate one rating history for each issuer. For each issuer with initial rating $i$, we simulate the waiting time for leaving this state with an exponential distribution having a mean equal to $\frac{1}{|\lambda_{ii}|}$, where $\lambda_{ii}$ are the elements of the generator matrix when $j = i$. If the waiting time is longer than period $T$, the issuer stays in its current rating for all the period. If the waiting time is shorter than $T$, we simulate a uniform distributed random variable between 0 and 1 to determine the issuer's next rating, using the migration intensities $\frac{\lambda_{ij}}{|\lambda_{ii}|}$ for all $j$ different from $i$ so that the migration intensity is different from zero. Then, we repeat the same task with the new rating until the cumulative of waiting times is greater than $T$ or the issuer gets default as a new rating. This procedure is carried out for each issuer having a rating at the beginning of the period. Using these rating histories for all issuers, a generator is estimated to obtain a term structure of default probabilities and an estimate of the average default-risk proportion in yield spreads for each of the maturities.

The second step is repeated 10,000 times to get 10,000 estimates of average default risk proportion in yield spreads. We then compute different statistics (mean, median, percentiles 2.5 and 97.5 used as our approximate confidence intervals, and minimum and maximum) of average default proportion for each rating and maturity.

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**Table 1: Measured Corporate Bond Spreads**

This table reports the average corporate bond spreads from government bonds for industrial Aa, A and Baa corporate bonds and for maturities from two to ten years. Spot rates were computed using the Nelson-and-Siegel model with the Levenberg-Marquardt algorithm. Treasuries spot rates are annualized. Corporate bond spreads are calculated as the difference between the corporate spot rates and treasury spot rates for a given maturity. Panel A contains the average treasury spot rates and corporate bond spreads over the entire 10-year period. Panel B contains the averages for the first five years and panel C contains the averages for the second five years. Our results are compared to those of Elton et al. (2001). It is easily seen that results from both studies are substantially similar.

| | Elton & al.(2001) | | | | Dionne & al.(2005) | | | |
|---|---|---|---|---|---|---|---|---|
| Maturity | Treasuries | Aa | A | Baa | Treasuries | Aa | A | Baa |
| Panel A: 1987-1996 | | | | | | | | |
| 2.00 | 6.414 | 0.414 | 0.621 | 1.167 | 6.454 | 0.413 | 0.612 | 1.180 |
| 3.00 | 6.689 | 0.419 | 0.680 | 1.205 | 6.709 | 0.416 | 0.672 | 1.206 |
| 4.00 | 6.925 | 0.455 | 0.715 | 1.210 | 6.920 | 0.447 | 0.722 | 1.229 |
| 5.00 | 7.108 | 0.493 | 0.738 | 1.205 | 7.090 | 0.477 | 0.752 | 1.237 |
| 6.00 | 7.246 | 0.526 | 0.753 | 1.199 | 7.226 | 0.502 | 0.769 | 1.234 |
| 7.00 | 7.351 | 0.552 | 0.764 | 1.193 | 7.337 | 0.526 | 0.776 | 1.224 |
| 8.00 | 7.432 | 0.573 | 0.773 | 1.188 | 7.426 | 0.548 | 0.779 | 1.210 |
| 9.00 | 7.496 | 0.589 | 0.779 | 1.184 | 7.500 | 0.569 | 0.778 | 1.193 |
| 10.00 | 7.548 | 0.603 | 0.785 | 1.180 | 7.562 | 0.590 | 0.776 | 1.174 |
| Panel B: 1987-1991 | | | | | | | | |
| 2.00 | 7.562 | 0.436 | 0.707 | 1.312 | 7.601 | 0.512 | 0.737 | 1.421 |
| 3.00 | 7.763 | 0.441 | 0.780 | 1.339 | 7.775 | 0.495 | 0.802 | 1.400 |
| 4.00 | 7.934 | 0.504 | 0.824 | 1.347 | 7.928 | 0.515 | 0.845 | 1.402 |
| 5.00 | 8.066 | 0.572 | 0.853 | 1.349 | 8.054 | 0.545 | 0.869 | 1.400 |
| 6.00 | 8.165 | 0.629 | 0.872 | 1.348 | 8.157 | 0.579 | 0.882 | 1.391 |
| 7.00 | 8.241 | 0.675 | 0.886 | 1.347 | 8.241 | 0.617 | 0.889 | 1.379 |
| 8.00 | 8.299 | 0.711 | 0.897 | 1.346 | 8.309 | 0.656 | 0.893 | 1.363 |
| 9.00 | 8.345 | 0.740 | 0.905 | 1.345 | 8.364 | 0.697 | 0.895 | 1.346 |
| 10.00 | 8.382 | 0.764 | 0.912 | 1.344 | 8.410 | 0.738 | 0.896 | 1.328 |
| Panel C: 1992-1996 | | | | | | | | |
| 2.00 | 5.265 | 0.392 | 0.536 | 1.022 | 5.306 | 0.315 | 0.487 | 0.939 |
| 3.00 | 5.616 | 0.396 | 0.580 | 1.070 | 5.643 | 0.336 | 0.543 | 1.012 |
| 4.00 | 5.916 | 0.406 | 0.606 | 1.072 | 5.912 | 0.379 | 0.599 | 1.056 |
| 5.00 | 6.15 | 0.415 | 0.623 | 1.062 | 6.126 | 0.409 | 0.635 | 1.074 |
| 6.00 | 6.326 | 0.423 | 0.634 | 1.049 | 6.296 | 0.425 | 0.655 | 1.077 |
| 7.00 | 6.461 | 0.429 | 0.642 | 1.039 | 6.433 | 0.434 | 0.664 | 1.070 |
| 8.00 | 6.565 | 0.434 | 0.649 | 1.030 | 6.544 | 0.439 | 0.665 | 1.057 |
| 9.00 | 6.647 | 0.438 | 0.653 | 1.022 | 6.636 | 0.441 | 0.661 | 1.040 |
| 10.00 | 6.713 | 0.441 | 0.657 | 1.016 | 6.713 | 0.442 | 0.655 | 1.019 |

**Table 2: Comparison of Average Root Mean Squared Errors**

This table presents the average root mean squared error (ARMSE) of the difference between theoretical bond prices computed using the Nelson-and-Siegel model and the actual bond prices for treasuries and industrial Aa, A and Baa corporate bonds. The estimation procedure is described in Section 2. Root mean squared error is measured in cents per dollar. For a given class of bonds, the root mean squared error is calculated once per period (month). The number reported is the average of all root mean squared errors within a given class over the months of the corresponding period. Our results are compared to those of Elton et al. (2001). Again, the results are similar. The ARMSE formula is given below:

$$ARMSE = \frac{\sum_{j=1}^{m} \sqrt{\frac{\sum_{i=1}^{n_j} (\hat{P}_i - P_i)^2}{n_j}}}{m}$$

where:

$m$: Number of months in the period.

$n_j$: Number of bonds in month $j$.

$\hat{P}_i$: The theoretical price of bond $i$.

$P_i$: The actual price of bond $i$.

| | Elton & al.(2001) | | | | Dionne & al.(2005) | | | |
|---|---|---|---|---|---|---|---|---|
| Period | Treasuries | Aa | A | Baa | Treasuries | Aa | A | Baa |
| 1987-1996 | 0.210 | 0.728 | 0.874 | 1.512 | 0.220 | 0.525 | 0.812 | 1.458 |
| 1987-1991 | 0.185 | 0.728 | 0.948 | 1.480 | 0.304 | 0.555 | 0.876 | 1.387 |
| 1992-1996 | 0.234 | 0.727 | 0.800 | 1.552 | 0.136 | 0.496 | 0.748 | 1.529 |

**Table 3a: Transition Matrix Information**

This table describes the matrix information used in different estimations. All cohort methods exclude withdrawn rating and entry firms information. The corresponding matrices were estimated by using annual cohorts. The MDC transition matrix includes entry firms and right censored information. With this estimation method, the transition matrices were continuously estimated over the corresponding five- or ten-year period. SPEG means Standard & Poor’s and Elton-Gruber. MDEG means Moody’s and Elton-Gruber. MDB1 is for data available on CD’s from Moody’s. T identifies the 1987-1996 period while 1 is for 1987-1991 and 2 for 1992-1996. MDB2 uses raw data from Moody’s. MDC is for the continuous-time method estimation.

| Matrix | Estimation Method | Period of estimation | Database | Data description |
|---|---|---|---|---|
| SPEG | Cohort | 1981-1995 | Standard & Poors 1996 | No entry and no right censoring |
| MDEG | Cohort | 1970-1993 | Moody's 1993 | No entry and no right censoring |
| MDB1_T | Cohort | 1987-1996 | CD Moody's 9-01-2003 | No entry and no right censoring |
| MDB1_1 | Cohort | 1987-1991 | CD Moody's 9-01-2003 | No entry and no right censoring |
| MDB1_2 | Cohort | 1992-1996 | CD Moody's 9-01-2003 | No entry and no right censoring |
| MDB2_T | Cohort | 1987-1996 | Raw Moody's 9-01-2002 | No entry and no right censoring |
| MDB2_1 | Cohort | 1987-1991 | Raw Moody's 9-01-2002 | No entry and no right censoring |
| MDB2_2 | Cohort | 1992-1996 | Raw Moody's 9-01-2002 | No entry and no right censoring |
| MDC_T | Continuous time | 1987-1996 | Raw Moody's 9-01-2002 | Entry firms and right censoring |
| MDC_1 | Continuous time | 1987-1991 | Raw Moody's 9-01-2002 | Entry firms and right censoring |
| MDC_2 | Continuous time | 1992-1996 | Raw Moody's 9-01-2002 | Entry firms and right censoring |

**Table 3b: Sensitivity of Ten-Year Average Default-Spread Proportion with Respect to Transition Matrices, Discrete- vs Continuous-Time Transition Probabilities and Discrete Theoretical Models**

This table reports the average of the ten-year default-spread proportion for different transition matrices and models. It also compares results between the discrete-time vs continuous-time analysis of rating transition probabilities. The two theoretical models are the Elton et al. (2001) and the model proposed in section 3.1. Both models are in discrete time. The first two transition matrices are Standard and Poor's (SPEG), and Moody's (MDEG), both as used in Elton et al. (2001) over the entire 10-year period. We also consider the Moody's matrices extracted (MDB1) and estimated (MDB2) for the entire 10-year period and the two 5-year periods from Moody's data base, and the continuous-time transition matrix estimated for the same periods (MDC). For the Elton et al. (2001) theoretical model, the spreads are estimated with bonds paying coupons and the same coupon rates as in their article. For our discrete model, the spreads were estimated with zero-coupon bonds. The recovery rates in both models are the same as in the Elton et al. (2001) paper: 59.59% for Aa, 60.63% for A and 49.42% for Baa. Panel A contains the average default-spread proportions for industrial Aa bonds. Panel B contains the averages for industrial A bonds and panel C contains the averages for industrial Baa bonds. For each class of bonds we report the results for the entire 10-year period and the sub-periods 1987-1991 and 1992-1996 in order to emphasize the default cycle effect on the average default-spread proportions.

| | Elton & al.(2001) | | | | | Dionne & al.(2005) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Period/Matrix | SPEG | MDEG | MDB1 | MDB2 | MDC | SPEG | MDEG | MDB1 | MDB2 | MDC |
| Panel A : Industrial Aa Bonds | | | | | | | | | | |
| 1987-1996 | 9.23 | 4.71 | 5.01 | 6.59 | 6.59 | 8.42 | 4.29 | 4.56 | 6.00 | 6.00 |
| 1987-1991 | 7.35 | 3.75 | 12.03 | 12.60 | 13.28 | 6.61 | 3.37 | 10.82 | 11.34 | 11.96 |
| 1992-1996 | 11.11 | 5.67 | 0.34 | 1.52 | 1.54 | 10.22 | 5.21 | 0.31 | 1.40 | 1.42 |
| Panel B : Industrial A Bonds | | | | | | | | | | |
| 1987-1996 | 19.45 | 12.23 | 10.87 | 13.50 | 11.01 | 17.76 | 11.12 | 9.88 | 12.28 | 10.00 |
| 1987-1991 | 16.80 | 10.56 | 25.34 | 26.09 | 23.35 | 15.13 | 9.47 | 22.85 | 23.54 | 21.05 |
| 1992-1996 | 22.11 | 13.90 | 1.26 | 4.53 | 3.82 | 20.40 | 12.77 | 1.15 | 4.15 | 3.50 |
| Panel C : Industrial Baa Bonds | | | | | | | | | | |
| 1987-1996 | 37.90 | 36.94 | 33.26 | 35.84 | 31.01 | 36.37 | 35.38 | 31.81 | 34.36 | 29.64 |
| 1987-1991 | 34.12 | 33.25 | 66.81 | 63.96 | 60.39 | 32.45 | 31.57 | 64.92 | 62.04 | 58.44 |
| 1992-1996 | 41.68 | 40.62 | 7.76 | 15.72 | 13.25 | 40.29 | 39.20 | 7.36 | 14.99 | 12.61 |

**Table 3c: Sensitivity of Two-Year Average Default-Spread Proportion with Respect to Transition Matrices, Discrete- vs Continuous-Time Transition Probabilities and Discrete Theoretical Models**

This table reports the average of two-year, default-spread proportion for different transition matrices and models. It also compares results between the discrete-time vs continuous-time analysis of rating transition probabilities. The two theoretical models are the Elton et al. (2001) and the model proposed in section 3.1. Both models are in discrete time. The first two transition matrices are Standard and Poor's (SPEG), and Moody's (MDEG), both as used in Elton et al. (2001) over the entire 10-year period. We also consider the Moody's matrices extracted (MDB1) and estimated (MDB2) for the entire 10-year period and the two 5-year periods from Moody's data base, and the continuous-time transition matrix estimated for the same periods (MDC). For the Elton et al. (2001) theoretical model, the spreads are estimated with bonds paying coupons and the same coupon rates as in their article. For our discrete model, the spreads were estimated with zero-coupon bonds. The recovery rates in both models are the same as in the Elton et al. (2001) paper: 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Panel A contains the average default-spread proportions for industrial Aa bonds. Panel B contains the averages for industrial A bonds and panel C contains the averages for industrial Baa bonds. For each class of bonds we report the results for the entire 10-year period and the sub-periods 1987-1991 and 1992-1996 in order to emphasize the default-cycle effect on the average default-spread proportions.

| | Elton & al.(2001) | | | | | Dionne & al.(2005) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Period/Matrix | SPEG | MDEG | MDB1 | MDB2 | MDC | SPEG | MDEG | MDB1 | MDB2 | MDC |
| Panel A : Industrial Aa Bonds | | | | | | | | | | |
| 1987-1996 | 1.03 | 0.22 | 0.34 | 0.45 | 1.17 | 0.94 | 0.20 | 0.31 | 0.40 | 1.06 |
| 1987-1991 | 0.83 | 0.18 | 0.83 | 0.89 | 2.50 | 0.74 | 0.16 | 0.74 | 0.79 | 2.22 |
| 1992-1996 | 1.24 | 0.27 | 0.00 | 0.01 | 0.10 | 1.13 | 0.24 | 0.00 | 0.01 | 0.09 |
| Panel B : Industrial A Bonds | | | | | | | | | | |
| 1987-1996 | 10.53 | 1.36 | 1.37 | 1.77 | 2.45 | 9.42 | 1.23 | 1.24 | 1.60 | 2.01 |
| 1987-1991 | 8.54 | 1.11 | 3.09 | 3.23 | 4.04 | 7.54 | 0.98 | 2.74 | 2.86 | 3.57 |
| 1992-1996 | 12.52 | 1.62 | 0.04 | 0.65 | 1.02 | 11.30 | 1.47 | 0.04 | 0.59 | 0.93 |
| Panel C : Industrial Baa Bonds | | | | | | | | | | |
| 1987-1996 | 15.49 | 10.37 | 10.14 | 14.31 | 13.59 | 14.39 | 9.64 | 9.43 | 13.29 | 12.63 |
| 1987-1991 | 11.90 | 7.97 | 16.55 | 16.98 | 17.60 | 10.95 | 7.34 | 15.18 | 15.58 | 16.15 |
| 1992-1996 | 19.08 | 12.78 | 1.72 | 11.17 | 10.84 | 17.82 | 11.94 | 1.62 | 10.46 | 10.15 |

**Table 4a: Implied Default Probability for Industrial Aa Bonds**

This table reports the implied default probabilities in percentages for industrial Aa bonds from different transition matrices. The implied default probability is the probability of default in year *n* conditional on a particular initial rating. The first two transition matrices are Standard and Poor's (SPEG), and Moody's (MDEG), both as used in Elton et al. (2001) over the entire 10-year period (1987-1996). We also consider the Moody's matrices extracted (MDB1) and estimated (MDB2) from Moody's data base (MDB1_T and MDB2_T for the entire 10-year period, MDB1_1 and MDB2_1 for the first 5-year period and MDB1_2 and MDB2_2 for the second 5-year period) and the continuous time transition matrix estimated for the same periods (MDC_T, MDC_1 and MDC_2). The implied default probability is given by the following formula: $q[\tau = n] = q[\tau \le n] - q[\tau \le n-1]$, where $\tau$ is the time of default.

| Year / Matrix | SPEG | MDEG | MDB1_T | MDB1_1 | MDB1_2 | MDB2_T | MDB2_1 | MDB2_2 | MDC_T | MDC_1 | MDC_2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.00 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00434 | 0.01157 | 0.00017 |
| 2.00 | 0.01673 | 0.00360 | 0.00558 | 0.01685 | 0.00001 | 0.00723 | 0.01800 | 0.00017 | 0.01467 | 0.03872 | 0.00120 |
| 3.00 | 0.03739 | 0.01085 | 0.01404 | 0.04309 | 0.00015 | 0.01875 | 0.04668 | 0.00163 | 0.02736 | 0.07120 | 0.00321 |
| 4.00 | 0.06105 | 0.02159 | 0.02544 | 0.07837 | 0.00053 | 0.03432 | 0.08483 | 0.00433 | 0.04235 | 0.10889 | 0.00614 |
| 5.00 | 0.08712 | 0.03562 | 0.03977 | 0.12227 | 0.00123 | 0.05365 | 0.13146 | 0.00821 | 0.05961 | 0.15192 | 0.00991 |
| 6.00 | 0.11521 | 0.05268 | 0.05695 | 0.17414 | 0.00231 | 0.07640 | 0.18554 | 0.01321 | 0.07907 | 0.20022 | 0.01447 |
| 7.00 | 0.14499 | 0.07251 | 0.07682 | 0.23308 | 0.00383 | 0.10220 | 0.24596 | 0.01923 | 0.10063 | 0.25351 | 0.01975 |
| 8.00 | 0.17615 | 0.09480 | 0.09918 | 0.29798 | 0.00579 | 0.13065 | 0.31155 | 0.02621 | 0.12415 | 0.31123 | 0.02568 |
| 9.00 | 0.20839 | 0.11924 | 0.12374 | 0.36764 | 0.00822 | 0.16133 | 0.38109 | 0.03403 | 0.14946 | 0.37268 | 0.03220 |
| 10.00 | 0.24143 | 0.14550 | 0.15022 | 0.44078 | 0.01110 | 0.19382 | 0.45336 | 0.04262 | 0.17634 | 0.43702 | 0.03925 |

## Table 4b: Implied Default Probability for Industrial A Bonds

This table reports the implied default probabilities in percentages for industrial A bonds from different transition matrices. The implied default probability is the probability of default in year *n* conditional on a particular initial rating. The first two transition matrices are Standard and Poor's (SPEG), and Moody's (MDEG), both as used in Elton et al. (2001) over the entire 10-year period (1987-1996). We also consider the Moody's matrices extracted (MDB1) and estimated (MDB2) from Moody's data base (MDB1_T and MDB2_T for the entire 10-year period, MDB1_1 and MDB2_1 for the first 5-year period and MDB1_2 and MDB2_2 for the second 5-year period) and the continuous-time transition matrix estimated for the same periods (MDC_T, MDC_1 and MDC_2). The implied default probability is given by the following formula: $q[\tau = n] = q[\tau \leq n] - q[\tau \leq n-1]$, where $\tau$ is the time of default.

| Year / Matrix | SPEG | MDEG | MDB1_T | MDB1_1 | MDB1_2 | MDB2_T | MDB2_1 | MDB2_2 | MDC_T | MDC_1 | MDC_2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.00 | 0.10300 | 0.00000 | 0.00163 | 0.00327 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.01280 | 0.02691 | 0.00521 |
| 2.00 | 0.15435 | 0.03354 | 0.03215 | 0.09025 | 0.00086 | 0.04365 | 0.09765 | 0.01353 | 0.04228 | 0.09498 | 0.01601 |
| 3.00 | 0.20388 | 0.07442 | 0.06830 | 0.19102 | 0.00332 | 0.09184 | 0.20538 | 0.02797 | 0.07670 | 0.17993 | 0.02724 |
| 4.00 | 0.25321 | 0.12086 | 0.10895 | 0.30226 | 0.00717 | 0.14331 | 0.32068 | 0.04308 | 0.11506 | 0.27688 | 0.03877 |
| 5.00 | 0.30273 | 0.17126 | 0.15296 | 0.42013 | 0.01219 | 0.19688 | 0.44035 | 0.05867 | 0.15643 | 0.38174 | 0.05053 |
| 6.00 | 0.35223 | 0.22417 | 0.19920 | 0.54090 | 0.01819 | 0.25151 | 0.56124 | 0.07457 | 0.19994 | 0.49100 | 0.06244 |
| 7.00 | 0.40119 | 0.27833 | 0.24662 | 0.66130 | 0.02500 | 0.30625 | 0.68055 | 0.09062 | 0.24483 | 0.60162 | 0.07447 |
| 8.00 | 0.44905 | 0.33264 | 0.29432 | 0.77858 | 0.03243 | 0.36033 | 0.79596 | 0.10668 | 0.29039 | 0.71108 | 0.08656 |
| 9.00 | 0.49526 | 0.38622 | 0.34154 | 0.89059 | 0.04034 | 0.41308 | 0.90567 | 0.12263 | 0.33597 | 0.81729 | 0.09866 |
| 10.00 | 0.53935 | 0.43834 | 0.38763 | 0.99573 | 0.04859 | 0.46399 | 1.00830 | 0.13837 | 0.38104 | 0.91864 | 0.11073 |

**Table 4c: Implied Default Probability for Industrial Baa Bonds**

This table reports the implied default probabilities in percentages for industrial Baa bonds from different transition matrices. The implied default probability is the probability of default in year *n* conditional on a particular initial rating. The first two transition matrices are Standard and Poor's (SPEG), and Moody's (MDEG), both as used in Elton et al. (2001) over the entire 10-year period (1987-1996). We also consider the Moody's matrices extracted (MDB1) and estimated (MDB2) from Moody's data base (MDB1_T and MDB2_T for the entire 10-year period, MDB1_1 and MDB2_1 for the first 5-year period and MDB1_2 and MDB2_2 for the second 5-year period) and the continuous-time transition matrix estimated for the same periods (MDC_T, MDC_1 and MDC_2). The implied default probability is given by the following formula: $q[\tau = n] = q[\tau \leq n] - q[\tau \leq n-1]$, where $\tau$ is the time of default.

| Year / Matrix | SPEG | MDEG | MDB1_T | MDB1_1 | MDB1_2 | MDB2_T | MDB2_1 | MDB2_2 | MDC_T | MDC_1 | MDC_2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.00 | 0.21200 | 0.10300 | 0.10401 | 0.19970 | 0.00831 | 0.18697 | 0.23364 | 0.14764 | 0.19801 | 0.28598 | 0.15003 |
| 2.00 | 0.34972 | 0.27384 | 0.26452 | 0.57829 | 0.04286 | 0.33216 | 0.56439 | 0.18253 | 0.29526 | 0.54118 | 0.17036 |
| 3.00 | 0.49065 | 0.43892 | 0.41309 | 0.92090 | 0.07638 | 0.46989 | 0.87617 | 0.21729 | 0.39681 | 0.80378 | 0.19142 |
| 4.00 | 0.62546 | 0.59276 | 0.54798 | 1.22160 | 0.10849 | 0.59698 | 1.15690 | 0.25110 | 0.49844 | 1.05700 | 0.21313 |
| 5.00 | 0.74871 | 0.73207 | 0.66802 | 1.47770 | 0.13882 | 0.71155 | 1.40080 | 0.28330 | 0.59664 | 1.28940 | 0.23514 |
| 6.00 | 0.85758 | 0.85517 | 0.77275 | 1.68930 | 0.16699 | 0.81270 | 1.60620 | 0.31335 | 0.68877 | 1.49460 | 0.25699 |
| 7.00 | 0.95105 | 0.96158 | 0.86232 | 1.85850 | 0.19274 | 0.90029 | 1.77400 | 0.34090 | 0.77304 | 1.66940 | 0.27824 |
| 8.00 | 1.02920 | 1.05160 | 0.93737 | 1.98880 | 0.21592 | 0.97471 | 1.90660 | 0.36574 | 0.84841 | 1.81350 | 0.29851 |
| 9.00 | 1.09290 | 1.12610 | 0.99885 | 2.08430 | 0.23647 | 1.03670 | 2.00730 | 0.38777 | 0.91441 | 1.92810 | 0.31751 |
| 10.00 | 1.14330 | 1.18620 | 1.04790 | 2.14950 | 0.25441 | 1.08720 | 2.07970 | 0.40702 | 0.97105 | 2.01530 | 0.33501 |

**Table 5: Sensitivity of Average Default-Spread Proportion in Our Discrete Theoretical Model With Respect to Recovery Rates**

This table reports the average of ten-year, default-spread proportions obtained by our discrete theoretical model for different transition matrices and recovery rates. The default spreads are estimated with zero-coupon bonds. The first two transition matrices are Standard and Poor's (SPEG) and Moody's (MDEG) both as used in Elton et al. (2001) over the entire 10-year period. We also consider the Moody's matrices estimated for the entire 10-year period and the two 5-year periods from Moody's data base (MDB2) and the continuous-time transition estimated for the same periods (MDC). We present three recovery rates: 40%, 50%, and 60%. Panel A contains the average of default-spread proportions for industrial Aa bonds. Panel B contains the averages for industrial A bonds and panel C contains the averages for industrial Baa bonds. For each class of bonds we report the results for the entire 10-year period and the sub-periods 1987-1991 and 1992-1996 in order to emphasize the default-cycle effect on the average of default-spread proportions.

| | Recovery Rate of 40% | | | | Recovery Rate of 50% | | | | Recovery Rate of 60% | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Period / Matrix | SPEG | MDEG | MDB2 | MDC | SPEG | MDEG | MDB2 | MDC | SPEG | MDEG | MDB2 | MDC |
| Panel A : Industrial Aa Bonds | | | | | | | | | | | | |
| 1987-1996 | 12.51 | 6.37 | 8.92 | 8.92 | 10.42 | 5.30 | 7.43 | 7.43 | 8.33 | 4.24 | 5.94 | 5.94 |
| 1987-1991 | 9.82 | 5.00 | 16.87 | 17.78 | 8.18 | 4.17 | 14.04 | 14.81 | 6.54 | 3.33 | 11.23 | 11.83 |
| 1992-1996 | 15.19 | 7.73 | 2.07 | 2.11 | 12.66 | 6.44 | 1.73 | 1.76 | 10.12 | 5.15 | 1.38 | 1.40 |
| Panel B : Industrial A Bonds | | | | | | | | | | | | |
| 1987-1996 | 27.12 | 16.97 | 18.74 | 15.27 | 22.58 | 14.13 | 15.61 | 12.71 | 18.05 | 11.30 | 12.47 | 10.16 |
| 1987-1991 | 23.10 | 14.45 | 36.04 | 32.20 | 19.23 | 12.03 | 29.97 | 26.79 | 15.37 | 9.62 | 23.92 | 21.39 |
| 1992-1996 | 31.15 | 19.49 | 6.33 | 5.34 | 25.93 | 16.23 | 5.27 | 4.45 | 20.73 | 12.97 | 4.22 | 3.56 |
| Panel C : Industrial Baa Bonds | | | | | | | | | | | | |
| 1987-1996 | 43.24 | 42.08 | 40.84 | 35.22 | 35.95 | 34.97 | 33.96 | 29.29 | 28.69 | 27.91 | 27.11 | 23.39 |
| 1987-1991 | 38.58 | 37.54 | 73.94 | 69.64 | 32.08 | 31.20 | 61.31 | 57.76 | 25.60 | 24.90 | 48.80 | 45.99 |
| 1992-1996 | 47.90 | 46.61 | 17.79 | 14.97 | 39.83 | 38.74 | 14.81 | 12.47 | 31.79 | 30.91 | 11.84 | 9.97 |

**Table 6: Comparison of Ten-Year Average Default Spread Proportions between Discrete and Continuous-Time Theoretical Models**

This table reports the average of ten-year default spread proportions for different transition matrices and models. It compares results between our discrete-time and continuous-time theoretical models. Recovery rates used are the same as in Elton et al. (2001): 59.59% for Aa. 60.63% for A, and 49.42% for Baa. The default spreads are estimated with zero-coupon bonds. The first transition matrix is Moody's (MDEG) used in Elton et al. (2001) over the entire 10-year period. We also consider the Moody's matrices extracted (MDB1) and estimated (MDB2) for the entire 10-year period and the two 5-year periods from Moody's data base and the continuous-time transition matrix estimated for the same periods (MDC). Panel A contains the average default spread proportions for industrial Aa bonds. Panel B contains the averages for industrial A bonds and panel C contains the averages for industrial Baa bonds. We observe that the continuous time model gives similar proportions of default risk in corporate bond spreads as the discrete-time model does for continuous time transition matrix.

| Period / Matrix | Discrete Time Model | | | | Continuous Time Model | | | |
|---|---|---|---|---|---|---|---|---|
| | MDEG | MDB1 | MDB2 | MDC | MDEG | MDB1 | MDB2 | MDC |
| | Panel A : Industrial Aa bonds | | | | | | | |
| 1987-1996 | 4.29 | 4.56 | 6.00 | 6.00 | 4.97 | 5.05 | 6.79 | 6.00 |
| 1987-1991 | 3.37 | 10.82 | 11.34 | 11.96 | 3.90 | 12.13 | 12.78 | 11.92 |
| 1992-1996 | 5.21 | 0.31 | 1.40 | 1.42 | 6.03 | 0.41 | 1.83 | 1.42 |
| | Panel B : Industrial A bonds | | | | | | | |
| 1987-1996 | 11.12 | 9.88 | 12.28 | 10.00 | 11.79 | 10.32 | 13.03 | 9.97 |
| 1987-1991 | 9.47 | 22.85 | 23.54 | 21.05 | 10.04 | 23.80 | 24.78 | 20.87 |
| 1992-1996 | 12.77 | 1.15 | 4.15 | 3.50 | 13.54 | 1.34 | 4.58 | 4.00 |
| | Panel C : Industrial Baa bonds | | | | | | | |
| 1987-1996 | 35.38 | 31.81 | 34.36 | 29.64 | 34.78 | 31.64 | 33.78 | 29.07 |
| 1987-1991 | 31.57 | 64.92 | 62.04 | 58.44 | 31.03 | 62.33 | 59.71 | 56.14 |
| 1992-1996 | 39.20 | 7.36 | 14.99 | 12.61 | 38.53 | 7.65 | 14.95 | 12.50 |

**Table 7: Rating Distributions Used to Compute Descriptive Statistics**

This table reports the distribution of issuers, by rating, at the starting date of the estimation period used to construct the confidence sets of average default-spread proportions. There are two periods of simulations: 1987-1991 and 1992-1996.

| **Period/Rating** | **Aaa** | **Aa** | **A** | **Baa** | **Ba** | **B** | **CCC-C** |
|---|---|---|---|---|---|---|---|
| 1987-1991 | 58 | 210 | 456 | 287 | 343 | 179 | 6 |
| 1992-1996 | 51 | 160 | 416 | 321 | 275 | 197 | 12 |

**Table 8a: Descriptive Statistics of Average Default-Spread Proportions in the Continuous-Time Model Using Withdrawn-Rating and Entry Firm Observations**

This table reports the mean, the standard error, the median, the percentiles 2.5 and 97.5, and the minimum and the maximum of average default-spread proportions for different maturities (2, 5, 8 and 10 years) obtained from Monte Carlo simulations using the continuous-time model and continuous-time transition matrix estimated for the two 5-year periods from Moody's data base (MDC). The transition matrix is estimated using withdrawn rating and entry firm observations. The recovery rates are the same as in the Elton et al. (2001) paper: 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Panel A contains the results for industrial Aa bonds. Panel B contains the results for industrial A bonds and panel C contains the results for industrial Baa bonds.

| | Period 1987-1991 | | | | Period 1992-1996 | | | |
|---|---|---|---|---|---|---|---|---|
| Maturity | 2 | 5 | 8 | 10 | 2 | 5 | 8 | 10 |
| Panel A: Industrial Aa Bonds | | | | | | | | |
| Mean | 2.24 | 6.19 | 9.39 | 11.95 | 0.09 | 0.42 | 0.93 | 1.41 |
| Standard Error | 1.28 | 2.43 | 2.64 | 2.79 | 0.04 | 0.17 | 0.34 | 0.49 |
| Percentile 2.5 | 0.42 | 2.44 | 5.06 | 7.25 | 0.02 | 0.14 | 0.35 | 0.58 |
| Median | 2.02 | 5.88 | 9.11 | 11.68 | 0.09 | 0.40 | 0.90 | 1.36 |
| Percentile 97.5 | 5.14 | 11.66 | 15.22 | 18.02 | 0.19 | 0.82 | 1.72 | 2.54 |
| Minimum | 0.17 | 1.18 | 2.71 | 4.09 | 0.00 | 0.03 | 0.10 | 0.19 |
| Maximum | 9.46 | 20.09 | 24.16 | 27.07 | 0.32 | 1.36 | 2.84 | 4.15 |
| Panel B: Industrial A Bonds | | | | | | | | |
| Mean | 3.58 | 8.96 | 15.81 | 20.91 | 0.93 | 1.76 | 2.72 | 3.48 |
| Standard Error | 0.79 | 1.49 | 2.22 | 2.71 | 0.40 | 0.66 | 0.91 | 1.09 |
| Percentile 2.5 | 2.15 | 6.26 | 11.70 | 15.88 | 0.27 | 0.64 | 1.17 | 1.61 |
| Median | 3.54 | 8.90 | 15.73 | 20.81 | 0.89 | 1.70 | 2.65 | 3.40 |
| Percentile 97.5 | 5.27 | 12.10 | 20.41 | 26.53 | 1.82 | 3.25 | 4.76 | 5.92 |
| Minimum | 1.15 | 4.43 | 8.99 | 12.12 | 0.04 | 0.18 | 0.45 | 0.70 |
| Maximum | 6.68 | 15.08 | 24.69 | 31.46 | 3.02 | 5.14 | 7.43 | 9.16 |
| Panel C: Industrial Baa Bonds | | | | | | | | |
| Mean | 16.18 | 30.89 | 46.11 | 56.21 | 10.12 | 9.75 | 11.10 | 12.44 |
| Standard Error | 4.08 | 4.60 | 5.58 | 6.30 | 5.46 | 3.99 | 3.65 | 3.64 |
| Percentile 2.5 | 9.27 | 22.54 | 35.82 | 44.50 | 1.45 | 3.15 | 4.96 | 6.27 |
| Median | 15.84 | 30.63 | 45.91 | 55.97 | 9.39 | 9.36 | 10.81 | 12.15 |
| Percentile 97.5 | 25.19 | 40.60 | 57.35 | 68.81 | 22.92 | 18.78 | 19.27 | 20.50 |
| Minimum | 5.57 | 15.47 | 25.69 | 32.53 | 0.40 | 1.27 | 2.29 | 3.09 |
| Maximum | 34.79 | 50.90 | 69.16 | 81.20 | 38.20 | 29.26 | 28.20 | 29.08 |

**Table 8b: Descriptive Statistics of Average Default-Spread Proportions in the Continuous-Time Model Excluding Withdrawn-Rating and Entry Firm Observations**

This table reports the mean, the standard error, the median, the percentiles 2.5 and 97.5, and the minimum and the maximum of average default-spread proportions, for different maturities (2, 5, 8 and 10 years) obtained from Monte Carlo simulations using the continuous-time model and continuous-time transition matrix estimated for the two 5-year periods from Moody's data base (MDC). The transition matrix is estimated by excluding withdrawn-rating and entered firm observations. The recovery rates are the same as in the Elton et al. (2001) paper: 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Panel A contains the results for industrial Aa bonds. Panel B contains the results for industrial A bonds and panel C contains the results for industrial Baa bonds.

| | Period 1987-1991 | | | | Period 1992-1996 | | | |
|---|---|---|---|---|---|---|---|---|
| Maturity | 2 | 5 | 8 | 10 | 2 | 5 | 8 | 10 |
| Panel A: Industrial Aa Bonds | | | | | | | | |
| Mean | 3.18 | 7.89 | 11.30 | 14.08 | 0.16 | 0.73 | 1.60 | 2.41 |
| Standard Error | 1.89 | 3.34 | 3.38 | 3.43 | 0.06 | 0.25 | 0.50 | 0.71 |
| Percentile 2.5 | 0.43 | 2.71 | 5.80 | 8.36 | 0.06 | 0.31 | 0.76 | 1.21 |
| Median | 3.08 | 7.58 | 10.96 | 13.78 | 0.15 | 0.71 | 1.56 | 2.36 |
| Percentile 97.5 | 7.49 | 15.40 | 18.87 | 21.63 | 0.30 | 1.28 | 2.70 | 3.96 |
| Minimum | 0.23 | 1.81 | 4.13 | 6.16 | 0.01 | 0.09 | 0.28 | 0.48 |
| Maximum | 12.90 | 24.64 | 27.65 | 30.13 | 0.48 | 2.02 | 4.12 | 5.92 |
| Panel B: Industrial A Bonds | | | | | | | | |
| Mean | 4.66 | 11.43 | 19.72 | 25.70 | 1.72 | 3.26 | 4.93 | 6.24 |
| Standard Error | 0.99 | 1.82 | 2.63 | 3.17 | 0.62 | 1.02 | 1.37 | 1.62 |
| Percentile 2.5 | 2.88 | 8.06 | 14.81 | 19.77 | 0.66 | 1.50 | 2.54 | 3.39 |
| Median | 4.59 | 11.34 | 19.63 | 25.62 | 1.67 | 3.18 | 4.84 | 6.14 |
| Percentile 97.5 | 6.77 | 15.20 | 25.14 | 32.15 | 3.08 | 5.46 | 7.89 | 9.73 |
| Minimum | 1.70 | 4.91 | 9.41 | 12.89 | 0.14 | 0.47 | 0.97 | 1.42 |
| Maximum | 9.62 | 19.15 | 30.86 | 39.08 | 4.87 | 8.35 | 11.71 | 14.19 |
| Panel C: Industrial Baa Bonds | | | | | | | | |
| Mean | 21.47 | 40.50 | 58.90 | 70.56 | 15.39 | 15.94 | 18.57 | 20.85 |
| Standard Error | 4.72 | 5.48 | 6.64 | 7.44 | 6.32 | 4.80 | 4.50 | 4.56 |
| Percentile 2.5 | 13.24 | 30.36 | 46.37 | 56.45 | 4.76 | 7.76 | 10.74 | 12.82 |
| Median | 21.16 | 40.33 | 58.78 | 70.42 | 14.84 | 15.60 | 18.32 | 20.62 |
| Percentile 97.5 | 31.56 | 51.66 | 72.41 | 85.63 | 29.36 | 26.45 | 28.21 | 30.53 |
| Minimum | 8.48 | 22.87 | 35.74 | 44.02 | 1.57 | 4.00 | 6.73 | 8.38 |
| Maximum | 44.94 | 62.55 | 82.73 | 97.71 | 47.66 | 40.29 | 41.05 | 43.26 |

**Table 8c: Descriptive Statistics of Average Default-Spread Proportions in the Continuous-Time Model Excluding Only Entry Firm Observations**

This table reports the mean, the standard error, the median, the percentiles 2.5 and 97.5, and the minimum and the maximum of average default-spread proportions for different maturities (2, 5, 8 and 10 years) obtained from Monte Carlo simulations using the continuous-time model and continuous-time transition matrix estimated for the two 5-year periods from Moody's data base (MDC). The transition matrix is estimated by excluding only entry firm observations. The recovery rates are the same as in the Elton et al. (2001) paper: 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Panel A contains the results for industrial Aa bonds. Panel B contains the results for industrial A bonds and panel C contains the results for industrial Baa bonds.

| | Period 1987-1991 | | | | Period 1992-1996 | | | |
|---|---|---|---|---|---|---|---|---|
| Maturity | 2 | 5 | 8 | 10 | 2 | 5 | 8 | 10 |
| Panel A: Industrial Aa Bonds | | | | | | | | |
| Mean | 2.42 | 6.58 | 9.89 | 12.53 | 0.04 | 0.14 | 0.33 | 0.45 |
| Standard Error | 1.37 | 2.57 | 2.76 | 2.89 | 0.02 | 0.06 | 0.13 | 0.17 |
| Percentile 2.5 | 0.43 | 2.60 | 5.42 | 7.74 | 0.01 | 0.05 | 0.12 | 0.18 |
| Median | 2.29 | 6.27 | 9.57 | 12.24 | 0.04 | 0.14 | 0.32 | 0.44 |
| Percentile 97.5 | 5.59 | 12.43 | 15.99 | 18.83 | 0.08 | 0.27 | 0.62 | 0.83 |
| Minimum | 0.14 | 1.06 | 2.55 | 3.93 | 0.00 | 0.01 | 0.02 | 0.04 |
| Maximum | 10.45 | 20.74 | 23.96 | 26.66 | 0.15 | 0.48 | 1.10 | 1.47 |
| Panel B: Industrial A Bonds | | | | | | | | |
| Mean | 4.18 | 10.24 | 17.72 | 23.15 | 1.39 | 2.63 | 3.99 | 5.07 |
| Standard Error | 0.86 | 1.61 | 2.37 | 2.88 | 0.52 | 0.85 | 1.16 | 1.37 |
| Percentile 2.5 | 2.63 | 7.35 | 13.44 | 17.91 | 0.52 | 1.18 | 2.00 | 2.68 |
| Median | 4.12 | 10.16 | 17.61 | 23.02 | 1.35 | 2.55 | 3.90 | 4.95 |
| Percentile 97.5 | 6.00 | 13.60 | 22.67 | 29.14 | 2.52 | 4.45 | 6.49 | 8.02 |
| Minimum | 1.62 | 4.87 | 9.48 | 13.08 | 0.08 | 0.35 | 0.79 | 1.21 |
| Maximum | 8.24 | 17.51 | 28.68 | 36.47 | 4.07 | 7.22 | 10.22 | 12.39 |
| Panel C: Industrial Baa Bonds | | | | | | | | |
| Mean | 18.24 | 34.01 | 49.69 | 59.85 | 13.20 | 13.24 | 15.22 | 17.03 |
| Standard Error | 4.26 | 4.81 | 5.80 | 6.50 | 5.92 | 4.40 | 4.07 | 4.09 |
| Percentile 2.5 | 10.75 | 25.15 | 38.76 | 47.61 | 3.04 | 5.66 | 8.19 | 9.92 |
| Median | 17.96 | 33.86 | 49.56 | 59.68 | 12.80 | 12.92 | 14.92 | 16.74 |
| Percentile 97.5 | 27.35 | 44.07 | 61.48 | 73.05 | 26.13 | 22.78 | 23.88 | 25.70 |
| Minimum | 6.74 | 19.57 | 30.40 | 37.58 | 0.95 | 2.62 | 4.48 | 5.77 |
| Maximum | 38.05 | 54.83 | 75.52 | 88.53 | 52.00 | 40.90 | 39.66 | 40.72 |

**Table 9a: Descriptive Statistics of Implied Default Probability Using Withdrawn-Rating and Entry Firm Observations**

This table reports the mean, the standard error, the median, the percentiles 2.5 and 97.5, and the minimum and the maximum of implied default probability $q_s$ for different maturities ($s$ = 2, 5, 8 and 10 years) obtained from Monte Carlo simulations using the continuous-time model and continuous-time transition matrix estimated for the two 5-year periods from Moody's data base (MDC). The transition matrix is estimated using withdrawn-rating and entry firm data. Panel A contains the results for industrial Aa bonds. Panel B contains the results for industrial A bonds and panel C contains the results for industrial Baa bonds.

| | Period 1987-1991 | | | | Period 1992-1996 | | | |
|---|---|---|---|---|---|---|---|---|
| Maturity | 2 | 5 | 8 | 10 | 2 | 5 | 8 | 10 |
| Panel A: Industrial Aa Bonds | | | | | | | | |
| Mean | 0.039 | 0.153 | 0.312 | 0.437 | 0.001 | 0.010 | 0.026 | 0.039 |
| Standard Error | 0.021 | 0.048 | 0.063 | 0.071 | 0.001 | 0.004 | 0.009 | 0.013 |
| Percentile 2.5 | 0.008 | 0.074 | 0.202 | 0.310 | 0.000 | 0.003 | 0.011 | 0.018 |
| Median | 0.036 | 0.147 | 0.307 | 0.433 | 0.001 | 0.009 | 0.025 | 0.038 |
| Percentile 97.5 | 0.087 | 0.260 | 0.446 | 0.586 | 0.002 | 0.019 | 0.046 | 0.067 |
| Minimum | 0.003 | 0.038 | 0.116 | 0.190 | 0.000 | 0.001 | 0.003 | 0.007 |
| Maximum | 0.160 | 0.424 | 0.632 | 0.764 | 0.004 | 0.031 | 0.075 | 0.108 |
| Panel B: Industrial A Bonds | | | | | | | | |
| Mean | 0.095 | 0.383 | 0.712 | 0.919 | 0.016 | 0.050 | 0.086 | 0.110 |
| Standard Error | 0.020 | 0.056 | 0.085 | 0.100 | 0.007 | 0.017 | 0.025 | 0.030 |
| Percentile 2.5 | 0.059 | 0.279 | 0.554 | 0.733 | 0.005 | 0.021 | 0.042 | 0.058 |
| Median | 0.094 | 0.380 | 0.709 | 0.916 | 0.015 | 0.049 | 0.084 | 0.108 |
| Percentile 97.5 | 0.138 | 0.499 | 0.886 | 1.121 | 0.031 | 0.090 | 0.143 | 0.175 |
| Minimum | 0.033 | 0.214 | 0.421 | 0.561 | 0.001 | 0.007 | 0.020 | 0.031 |
| Maximum | 0.176 | 0.610 | 1.036 | 1.287 | 0.051 | 0.141 | 0.219 | 0.264 |
| Panel C: Industrial Baa Bonds | | | | | | | | |
| Mean | 0.542 | 1.291 | 1.813 | 2.014 | 0.170 | 0.234 | 0.297 | 0.333 |
| Standard Error | 0.109 | 0.148 | 0.169 | 0.169 | 0.083 | 0.071 | 0.067 | 0.067 |
| Percentile 2.5 | 0.351 | 1.016 | 1.491 | 1.689 | 0.035 | 0.112 | 0.179 | 0.213 |
| Median | 0.535 | 1.287 | 1.810 | 2.012 | 0.160 | 0.229 | 0.293 | 0.329 |
| Percentile 97.5 | 0.778 | 1.587 | 2.147 | 2.347 | 0.359 | 0.390 | 0.440 | 0.472 |
| Minimum | 0.221 | 0.745 | 1.152 | 1.335 | 0.010 | 0.051 | 0.098 | 0.127 |
| Maximum | 1.008 | 1.862 | 2.467 | 2.682 | 0.582 | 0.566 | 0.575 | 0.603 |

**Table 9b: Descriptive Statistics of Implied Default Probability Excluding Withdrawn-Rating and Entry Firm Observations**

This table reports the mean, the standard error, the median, the percentiles 2.5 and 97.5, and the minimum and the maximum of implied default probability $q_s$ for different maturities ($s$ = 2, 5, 8 and 10 years) obtained from Monte Carlo simulations using the continuous-time model and continuous-time transition matrix estimated for the two 5-year periods from Moody's data base (MDC). The transition matrix is estimated by excluding withdrawn-rating and entry firm observations. Panel A contains the results for industrial Aa bonds. Panel B contains the results for industrial A bonds and panel C contains the results for industrial Baa bonds.

| | Period 1987-1991 | | | | Period 1992-1996 | | | |
|---|---|---|---|---|---|---|---|---|
| Maturity | 2 | 5 | 8 | 10 | 2 | 5 | 8 | 10 |
| Panel A: Industrial Aa Bonds | | | | | | | | |
| Mean | 0.055 | 0.185 | 0.360 | 0.497 | 0.002 | 0.017 | 0.044 | 0.066 |
| Standard Error | 0.031 | 0.062 | 0.073 | 0.078 | 0.001 | 0.006 | 0.013 | 0.018 |
| Percentile 2.5 | 0.009 | 0.085 | 0.233 | 0.355 | 0.001 | 0.008 | 0.022 | 0.035 |
| Median | 0.053 | 0.179 | 0.355 | 0.493 | 0.002 | 0.017 | 0.043 | 0.065 |
| Percentile 97.5 | 0.126 | 0.325 | 0.517 | 0.661 | 0.004 | 0.030 | 0.071 | 0.105 |
| Minimum | 0.005 | 0.058 | 0.174 | 0.270 | 0.000 | 0.002 | 0.009 | 0.016 |
| Maximum | 0.215 | 0.487 | 0.670 | 0.821 | 0.006 | 0.046 | 0.106 | 0.151 |
| Panel B: Industrial A Bonds | | | | | | | | |
| Mean | 0.124 | 0.481 | 0.863 | 1.089 | 0.030 | 0.092 | 0.153 | 0.191 |
| Standard Error | 0.025 | 0.067 | 0.097 | 0.111 | 0.011 | 0.027 | 0.037 | 0.042 |
| Percentile 2.5 | 0.078 | 0.357 | 0.680 | 0.878 | 0.012 | 0.046 | 0.087 | 0.116 |
| Median | 0.122 | 0.479 | 0.861 | 1.087 | 0.029 | 0.090 | 0.151 | 0.189 |
| Percentile 97.5 | 0.177 | 0.619 | 1.058 | 1.312 | 0.053 | 0.150 | 0.233 | 0.282 |
| Minimum | 0.046 | 0.223 | 0.455 | 0.611 | 0.003 | 0.017 | 0.039 | 0.056 |
| Maximum | 0.248 | 0.763 | 1.261 | 1.531 | 0.083 | 0.223 | 0.332 | 0.391 |
| Panel C: Industrial Baa Bonds | | | | | | | | |
| Mean | 0.722 | 1.650 | 2.186 | 2.337 | 0.270 | 0.399 | 0.501 | 0.549 |
| Standard Error | 0.129 | 0.174 | 0.187 | 0.179 | 0.098 | 0.089 | 0.086 | 0.086 |
| Percentile 2.5 | 0.491 | 1.321 | 1.823 | 1.985 | 0.103 | 0.240 | 0.342 | 0.390 |
| Median | 0.716 | 1.648 | 2.184 | 2.335 | 0.262 | 0.394 | 0.497 | 0.545 |
| Percentile 97.5 | 0.994 | 1.997 | 2.558 | 2.689 | 0.487 | 0.588 | 0.683 | 0.727 |
| Minimum | 0.336 | 1.020 | 1.476 | 1.647 | 0.039 | 0.155 | 0.242 | 0.282 |
| Maximum | 1.288 | 2.275 | 2.952 | 3.054 | 0.764 | 0.818 | 0.866 | 0.901 |

**Table 9c: Descriptive Statistics of Implied Default Probability Excluding Entry Firm Data**

This table reports the mean, the standard error, the median, the percentiles 2.5 and 97.5, the minimum and the maximum of implied default probability $q_s$ for different maturities ( $s$ = 2, 5, 8 and 10 years) obtained from Monte Carlo simulations using the continuous-time model and continuous-time transition matrix estimated for the two 5-year periods from Moody's data base (MDC). The transition matrix is estimated by excluding only entry firm data. Panel A contains the results for industrial Aa bonds. Panel B contains the results for industrial A bonds and panel C contains the results for industrial Baa bonds.

| | Period 1987-1991 | | | | Period 1992-1996 | | | |
|---|---|---|---|---|---|---|---|---|
| Maturity | 2 | 5 | 8 | 10 | 2 | 5 | 8 | 10 |
| Panel A: Industrial Aa Bonds | | | | | | | | |
| Mean | 0.042 | 0.161 | 0.326 | 0.455 | 0.002 | 0.015 | 0.038 | 0.057 |
| Standard Error | 0.023 | 0.051 | 0.064 | 0.072 | 0.001 | 0.005 | 0.011 | 0.016 |
| Percentile 2.5 | 0.009 | 0.080 | 0.214 | 0.324 | 0.001 | 0.006 | 0.019 | 0.030 |
| Median | 0.040 | 0.155 | 0.321 | 0.451 | 0.002 | 0.014 | 0.037 | 0.056 |
| Percentile 97.5 | 0.095 | 0.274 | 0.463 | 0.606 | 0.003 | 0.026 | 0.063 | 0.092 |
| Minimum | 0.003 | 0.035 | 0.112 | 0.188 | 0.000 | 0.002 | 0.007 | 0.013 |
| Maximum | 0.175 | 0.421 | 0.622 | 0.773 | 0.006 | 0.046 | 0.106 | 0.151 |
| Panel B: Industrial A Bonds | | | | | | | | |
| Mean | 0.111 | 0.432 | 0.781 | 0.990 | 0.024 | 0.075 | 0.124 | 0.156 |
| Standard Error | 0.022 | 0.060 | 0.089 | 0.103 | 0.009 | 0.022 | 0.032 | 0.036 |
| Percentile 2.5 | 0.071 | 0.324 | 0.616 | 0.797 | 0.009 | 0.036 | 0.069 | 0.092 |
| Median | 0.109 | 0.429 | 0.778 | 0.987 | 0.023 | 0.073 | 0.122 | 0.154 |
| Percentile 97.5 | 0.158 | 0.558 | 0.966 | 1.202 | 0.043 | 0.123 | 0.192 | 0.234 |
| Minimum | 0.044 | 0.223 | 0.464 | 0.628 | 0.002 | 0.013 | 0.034 | 0.051 |
| Maximum | 0.214 | 0.709 | 1.186 | 1.443 | 0.070 | 0.196 | 0.291 | 0.342 |
| Panel C: Industrial Baa Bonds | | | | | | | | |
| Mean | 0.609 | 1.393 | 1.892 | 2.062 | 0.227 | 0.325 | 0.406 | 0.447 |
| Standard Error | 0.114 | 0.153 | 0.170 | 0.167 | 0.091 | 0.080 | 0.076 | 0.076 |
| Percentile 2.5 | 0.404 | 1.105 | 1.567 | 1.739 | 0.071 | 0.185 | 0.269 | 0.308 |
| Median | 0.604 | 1.391 | 1.891 | 2.061 | 0.221 | 0.320 | 0.403 | 0.443 |
| Percentile 97.5 | 0.853 | 1.702 | 2.230 | 2.397 | 0.425 | 0.494 | 0.563 | 0.603 |
| Minimum | 0.272 | 0.872 | 1.277 | 1.449 | 0.024 | 0.103 | 0.177 | 0.216 |
| Maximum | 1.106 | 2.079 | 2.578 | 2.747 | 0.802 | 0.766 | 0.760 | 0.799 |

**Table 10: Data Composition of Tables 8 and 9**

This table reports the number of firms, transitions, and defaults used to estimate the continuous-time transition matrices. There are three cases. First, we excluded entry firm and right censored data (panel b). Second, we excluded only entry firm data (panel c). Finally, we included entry firm and withdrawn-rating data (panel a). The analysis was done for the 1987-1996 period and for both the 1987-1991 and 1992-1996 sub-periods. We observe that the proportion of default issuers (Defaults/Issuers) decreases when we include censored and entry firms data and so do the average default-spread proportions (Tables 8a, 8b, and 8c) and the corresponding implied default probabilities (Tables 9a, 9b, and 9c).

| | Including Withdrawals and Entry Firm Data (a) | | | Standard (b) | | | Including Withdrawals (c) | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1987-1996 | 1987-1991 | 1992-1996 | 1987-1996 | 1987-1991 | 1992-1996 | 1987-1996 | 1987-1991 | 1992-1996 |
| Issuers | 3,879 | 2,656 | 3,090 | 1,239 | 1,539 | 1,432 | 1,977 | 1,977 | 1,867 |
| Rating observations | 7,652 | 4,690 | 4,829 | 2,731 | 2,672 | 2,236 | 4,590 | 3,667 | 3,213 |
| Defaults | 399 | 267 | 132 | 250 | 196 | 92 | 250 | 196 | 92 |
| Defaults/ Issuers | 10.29% | 10.05% | 4.27% | 20.18% | 12.74% | 6.42% | 12.65% | 9.91% | 4.93% |

**Table 11: Average Default-Spread Proportions for the 1992-1996 Period with Three Different Benchmarks**

Panel A reports the default-spread proportion using the Hull et al. (2005) risk-free rate benchmark. The latter is equal to the Treasury rate plus 83% of the difference between the interest-rate swap and the Treasury rate, as suggested by the authors. The interest-rate-swap term structure is constructed using interest-rate swap rates of 2, 4, 5, 7, and 10 years from Datastream database and Libor rates for 3 and 6 months from the Federal Reserve historical data. We applied our discrete theoretical model with the continuous-time transition matrix estimated for the 5-year period 1992-1996 and the recovery rates from the Elton et al. (2001) paper: 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Panel B reports the average default-spread proportions using the industrial Aaa benchmark. This method was suggested in the literature to isolate the credit risk from the tax and liquidity factors in observed spreads. The results in the two panels are compared with those in Table 3a with the MDC transition matrix for the 1992-1996 period (Panel C). Both new benchmarks generate higher average default-spread proportions for that period but still lower than those for the 1987-1991 period.

| | A: Hull *et al*. (2005) Benchmark | | | | B: Aaa Benchmark | | | | C: Treasury Rate Benchmark |
|---|---|---|---|---|---|---|---|---|---|
| | T=7 | T=8 | T=9 | T=10 | T=7 | T=8 | T=9 | T=10 | T=10 |
| Industrial Aa Bonds | 2.08 | 2.68 | 3.50 | 4.71 | 3.42 | 4.40 | 4.75 | 7.28 | 1.42 |
| Industrial A Bonds | 4.12 | 4.78 | 5.52 | 6.37 | 4.81 | 5.49 | 6.36 | 7.57 | 3.50 |
| Industrial Baa Bonds | 14.56 | 15.43 | 16.44 | 17.62 | 15.73 | 16.48 | 17.47 | 18.76 | 12.61 |

**Table A1: Mean, Minimum, and Maximum Default Spreads and Default Spread Proportions**

This table reports the average, minimum and maximum of default spreads (panels A, B and C) and the proportion of average default spreads (in percentage, panel D) obtained using the Elton et al. (2001) model with the same assumptions for recovery rates, coupons rates, and Standard and Poor's transition matrix, as described in the text. The recovery rates are 59.59% for Aa, 60.63% for A, and 49.42% for Baa. Our results are compared to those of Elton et al. (2001). We observe that they are equivalent. The proportion of average default spread (DSP) is obtained from the average of default spreads (DS) and the average of corporate bond spreads (CBS) by the following formula: $DSP = 100 \frac{DS}{CBS}$ .

| | Elton & al.(2001) | | | Dionne & al.(2005) | | |
|---|---|---|---|---|---|---|
| Years | Aa | A | Baa | Aa | A | Baa |
| | Panel A: Mean Default Spreads | | | | | |
| 1.00 | 0.000 | 0.043 | 0.110 | 0.000 | 0.046 | 0.117 |
| 2.00 | 0.004 | 0.053 | 0.145 | 0.004 | 0.057 | 0.154 |
| 3.00 | 0.008 | 0.063 | 0.181 | 0.008 | 0.068 | 0.192 |
| 4.00 | 0.012 | 0.074 | 0.217 | 0.013 | 0.078 | 0.229 |
| 5.00 | 0.017 | 0.084 | 0.252 | 0.018 | 0.089 | 0.265 |
| 6.00 | 0.023 | 0.095 | 0.286 | 0.023 | 0.100 | 0.300 |
| 7.00 | 0.028 | 0.106 | 0.319 | 0.029 | 0.111 | 0.333 |
| 8.00 | 0.034 | 0.117 | 0.351 | 0.036 | 0.122 | 0.364 |
| 9.00 | 0.041 | 0.128 | 0.380 | 0.042 | 0.133 | 0.394 |
| 10.00 | 0.048 | 0.140 | 0.409 | 0.049 | 0.144 | 0.421 |
| | Panel B: Minimum Default Spreads | | | | | |
| 1.00 | 0.000 | 0.038 | 0.101 | 0.000 | 0.045 | 0.115 |
| 2.00 | 0.003 | 0.046 | 0.132 | 0.004 | 0.055 | 0.151 |
| 3.00 | 0.007 | 0.055 | 0.164 | 0.008 | 0.066 | 0.188 |
| 4.00 | 0.011 | 0.063 | 0.197 | 0.012 | 0.076 | 0.225 |
| 5.00 | 0.015 | 0.073 | 0.229 | 0.017 | 0.086 | 0.260 |
| 6.00 | 0.020 | 0.083 | 0.262 | 0.023 | 0.097 | 0.294 |
| 7.00 | 0.025 | 0.093 | 0.294 | 0.028 | 0.107 | 0.326 |
| 8.00 | 0.031 | 0.104 | 0.326 | 0.034 | 0.118 | 0.356 |
| 9.00 | 0.038 | 0.116 | 0.356 | 0.041 | 0.129 | 0.385 |
| 10.00 | 0.044 | 0.128 | 0.385 | 0.047 | 0.140 | 0.412 |
| | Panel C: Maximum Default Spreads | | | | | |
| 1.00 | 0.000 | 0.047 | 0.118 | 0.000 | 0.047 | 0.119 |
| 2.00 | 0.004 | 0.059 | 0.156 | 0.004 | 0.058 | 0.157 |
| 3.00 | 0.009 | 0.071 | 0.196 | 0.008 | 0.069 | 0.195 |
| 4.00 | 0.014 | 0.083 | 0.235 | 0.013 | 0.080 | 0.233 |
| 5.00 | 0.019 | 0.094 | 0.273 | 0.019 | 0.092 | 0.270 |
| 6.00 | 0.025 | 0.106 | 0.309 | 0.024 | 0.103 | 0.306 |
| 7.00 | 0.031 | 0.117 | 0.342 | 0.030 | 0.114 | 0.340 |
| 8.00 | 0.038 | 0.129 | 0.374 | 0.037 | 0.126 | 0.372 |
| 9.00 | 0.044 | 0.140 | 0.403 | 0.043 | 0.137 | 0.401 |
| 10.00 | 0.051 | 0.151 | 0.431 | 0.050 | 0.148 | 0.429 |
| | Panel D: Default-Spread Proportions | | | | | |
| 2.00 | 0.966 | 8.535 | 12.425 | 0.969 | 9.314 | 13.051 |
| 3.00 | 1.909 | 9.265 | 15.021 | 1.923 | 10.119 | 15.920 |
| 4.00 | 2.637 | 10.350 | 17.934 | 2.908 | 10.803 | 18.633 |
| 5.00 | 3.448 | 11.382 | 20.913 | 3.774 | 11.835 | 21.423 |
| 6.00 | 4.373 | 12.616 | 23.853 | 4.582 | 13.004 | 24.311 |
| 7.00 | 5.072 | 13.874 | 26.739 | 5.513 | 14.304 | 27.206 |
| 8.00 | 5.934 | 15.136 | 29.545 | 6.569 | 15.661 | 30.083 |
| 9.00 | 6.961 | 16.431 | 32.095 | 7.381 | 17.095 | 33.026 |
| 10.00 | 7.960 | 17.834 | 34.661 | 8.305 | 18.557 | 35.860 |

## Table A2: Estimated Generators

### Table A2-a: Estimated Generator for the 1987-1991 Period

| | Aaa | Aa | A | Baa | Ba | B | CCC-C | D |
|---|---|---|---|---|---|---|---|---|
| Aaa | -0.0608 | 0.0582 | 0.0000 | 0.0000 | 0.0026 | 0.0000 | 0.0000 | 0.0000 |
| Aa | 0.0134 | -0.1355 | 0.1168 | 0.0018 | 0.0018 | 0.0018 | 0.0000 | 0.0000 |
| A | 0.0008 | 0.0138 | -0.0946 | 0.0678 | 0.0100 | 0.0023 | 0.0000 | 0.0000 |
| Baa | 0.0006 | 0.0044 | 0.0404 | -0.1356 | 0.0758 | 0.0122 | 0.0006 | 0.0017 |
| Ba | 0.0004 | 0.0008 | 0.0041 | 0.0382 | -0.1729 | 0.1174 | 0.0016 | 0.0103 |
| B | 0.0000 | 0.0012 | 0.0012 | 0.0047 | 0.0507 | -0.2081 | 0.0461 | 0.1043 |
| CCC-C | 0.0000 | 0.0000 | 0.0139 | 0.0418 | 0.0139 | 0.0279 | -0.9341 | 0.8365 |
| D | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

### Table A2-b: Estimated Generator for the 1992-1996 Period

| | Aaa | Aa | A | Baa | Ba | B | CCC-C | D |
|---|---|---|---|---|---|---|---|---|
| Aaa | -0.0752 | 0.0716 | 0.0036 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Aa | 0.0035 | -0.1071 | 0.1036 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| A | 0.0000 | 0.0113 | -0.0552 | 0.0418 | 0.0011 | 0.0011 | 0.0000 | 0.0000 |
| Baa | 0.0005 | 0.0014 | 0.0737 | -0.1120 | 0.0327 | 0.0014 | 0.0009 | 0.0014 |
| Ba | 0.0000 | 0.0000 | 0.0068 | 0.0610 | -0.1395 | 0.0683 | 0.0000 | 0.0034 |
| B | 0.0005 | 0.0005 | 0.0036 | 0.0047 | 0.0738 | -0.1616 | 0.0410 | 0.0374 |
| CCC-C | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0124 | 0.1244 | -0.3443 | 0.2074 |
| D | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

### Table A2-c: Estimated Generator for the 1987-1996 Period

| | Aaa | Aa | A | Baa | Ba | B | CCC-C | D |
|---|---|---|---|---|---|---|---|---|
| Aaa | -0.0669 | 0.0639 | 0.0015 | 0.0000 | 0.0015 | 0.0000 | 0.0000 | 0.0000 |
| Aa | 0.0091 | -0.1232 | 0.1111 | 0.0010 | 0.0010 | 0.0010 | 0.0000 | 0.0000 |
| A | 0.0004 | 0.0125 | -0.0741 | 0.0543 | 0.0053 | 0.0017 | 0.0000 | 0.0000 |
| Baa | 0.0005 | 0.0028 | 0.0585 | -0.1228 | 0.0524 | 0.0063 | 0.0008 | 0.0015 |
| Ba | 0.0002 | 0.0004 | 0.0053 | 0.0487 | -0.1576 | 0.0949 | 0.0009 | 0.0071 |
| B | 0.0003 | 0.0008 | 0.0025 | 0.0047 | 0.0629 | -0.1835 | 0.0434 | 0.0690 |
| CCC-C | 0.0000 | 0.0000 | 0.0032 | 0.0096 | 0.0128 | 0.1023 | -0.4794 | 0.3516 |
| D | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

**Table A3**
**Comparison with Christensen et al (2004) database**

Christensen et al. (2004) used issuers having senior unsecured bonds rated by Moody's only, while we used issuers having an "Estimated senior unsecured rating" by Moody's, even if they have not issued senior unsecured bonds. In both studies, the data source is Moody's Corporate Bond Default Database (January 9, 2002).

| | Christensen et al. (2004) | Dionne et al. (2005) |
|---|---|---|
| Initial number of issuers | 3,405 | 5,719 |
| Initial ratings (when irrelevant withdrawals were eliminated) | 13,390 | 22,938 |
| Final number of issuers | 3,446 | 5,821 |
| Defaults | 305 | 943 |
| Refreshed firms | 41 | 102 |
| Final ratings | 9,991 | 15,564 |

**Figure 1: Empirical spreads on industrial bonds of six years maturity**

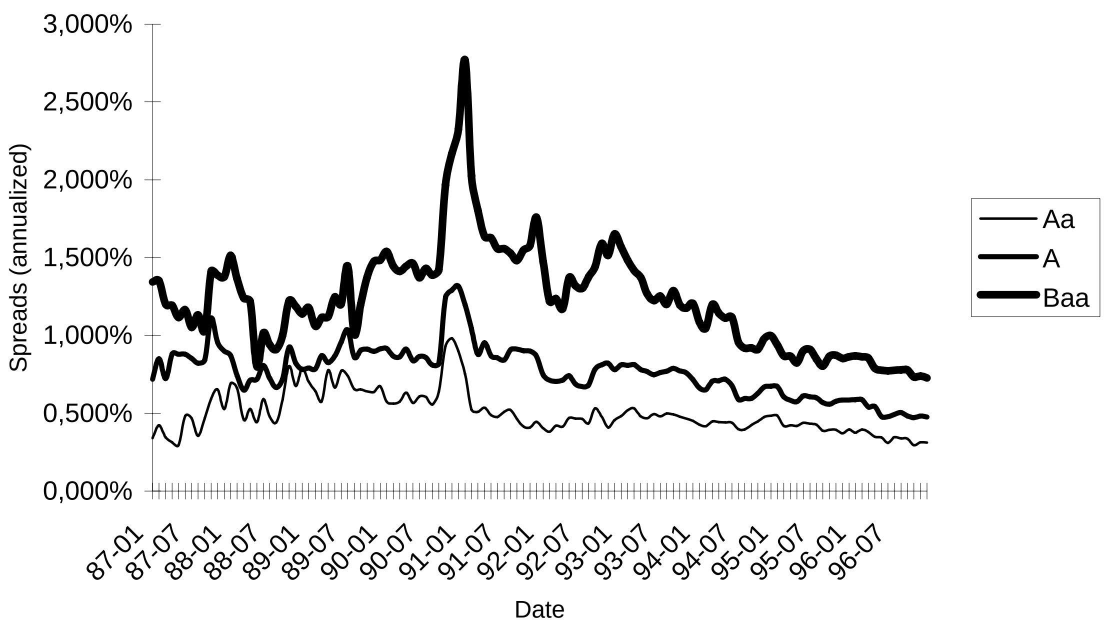


Figure 2: Annual Issuer Weighted Default Rates, 1987-1996

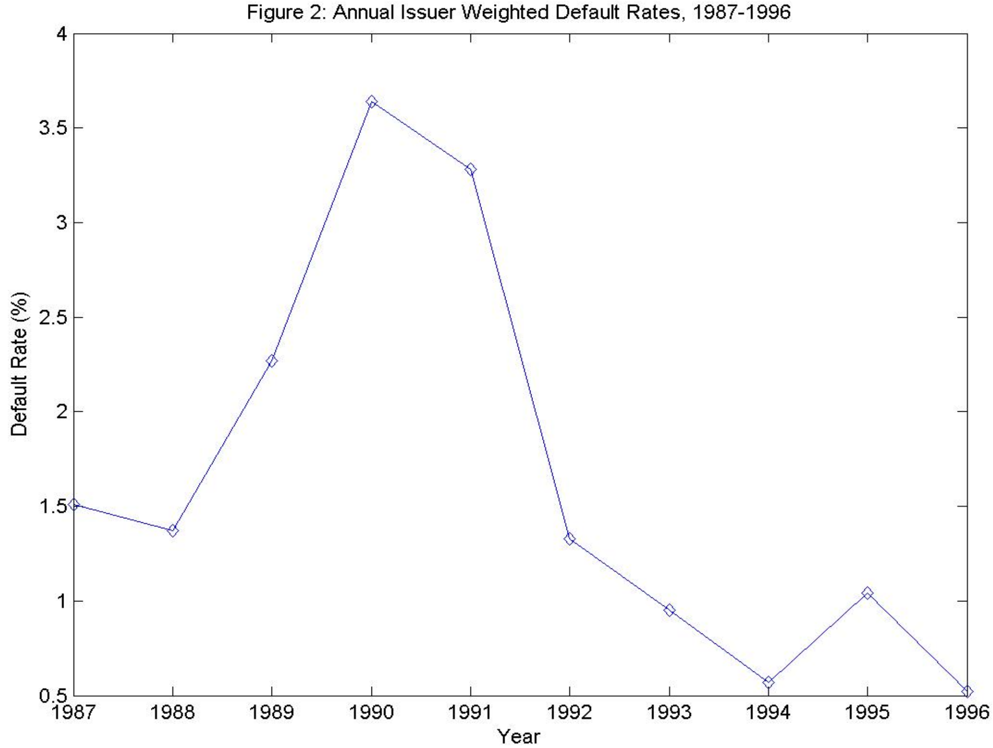
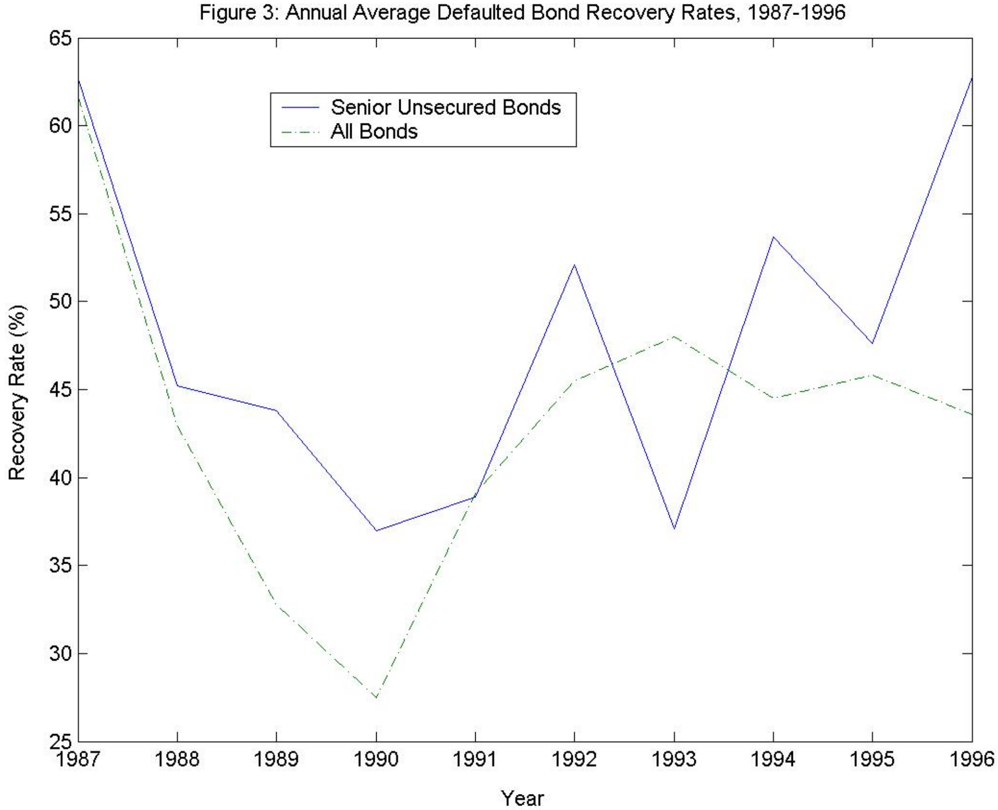



Figure 3: Annual Average Defaulted Bond Recovery Rates, 1987-1996



# Default Risk, Default Risk Premium and Corporate Yield Spreads

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**Abstract**

An important research area of the corporate yield spread literature seeks to measure the proportion of the spread explained by various factors such as the possibility of default, liquidity or tax differentials. We contribute to this literature by assessing the ability of observed macroeconomic factors and the possibility of changes in regimes to explain the default risk proportion in yield spreads. For this purpose, we extend the Markov Switching risk-free term structure model of Bansal and Zhou (2002) to the corporate bond setting and develop recursive formulas for default probabilities, risk-free and risky zero-coupon bond prices. The model is calibrated with consumption, inflation, risk-free yield and default data over the 1987-1996 period. Preliminary results show that default probabilities and default spreads exhibit a regime shift. On average the default spread explains 41% of the observed corporate yield spread for industrial Baa during the 1987-1996 period for ten year zero-coupon bonds. The average default spread proportion is 48.5% in the high default regime and 35.5% in the low default regime. More interesting, the default spread proportion can attain 72.4% in the high default regime and 25% in the low default regime.

Keywords: Corporate yield spread, default risk, default risk premium, credit spread level puzzle, macroeconomic risk premium, regime shift.

JEL classification: G11, G12, G13, G21, G23.

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# 1 Introduction

Several empirical studies have been recently performed on corporate yield spreads, the difference between the yields of corporate and risk-free bonds. These studies try to explain the average level and volatility of corporate yield spreads through time. Many factors are believed to be at the source of this yield spread : the possibility of defaulting on the loan, the low liquidity of corporate bonds and taxes are the usual factors invoked. A surprising finding from this literature is that the possibility of default explains a small proportion of the spread. For example, using structural bond pricing models calibrated to historical default and recovery, Huang and Huang (2003) find that only 20% of the Baa-treasury spread is explained by the possibility of default. Elton et al. (2001) reach a similar conclusion using a reduced form approach. This result is, to some extent, counterintuitive. One should expect the possibility of default to be the key driver behind the corporate spread level because this is the central feature distinguishing corporate from risk free bonds. This intuition is confirmed by looking at the yield differences between corporate bonds of different rating classes. If the effect of taxes and liquidity are similar from one rating to the other, this yield difference is now mostly caused by default. As reported in Chen et al. (2005), these observed corporate yield differentials are higher than what theoretical models of corporate bond price can generate, once calibrated to historical default and recovery.

Several authors have tried adjusting pricing models, either empirically or theoretically, to generate higher default spread levels. A first research direction explored in the literature examines if alternative empirical implementation of existing models could generate higher default spread levels. For example, Ericsson and Reneby (2004) and Bruche (2005) investigate if maximum likelihood estimation method applied to structural bond pricing model can produce better empirical results. Dionne et al. (2005) look at alternative approaches for the calibration of default probabilities and their influence on results got with a reduced form model. These attempts generate some improvements but lack in producing high enough default spread levels. Another research direction examined in this literature introduces, in the theoretical models, the possibility of sudden changes in default risk. These are usually introduced with the addition of jumps and jump risk premia in the stochastic processes specified in the theoretical model. Zhou (1997), Driessen (2005) and Collin-Dufresne, Goldstein and Helwege (2003) are examples of this approach. The empirical evidences on jump risk are however mixed and statistically inconclusive. More recently, a third research direction looks at the links between default and business cycle or macroeconomic conditions. For example, Chen et al. (2005) have verified if the level and the dynamic variations of credit spreads can be explained by a pricing kernel solving the equity premium puzzle. They found that large variations in time-varying risk premiums are essentials for explaining the spreads. They also found that macroeconomic variables have a certain explanatory power after some firm-specific factors are taken into account. In Tang and Yan (2005) the link with aggregate factors is explored further by modeling firm characteristics to directly depend on macroeconomic conditions.

In this article we contribute to the corporate rate spread literature by merging the last two research directions outlined above. More specifically, we investigate if macroeconomic factors coupled with the possibility of sudden changes in economic conditions modeled through a Markov Switching approach can help in explaining the spread levels. It is reasonable to assume that macroeconomic fundamentals should play a role in the spread level because interest rates and output from firms fluctuate over the business cycle. Also, many empirical evidences suggest that switching regimes are better descriptions of macroeconomic variables and risk-free interest behavior than single regime models. See for example, Ang and Bekeart (2003) and Bansal and Zhou (2002). Because the possibility of changes in regime might influence macroeconomic factors and risk-free interest rates, it is only natural to assume that this might also affect the corporate yield spreads.

To introduce macroeconomic factors and the possibility of changes in regime, we extend the switching regime risk-free term structure model of Bansal and Zhou (2002) to the risky corporate setting. Starting from the first order condition of the intertemporal consumption problem with a power utility function, we postulate that consumption and inflation dynamics are governed by two independent Markov chains. Using the log linear approximation proposed in Bansal and Zhou (2002), we derive a convenient recursive formula for risk-free and risky bond prices as well as default probabilities which are all functions of the growth rates of our two observed factors. The model developed here also allows risk aversion parameters to change in the different regimes. This enables us to capture some flight to quality phenomena reported in the litterature by allowing higher risk aversion in states of economic downturn.

The models and the empirical study proposed here can also be seen as an extension of the Elton et al. (2001) study to a risk averse setting. In Elton et al. (2001), risk neutrality was required in order to use objective default probabilities in a bond pricing model specified under the risk neutral measure. The model developed here is entirely specified under the objective measure in a risk averse setting. The risk of default is caused by macroeconomic factors and the possibility of sudden changes in regime. Also, as in Elton et al. (2001), we attempt to explain default spread for entire rating categories instead of corporate spreads at the firm level. We are thus avoiding the need to include firm specific features which results in a simpler model to calibrate.

To assess the capacity of the proposed approach to generate default spreads, we calibrate the model using aggregate consumption, inflation, risk free rate and default data. The calibration procedure proceeds in three steps. In a first step, we estimate the Markov switching parameters using aggregate consumption and inflation. Using the parameters obtained in the first step, we then extract, each quarter, implied utility parameters enabling us to produce realistic theoretical term structure of risk free rates. In a third step, using parameter values obtained in the first two steps, we calibrate the parameters linking our theoretical default probabilities with macroeconomic variables to closely match the observed default probabilities obtained from default data.

Our preliminary results show that this corporate bond pricing model calibrated as above can generate default probabilities and default spreads that exhibit a regime shift. On average the default spread explains 41% of the observed corporate yield spread for industrial Baa during the 1987-1996 period for ten year zero-coupon bonds. The average default spread proportion is 48.5% in the high default regime and 35.5% in the low default regime. More interesting, the default spread proportion can attain 72.4% in the high default regime and 25% in the low default regime. Section 2 presents our theoretical models of bond pricing: one for the default-risk free-zero-coupon bond and one for the risky zero-coupon bond. Section 3 presents our estimation results and calibration procedure. Section 4 computes the default spread, decomposes the default spread into the default risk and the default risk premium, and sets their proportions in corporate yield spreads for industrial Baa bonds. Section 5 concludes.

## 2 Models

The model developped here starts from the well known first order condition of the intertemporal consumption problem as described in, for example, Cochrane (2005). Because we attempt to model nominal bond prices, we must account for the future growth rates of the price level and real consumption. We assume that the future evolution of these variables is well described by a Markov Switching process.

Let $C_t$ denote the real personal consumption expenditures per capita at time $t$, and $\Pi_t$ the ratio of nominal over real consumptions (consumption price index) at time $t$. Here, the time variable is expressed in quarters and the continuously compounded quarterly growth rate are defined as:

$$\Delta c_t = \ln C_t - \ln C_{t-1},$$

$$\Delta \pi_t = \ln \Pi_t - \ln \Pi_{t-1}.$$

We assume that $\{\Delta\pi_t, \Delta c_t : t \in \mathcal{N}\}$ follow an autoregressive model with switching regimes

$$\Delta c_t = a^c_{S^C_t} + b^c_{S^C_t}\Delta c_{t-1} + e^c_t \tag{1a}$$

$$\Delta \pi_t = a^\pi_{S^\Pi_t} + b^\pi_{S^\Pi_t}\Delta \pi_{t-1} + e^\pi_t \tag{1b}$$

where $S^C_t \in \{1,2\}$ is the state of the consumption at time $t$ and $S^\Pi_t \in \{1,2\}$ is the state of the inflation at time $t$. $\left\{(e^c_t, e^\pi_t)' : t \in \mathcal{N}\right\}$ is a sequence of independent Gaussian vectors with mean $(0,0)'$ and covariance matrix

$$\mathrm{Var}\left(\left(\begin{array}{c} e^c_t \\ e^\pi_t \end{array}\right)\right) = \left(\begin{array}{cc} \left(\sigma^c_{S^C_t}\right)^2 & \rho_{S^C_t,S^\Pi_t}\sigma^c_{S^C_t}\sigma^\pi_{S^\Pi_t} \\ \rho_{S^C_t,S^\Pi_t}\sigma^c_{S^C_t}\sigma^\pi_{S^\Pi_t} & \left(\sigma^\pi_{S^\Pi_t}\right)^2 \end{array}\right).$$

The state of consumption $\left\{S^C_t : t \in \mathcal{N}\right\}$ and the state of inflation $\left\{S^\Pi_t : t \in \mathcal{N}\right\}$ are assumed to follow two independent Markov chains (Hamilton, 1994) with transition matrices

$$\phi^C = \left(\begin{array}{cc} \phi^C_{11} & 1-\phi^C_{11} \\ 1-\phi^C_{22} & \phi^C_{22} \end{array}\right), \qquad \phi^\Pi = \left(\begin{array}{cc} \phi^\Pi_{11} & 1-\phi^\Pi_{11} \\ 1-\phi^\Pi_{22} & \phi^\Pi_{22} \end{array}\right).$$

These Markov chains are presumed to be independent of the past values of $\Delta c$ and $\Delta\pi$. More precisely,

$$\begin{aligned} &P\left(S^C_t = i, S^\Pi_t = j \left| S^C_{t-1}, S^\Pi_{t-1}, \Delta c_{t-1}, \Delta\pi_{t-1}, ..., \Delta c_1, \Delta\pi_1\right.\right) \\ &= P\left(S^C_t = i \left| S^C_{t-1}\right.\right) P\left(S^\Pi_t = j \left| S^\Pi_{t-1}\right.\right). \end{aligned}$$

The parameters

$$\begin{aligned}\boldsymbol{\theta} &= \left(a_1^c, a_2^c, b_1^c, b_2^c, a_1^\pi, a_2^\pi, b_1^\pi, b_2^\pi, \sigma_1^c, \sigma_2^c, \sigma_1^\pi, \sigma_2^\pi, \rho_{1,1}, \rho_{1,2}, \rho_{2,1}, \rho_{2,2}\right) \\ \text{and } \boldsymbol{\phi} &= \left(\phi_{11}^C, \phi_{22}^C, \phi_{11}^\Pi, \phi_{22}^\Pi\right)\end{aligned}$$

are unknown and need to be estimated. As described in Section 3, the 20 parameters will be estimated using time series of consumption and inflation.

Define the $\sigma-$field $\mathcal{G}_t = \sigma\left(\left(C_u, \Pi_u, S_u^C, S_u^\Pi\right) : u \in \{0, 1, 2, ..., t\}\right)$. It may be interpreted as the information available at time $t$ if one observe the evolution of consumption, inflation, and the state of consumption and inflation up to time $t$. Using the first order condition of the intertemporal consumption problem with the assumption of a power utility function, the time $t$ value of a security worth $X_{t+1}$ at time $t+1$ is given by

$$V_t = \mathrm{E}_{\mathcal{G}_t}^P\left[\beta_{S_{t+1}^C, S_{t+1}^\Pi}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma_{S_{t+1}^C, S_{t+1}^\Pi}}\frac{\Pi_t}{\Pi_{t+1}}X_{t+1}\right] = \mathrm{E}_{\mathcal{G}_t}^P\left[M_{t,t+1}X_{t+1}\right] \tag{2}$$

where

$$M_{t,t+1} = \exp\left(\ln\beta_{S_{t+1}^C, S_{t+1}^\Pi} - \gamma_{S_{t+1}^C, S_{t+1}^\Pi}\Delta c_{t+1} - \Delta\pi_{t+1}\right)$$

is the nominal discount factor or the pricing kernel for the time period $]t, t+1]$, $\beta_{1,1}, \beta_{1,2}, \beta_{2,1}, \beta_{2,2}$ are the impatience coefficients and $\gamma_{1,1}, \gamma_{1,2}, \gamma_{2,1}, \gamma_{2,2}$ the risk aversion coefficients. $\mathrm{E}_{\mathcal{G}_t}^P[\bullet] = \mathrm{E}^P[\bullet|\mathcal{G}_t]$ is the conditional expectation with respect to available information at time $t$. Note that we use a power utility function instead of the logarithmic specification of Bansal and Zhou (2002). Equation (2) thus proposes a pricing kernel which is a function of the consumption and inflation processes. This pricing kernel must take into account regime shifts uncertainty and we assume that this uncertainty should affect the preference parameters. We are thus allowing the preference parameters to be different in the different possible regimes i.e. $\boldsymbol{\beta} = \left(\beta_{1,1}, \beta_{1,2}, \beta_{2,1}, \beta_{2,2}\right)$ and $\boldsymbol{\gamma} = \left(\gamma_{1,1}, \gamma_{1,2}, \gamma_{2,1}, \gamma_{2,2}\right)$. As in Bansal and Zhou (2002) and to be consistent with Hamilton methodology (1994), we assume that $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$ depend of $t+1$ instead of $t$ in order to incorporate the regime shifts uncertainty related to the conditional distribution of $X_{t+1}$. In fact, we implicitly assume that the economic agents anticipate the regime shift effects. The same argument of future regime shifts uncertainty is also used to justify why parameters $a$ and $b$ in equations (1a) and (1b) above are function of $t$ instead of $t-1$.

## 2.1 The default-risk free zero-coupon bond

As shown in Appendix A.1, an exact formula for the time $t$ value of a default-risk free zero-coupon bond paying one dollar at time $T$ can be obtained using the framework described above. However, using such a solution is not practical. For example, with quarterly time steps, the value of a zero-coupon bond maturing in ten years would contain $4^{40} = 1.208\,9 \times 10^{24}$ terms to compute. This would make the numerical implementation of the exact solution unmanageable. For this reason, we do not use this exact formula and instead rely on an approximation assuming that a sum of exponential functions may be well estimated by a single exponential function. More precisely, define

$$\widetilde{P}(t,T) = \exp\left(\widetilde{A}_{T-t,S_t^C,S_t^\Pi} - \widetilde{B}^c_{T-t,S_t^C,S_t^\Pi}\Delta c_t - \widetilde{B}^\pi_{T-t,S_t^C,S_t^\Pi}\Delta\pi_t\right) \tag{3}$$

where expressions for $\widetilde{A}_{T-t,S_t^C,S_t^\Pi}$, $\widetilde{B}^c_{T-t,S_t^C,S_t^\Pi}$, and $\widetilde{B}^\pi_{T-t,S_t^C,S_t^\Pi}$ are given in Appendix A.2. These expressions are function of the time to maturity $T-t$, and the actual states of consumption and of inflation. As shown in Appendix A.2, $\widetilde{A}_{T-t,S_t^C,S_t^\Pi}$, $\widetilde{B}^c_{T-t,S_t^C,S_t^\Pi}$, and $\widetilde{B}^\pi_{T-t,S_t^C,S_t^\Pi}$ are determined recursively using a backward induction and the terminal condition $\widetilde{A}_{0,S_T^C,S_T^\Pi} = \widetilde{B}^c_{0,S_T^C,S_T^\Pi} = \widetilde{B}^\pi_{0,S_T^C,S_T^\Pi} = 0$. We are thus assuming that the exact time $t$ theoretical value of the default-risk free zero-coupon bond knowing the state of consumption and the state of inflation, $P\left(t,T,S_t^C,S_t^\Pi,\Delta c_t,\Delta\pi_t\right)$, is well approximated by the function $\widetilde{P}\left(t,T,S_t^C,S_t^\Pi,\Delta c_t,\Delta\pi_t\right)$.

## 2.2 The risky zero-coupon bond

We consider a risky zero-coupon bond that pays one dollar at time $T$ if it has not defaulted before. In case of default, the bondholder receives at the default time $\tau$, a fraction of its market value just before default.

More precisely, let $\lambda$ be a nonnegative (measurable) function from $\mathcal{R}^n$ to $\mathcal{R}$, $\mathbf{X}_t = \left(C_t,\Pi_t,S_t^C,S_t^\Pi\right)$ be the set of risk factors at time $t$, and $E_1$ be an exponential random variable with expectation equal to 1 independent of the filtration $\{\mathcal{G}_t : t\in\mathbb{N}_0\}$. The default time is the first time for which the stochastic process $\left\{\sum_{s=1}^t \lambda(\mathbf{X}_s) : t\in\{0,1,2,...\}\right\}$ is larger that $E_1$. This setting is a discrete time version of the intensity based model using Cox processes. To model the information given by the risk factors (up to time $t$) and whether or not the default arise before time $t$, we define a stochastic process $\{\mathbf{1}_{\tau\le t} : t\in\{0,1,2,...\}\}$ worth zero before the default time and one after, and the

$\sigma-$field $\mathcal{F}_t = \sigma\left(\mathbf{X}_u, \mathbf{1}_{\tau \leq u} : u \in \{0, 1, 2, ..., t\}\right)$.

In this well studied context, the time $t$ value of the survived risky zero-coupon bond (defined on the set $\tau > t$) is

$$V(t,T) = \mathrm{E}_{\mathcal{G}_t}\left[M_{t,t+1}\left(1 - Lh_{t+1}\right)V(t+1,T)\ \right] \tag{4}$$

where $L = (1-\rho)$ and $\rho$ is the recovery rate that is assumed independent of $t$ for the moment. $h_{t+1} = \left(1 - e^{-\lambda(\mathbf{X}_{t+1})}\right) = P_{\mathcal{G}_{t+1}}\left[\tau = t+1 |\ \tau > t\right]$ is the conditional probability that the default arises within the next period of time knowing that the firm as survived at time $t$ and having the information available at time $t+1$. Since the default probabilities are usually small (especially when the length of a time period is small), it is reasonable to approximate[1] $1 - Lh_{t+1}$ by $\exp\left(-Lh_{t+1}\right)$. Hence

$$V(t,T) \cong \mathrm{E}_{\mathcal{G}_t}\left[M_{t,t+1}\exp\left(-Lh_{t+1}\right)V(t+1,T)\right]\mathbf{1}_{\tau>t}. \tag{5}$$

We assume that the conditional default probability $h_{t+1}$ is approximated by an affine function of $\Delta c_{t+1}$ and $\Delta\pi_{t+1}$, that is,

$$h_{t+1} \cong \alpha_{S^C_{t+1}, S^\Pi_{t+1}} + \alpha^c_{S^C_{t+1}, S^\Pi_{t+1}}\Delta c_{t+1} + \alpha^\pi_{S^C_{t+1}, S^\Pi_{t+1}}\Delta\pi_{t+1}. \tag{6}$$

The parameters

$$\boldsymbol{\alpha} = \left(\alpha_{1,1}, \alpha_{1,2}, \alpha_{2,1}, \alpha_{2,2}, \alpha^c_{1,1}, \alpha^c_{1,2}, \alpha^c_{2,1}, \alpha^c_{2,2}, \alpha^\pi_{1,1}, \alpha^\pi_{1,2}, \alpha^\pi_{2,1}, \alpha^\pi_{2,2}\right)$$

are unknown and need to be estimated. Note that the approximation of (6) can produce negative probabilities as well as probabilities larger than one. However, if the parameter $\boldsymbol{\alpha}$ is chosen wisely, the chances of obtaining such abberations should be small.

Under this assumption, the time $t$ value of the survived risky zero-coupon bond is approximated by

$$\begin{aligned} V(t,T) &\cong \mathrm{E}_{\mathcal{G}_t}\left[M_{t,t+1}\exp\left(-Lh_{t,t+1}\right)V(t+1,T)\right]\mathbf{1}_{\tau>t} \\ &\cong \mathrm{E}_{\mathcal{G}_t}\left[\exp\left(\begin{array}{c} \ln\beta_{S^C_{t+1}, S^\Pi_{t+1}} - L\alpha_{S^C_{t+1}, S^\Pi_{t+1}} \\ -\left(\gamma_{S^C_{t+1}, S^\Pi_{t+1}} + L\alpha^c_{S^C_{t+1}, S^\Pi_{t+1}}\right)\Delta c_{t+1} \\ -\left(1 + L\alpha^\pi_{S^C_{t+1}, S^\Pi_{t+1}}\right)\Delta\pi_{t+1} \end{array}\right)V(t+1,T)\right]\mathbf{1}_{\tau>t}. \end{aligned} \tag{7}$$

[1] The Taylor-Lagrange expansion of $\exp\left(-Lh_{t+1}\right)$ around zero is $1 - Lh_{t+1} + \frac{1}{2}L^2h^2_{t+1}\exp\left(-\xi\right)$ where $\xi \in \left[0, Lh_{t+1}\right]$.

As we did in the case of the default-risk free bond, we assume that $V(t,T)$ may be well approximated by a single exponential function,

$$V^*(t,T) = \exp\left(A^*_{T-t,S^C_t,S^\Pi_t} - B^{*c}_{T-t,S^C_t,S^\Pi_t}\Delta c_t - B^{*\pi}_{T-t,S^C_t,S^\Pi_t}\Delta\pi_t\right). \tag{8}$$

As shown in Appendix B, the coefficients $A^*_{T-t,S^C_t,S^\Pi_t}$, $B^{*c}_{T-t,S^C_t,S^\Pi_t}$ and $B^{*\pi}_{T-t,S^C_t,S^\Pi_t}$ are obtained recursively starting with $A^*_{0,S^C_T,S^\Pi_T} = B^{*c}_{0,S^C_T,S^\Pi_T} = B^{*\pi}_{0,S^C_T,S^\Pi_T} = 0$ . Notice that in the above formulation, we have used several approximations: (i) the conditional default probability $h_{t+1}$ is assumed to satisfy equation (6); (ii) $1 - Lh_{t+1}$ is well approximated by $\exp(-Lh_{t+1})$; (iii) $V(t,T)$ may be expressed as an exponential function, that is $V^*(t,T)$ is a good proxy for $V(t,T)$; and (iv) $\exp(x) \cong 1 + x$. In this setting the default spread $DS(t,T)$ is given by

$$\begin{aligned}
&DS(t,T) \\
&= \frac{-\ln V(t,T)}{T-t} + \frac{\ln P(t,T)}{T-t} \\
&\cong \frac{-\ln V^*(t,T)}{T-t} + \frac{\ln \widetilde{P}(t,T)}{T-t} \\
&= \frac{\widetilde{A}_{T-t,S^C_t,S^\Pi_t} - A^*_{T-t,S^C_t,S^\Pi_t} + \left(B^{*c}_{T-t,S^C_t,S^\Pi_t} - \widetilde{B}^c_{T-t,S^C_t,S^\Pi_t}\right)\Delta c_t + \left(B^{*\pi}_{T-t,S^C_t,S^\Pi_t} - \widetilde{B}^\pi_{T-t,S^C_t,S^\Pi_t}\right)\Delta\pi_t}{T-t}.
\end{aligned}$$

In appendix C we decompose the default spread into the default risk and the default risk premium. We obtain the default risk $DS^*(t,T)$ from the risky bond value in absence of the default risk premium. It is approximated by

$$DS^*(t,T) = \frac{-\widehat{A}_{T-t,S^C_t,S^\Pi_t} + \widehat{B}^c_{T-t,S^C_t,S^\Pi_t}\Delta c_t + \widehat{B}^\pi_{T-t,S^C_t,S^\Pi_t}\Delta\pi_t}{T-t}$$

where expressions for $\widehat{A}_{T-t,S^C_t,S^\Pi_t}$, $\widehat{B}^c_{T-t,S^C_t,S^\Pi_t}$, and $\widehat{B}^\pi_{T-t,S^C_t,S^\Pi_t}$ are given in Appendix C.

# 3 Parameters estimation

## 3.1 The empirical yield curves

Quaterly estimates of the yield-spread curves on zero-coupon bonds for each rating class are needed. These yield-spread curves are obtained by first estimating the parameters associated with the Nelson and Siegel curve fitting approach. Appendix D.1 provides a brief summary of this approach.

The data comes from the Lehman Brothers Fixed Income Database (Warga, 1998). We choose this data to enable comparisons with other articles in this literature using the same database. Moreover, the database covers two default cycles an issue that will become important in the analysis (see Dionne et al., 2005, for more details). The data contains information on monthly prices (quote and matrix), accrued interest, coupons, ratings, callability, and returns on all investment-grade corporate and government bonds for the period from January 1987 to December 1996. All bonds with matrix prices and options were eliminated; bonds not included in Lehman Brothers' bond indexes and bonds with an odd frequency of coupon payments were also dropped.[2] Appendix D.2, provides details on the treatment of accrued interest. All bonds with a pricing error higher than $5 are dropped. We then repeat this estimation and data removal procedure until all bonds with a pricing error larger than $5 have been eliminated. Using this procedure, 695 bonds were eliminated out of a total of 12,849 bonds found in the Baa industrial sector, which is the focus of this study. For government bonds, four bonds were eliminated out of a total of 13,552.

Let $y_{NS}^{R}(t,\bullet)$ be the Nelson-Siegel yield curve obtained for the $t$th quarter using a sample of bond prices with credit rating $R$ :

$$y_{NS}^{R}(t,T) = -\zeta_t (T-t) - \omega_t \eta_t \left(1 - \exp\left(-\frac{T-t}{\eta_t}\right)\right) - \psi_t (T-t) \exp\left(-\frac{T-t}{\eta_t}\right) \tag{9a}$$

where $T$ is the maturity date of the underlying bond and $\zeta_t$, $\omega_t$, $\psi_t$ and $\eta_t$ are some constants estimated for every quarter. Table 1 presents the measured bond spreads for industrial Baa during the 1987-1996 period.

### 3.2 Estimation of the Markov Switching parameters

In this section, we describe how we estimate the 20 parameters of the Markov Switching model where $\boldsymbol{\phi}$ denotes the set of transition probabilities and $\boldsymbol{\theta}$ contains the regime switching parameters :

$$\boldsymbol{\theta} = \left(a_1^c, a_2^c, b_1^c, b_2^c, a_1^\pi, a_2^\pi, b_1^\pi, b_2^\pi, \sigma_1^c, \sigma_2^c, \sigma_1^\pi, \sigma_2^\pi, \rho_{1,1}, \rho_{1,2}, \rho_{2,1}, \rho_{2,2}\right)$$
$$\boldsymbol{\phi} = \left(\phi_{11}^C, \phi_{22}^C, \phi_{11}^\Pi, \phi_{22}^\Pi\right).$$

[2] We did, however, keep three categories from the list of eliminations in Elton *et al.*(2001), because we lacked the information needed to identify them: government flower bonds, inflation-indexed government bonds, and bonds with floating rate debt.

We observe a time series of consumption $C_0, ..., C_T$ and a time series of inflation $\Pi_0, ..., \Pi_T$. We create the sample $\Delta c_1, ...\Delta c_T, \Delta\pi_1, ..., \Delta\pi_T$. Let

$$\mathbf{V}_t = (\mathbf{Y}_t, ..., \mathbf{Y}_1) \text{ where } \mathbf{Y}_t = (\Delta c_t, \Delta\pi_t)$$

denotes the observable sample up to time $t$, and

$$\mathbf{U}_t = (\mathbf{S}_t, ..., \mathbf{S}_1) \text{ where } \mathbf{S}_t = \left(S_t^C, S_t^\Pi\right)$$

be the unobservable states of consumption and inflation up to time $t$.

The log-likelihood function based on the observed sample $\mathbf{v}_T$ up to time $T$ is

$$\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\phi}; \mathbf{v}_T) = \ln f_{\mathbf{V}_T}(\mathbf{v}_T; \boldsymbol{\theta}, \boldsymbol{\phi}) = \sum_{t=2}^{T} \ln f_{\mathbf{Y}_t|\mathbf{V}_{t-1}}(\mathbf{y}_t | \mathbf{v}_{t-1}; \boldsymbol{\theta}, \boldsymbol{\phi}) \tag{10a}$$

where $f_{\mathbf{V}}(\mathbf{v})$ denotes the joint density function of the random vector $\mathbf{V}$ and $f_{\mathbf{Y}|\mathbf{V}}(\mathbf{y}|\mathbf{v})$ represents the conditional density function of the random vector $\mathbf{Y}$ given the random vector $\mathbf{V} = \mathbf{v}$. It is assumed that $f_{\mathbf{Y}_1}(\mathbf{y}_1; \boldsymbol{\theta}, \boldsymbol{\phi}) = 1$. As shown in Appendix E,

$$f_{\mathbf{Y}_t|\mathbf{V}_{t-1}}(\mathbf{y}_t | \mathbf{v}_{t-1}; \boldsymbol{\theta}, \boldsymbol{\phi}) = \boldsymbol{\eta}'_{t;\boldsymbol{\theta}} \boldsymbol{\xi}_{t|t-1;\boldsymbol{\theta},\boldsymbol{\phi}} \tag{10b}$$

where

$$\boldsymbol{\eta}_{t;\boldsymbol{\theta}} = \begin{pmatrix} f_{\mathbf{Y}_t|\mathbf{S}_t,\mathbf{V}_{t-1}}(\mathbf{y}_t | (1,1), \mathbf{v}_{t-1}; \boldsymbol{\theta}) \\ f_{\mathbf{Y}_t|\mathbf{S}_t,\mathbf{V}_{t-1}}(\mathbf{y}_t | (1,2), \mathbf{v}_{t-1}; \boldsymbol{\theta}) \\ f_{\mathbf{Y}_t|\mathbf{S}_t,\mathbf{V}_{t-1}}(\mathbf{y}_t | (2,1), \mathbf{v}_{t-1}; \boldsymbol{\theta}) \\ f_{\mathbf{Y}_t|\mathbf{S}_t,\mathbf{V}_{t-1}}(\mathbf{y}_t | (2,2), \mathbf{v}_{t-1}; \boldsymbol{\theta}) \end{pmatrix} \text{ and } \boldsymbol{\xi}_{t|u;\boldsymbol{\theta},\boldsymbol{\phi}} = \begin{pmatrix} f_{\mathbf{S}_t|\mathbf{V}_u}((1,1) | \mathbf{v}_u; \boldsymbol{\theta}, \boldsymbol{\phi}) \\ f_{\mathbf{S}_t|\mathbf{V}_u}((1,2) | \mathbf{v}_u; \boldsymbol{\theta}, \boldsymbol{\phi}) \\ f_{\mathbf{S}_t|\mathbf{V}_u}((2,1) | \mathbf{v}_u; \boldsymbol{\theta}, \boldsymbol{\phi}) \\ f_{\mathbf{S}_t|\mathbf{V}_u}((1,2) | \mathbf{v}_u; \boldsymbol{\theta}, \boldsymbol{\phi}) \end{pmatrix}.$$

The components of $\boldsymbol{\eta}_{t;\boldsymbol{\theta}}$ are computed analytically using the bivariate Gaussian density function. Indeed, from Markov Switching model, the conditional density function of $\mathbf{Y}_t = (\Delta c_t, \Delta\pi_t)$ given $\mathbf{y}_{t-1} = (\Delta c_{t-1}, \Delta\pi_{t-1})$ and the actual states of consumption and inflation $\mathbf{s}_t = \left(s_t^C, s_t^\Pi\right)$ is

$$f_{\mathbf{Y}_t|\mathbf{S}_t,\mathbf{Y}_{t-1}}(\mathbf{y}_t | \mathbf{s}_t, \mathbf{y}_{t-1}; \boldsymbol{\theta}) = \frac{1}{2\pi} \frac{\exp\left(-\frac{1}{2}(z_t^c)^2 - \frac{1}{2}(z_t^\pi)^2 + \rho_{s_t^C, s_t^\Pi} z_t^c z_t^\pi\right)}{\sigma^c_{s_t^C} \sigma^\pi_{s_t^\Pi} \sqrt{1 - \rho^2_{s_t^C, s_t^\Pi}}} \tag{10c}$$

with $z_t^c = \frac{\Delta c_t - a^c_{s_t^C} - b^c_{s_t^C} \Delta c_{t-1}}{\sigma^c_{s_t^C} \sqrt{1 - \rho^2_{s_t^C, s_t^\Pi}}}$ and $z_t^\pi = \frac{\Delta\pi_t - a^\pi_{s_t^\Pi} - b^\pi_{s_t^\Pi} \Delta\pi_{t-1}}{\sigma^\pi_{s_t^\Pi} \sqrt{1 - \rho^2_{s_t^C, s_t^\Pi}}}$. The vectors $\boldsymbol{\xi}_{2|1;\boldsymbol{\theta},\boldsymbol{\phi}}, ..., \boldsymbol{\xi}_{T|T-1;\boldsymbol{\theta},\boldsymbol{\phi}}$ are obtained recursively. More precisely, as shown in Appendix E,

$$\boldsymbol{\xi}_{t|t-1;\boldsymbol{\theta},\boldsymbol{\phi}} = \mathbf{P}'_{\boldsymbol{\phi}} \frac{\boldsymbol{\eta}'_{t-1;\boldsymbol{\theta}} \odot \boldsymbol{\xi}_{t-1|t-2;\boldsymbol{\theta},\boldsymbol{\phi}}}{\boldsymbol{\eta}'_{t-1;\boldsymbol{\theta}} \boldsymbol{\xi}_{t-1|t-2;\boldsymbol{\theta},\boldsymbol{\phi}}} \tag{10d}$$

where $\odot$ denotes element-by-element multiplication and $\mathbf{P}_{\boldsymbol{\phi}}$ is the transition matrix

$$\mathbf{P}_{\boldsymbol{\phi}} = \begin{pmatrix} \phi^C_{1,1}\phi^\Pi_{1,1} & \phi^C_{1,1}\phi^\Pi_{1,2} & \phi^C_{1,2}\phi^\Pi_{1,1} & \phi^C_{1,2}\phi^\Pi_{1,2} \\ \phi^C_{1,1}\phi^\Pi_{2,1} & \phi^C_{1,1}\phi^\Pi_{2,2} & \phi^C_{1,2}\phi^\Pi_{2,1} & \phi^C_{1,2}\phi^\Pi_{2,2} \\ \phi^C_{2,1}\phi^\Pi_{1,1} & \phi^C_{2,1}\phi^\Pi_{1,2} & \phi^C_{2,2}\phi^\Pi_{1,1} & \phi^C_{2,2}\phi^\Pi_{1,2} \\ \phi^C_{2,1}\phi^\Pi_{2,1} & \phi^C_{2,1}\phi^\Pi_{2,2} & \phi^C_{2,2}\phi^\Pi_{2,1} & \phi^C_{2,2}\phi^\Pi_{2,2} \end{pmatrix}.$$

To initialize the recursion, we let $\boldsymbol{\eta}_{1;\boldsymbol{\theta}} = (1,1,1,1)'$ and $\boldsymbol{\xi}_{1|0;\boldsymbol{\theta},\boldsymbol{\phi}}$ is set to the stationary distribution of the Markov chain associated with $\mathbf{P}_{\boldsymbol{\phi}}$. Using the independence between the evolution of the state of consumption and the state of inflation, the stationary distribution is obtained as the product of the stationary distribution of $S^C$ and $S^\Pi$ :

$$\boldsymbol{\xi}_{1|0;\boldsymbol{\theta},\boldsymbol{\phi}} = \boldsymbol{\xi}_{1|0;\boldsymbol{\phi}} = \begin{pmatrix} \frac{1-\phi^C_{2,2}}{1-\phi^C_{1,1}+1-\phi^C_{2,2}} \frac{1-\phi^\Pi_{2,2}}{1-\phi^\Pi_{1,1}+1-\phi^\Pi_{2,2}} \\ \frac{1-\phi^C_{2,2}}{1-\phi^C_{1,1}+1-\phi^C_{2,2}} \frac{1-\phi^\Pi_{1,1}}{1-\phi^\Pi_{1,1}+1-\phi^\Pi_{2,2}} \\ \frac{1-\phi^C_{1,1}}{1-\phi^C_{1,1}+1-\phi^C_{2,2}} \frac{1-\phi^\Pi_{2,2}}{1-\phi^\Pi_{1,1}+1-\phi^\Pi_{2,2}} \\ \frac{1-\phi^C_{1,1}}{1-\phi^C_{1,1}+1-\phi^C_{2,2}} \frac{1-\phi^\Pi_{1,1}}{1-\phi^\Pi_{1,1}+1-\phi^\Pi_{2,2}} \end{pmatrix}. \tag{10e}$$

The maximization of the log-likelihood function $\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\phi}; \mathbf{v}_T)$ given by Equation (10a) is done numerically using some starting point $(\boldsymbol{\theta}_0, \boldsymbol{\phi}_0)$ under the constraint that the four parameters of the vector $\boldsymbol{\phi}$ are between zero and one (probabilities). The estimates that maximize the log-likelihood function $\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\phi}; \mathbf{v}_T)$ are denoted $\left(\widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\phi}}\right)$.

We have observations on the growth rate of non-durable personal consumption expenditures per capita (real) from the first quarter of 1957 to the last quarter of 1996 and the growth rate of consumption price index for the same period (160 quarters). The data comes from the U.S. Department of Commerce : Bureau of Economic Analysis. The data period contains seven recessions according to the NBER and many of them should be important enough to generate regime shifts.[3] We only consider data up to the last quarter of 1996 because our data for the risky bonds covers the I-1987 to IV-1996 period. Figures 1 and 2 illustrate the temporal evolution of the two growth rates. It is interesting to observe that they are negatively related, particularly in recession periods.

[3] The official recession periods during our research period, according to the NBER, are: 1957-III to 1958-II, 1960-II to 1961-I, 1969-IV to 1970-IV, 1973-IV to 1975-I, 1980-I to 1980-III, 1981-III to 1982-IV, and 1990-III to 1991-I.

The result of the estimation procedure are presented in Table 2. Many parameters are statistically different from zero but are not necessarily different two by two, as shown in Table 3. For consumption, the regime switching seems to appear only in the volatility. For inflation, the autoregressive parameters as well as the volatility parameters are modified with the regime shifts. We also observe, in Table 2, that $\rho_{12}$ and $\rho_{22}$ are negative and statistically different from zero, which confirms a negative empirical correlation between consumption and inflation. As reported in Table 2, it seems reasonnable to assume that $a_1^c = a_2^c = a^c$, $b_1^c = b_2^c = b^c$, $a_1^\pi = a_2^\pi = a^\pi$. We also see in Table 2 that the correlation coefficients are not statistically different two by two. It is not clear, however, that we have applied the appropriate tests in Table 3 and the results should be interpreted with precaution as additional work is needed on this issue.

### 3.3 Estimation of the states of consumption and inflation

In estimating regime-switching models, two different conditional probabilities are of interest. The ex-ante probability, $\boldsymbol{\xi}_{t|t-\Delta t;\theta;\phi}$, is useful in forecasting future inflation and consumption rates based on an evolving information set. The smoothed probability, $\boldsymbol{\xi}_{t|T;\boldsymbol{\theta};\boldsymbol{\phi}}$, estimated using the entire information set available, is of interest in determining when regime shifts occurred in the sample period. We are more concerned with this second conditional probability.

To estimate the mass function $\boldsymbol{\xi}_{t|T;\boldsymbol{\theta},\boldsymbol{\phi}} = \left(f_{\mathbf{S}_t|\mathbf{V}_T}\left(\mathbf{s}_t\,|\mathbf{v}_T\,;\boldsymbol{\theta},\boldsymbol{\phi}\right)\right)_{\mathbf{s}_t\in\{(1,1),(1,2),(2,1),(2,2)\}}$ of $\mathbf{S}_t$ conditionnal upon the observed sample $\mathbf{v}_T$, we used an algorithm developed by Kim (1994) and described in Hamilton (1994) :

$$\widehat{\boldsymbol{\xi}}_{t|T;\widehat{\boldsymbol{\theta}},\widehat{\boldsymbol{\phi}}} = \widehat{\boldsymbol{\xi}}_{t|t;\widehat{\boldsymbol{\theta}},\widehat{\boldsymbol{\phi}}} \odot \left[\mathbf{P}_\phi\left(\widehat{\boldsymbol{\xi}}_{t+1|T;\widehat{\boldsymbol{\theta}},\widehat{\boldsymbol{\phi}}}\,(\div)\,\widehat{\boldsymbol{\xi}}_{t+1|t;\widehat{\boldsymbol{\theta}},\widehat{\boldsymbol{\phi}}}\right)\right]$$

where $(\div)$ means an element-by-element division.

We applied the estimation procedure to the same time series as for the estimation of the parameters of the Markov Switching process but restricted the analysis to the period I-1987 to IV-1996 which contains 40 quarters. This period corresponds to the data period we shall use for risky bonds information: for transition matrices and for recovery rates in the analysis of default risk. Note that we used the estimates of $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$ that has been determined in Section 3.2.

The results of the estimation procedure are presented in Figure 3. There are only two quarters for which there is not one of the four values of the mass function that clearly dominates the others. Indeed, at the third quarter of 1990, we obtain 0.5046 for the state (1,2) and 0.4950 for the state (2,2). At the second quarter of 1993, the mass function is 0.5439 for the state (1,1) and 0.4511 for the state (2,1). The interpretations of the estimated states are as follows: State (1,1) corresponds to less volatile consumption rates and lower level and less volatile inflation rates. State (1,2) is associated to less volatile consumption rates and higher level and more volatile inflation rates. State (2,1) corresponds to more volatile consumption rates and lower level and less volatile inflation rates. Finally, state (2,2) is associated to more volatile consumption rates and higher level and more volatile inflation rates.

The estimated states $\widehat{\mathbf{s}}_t$ at time $t$ maximize the conditional likelihood $f_{\mathbf{S}_t|\mathbf{V}_T}\left(\mathbf{s}_t\,|\mathbf{v}_T\,;\widehat{\boldsymbol{\theta}},\widehat{\boldsymbol{\phi}}\right)$, that is,

$$f_{\mathbf{S}_t|\mathbf{V}_T}\left(\widehat{\mathbf{s}}_t\,|\mathbf{v}_T\,;\widehat{\boldsymbol{\theta}},\widehat{\boldsymbol{\phi}}\right) \geq f_{\mathbf{S}_t|\mathbf{V}_T}\left(\mathbf{s}_t\,|\mathbf{v}_T\,;\widehat{\boldsymbol{\theta}},\widehat{\boldsymbol{\phi}}\right) \text{ for all } \mathbf{s}_t \in \{(1,1)\,;(1,2)\,,(2,1)\,,(2,2)\}\,.$$

The results are reported in Figure 4. The estimated state of consumption is 1 for two distinct time periods : 1987-I to 1990-III (15 quarters) and for 1993-II to 1996-IV (15 quarters). In between, the consumption's estimated state is 2 for the period 1990-IV to 1993-I (10 quarters). For the inflation, we note only one change of regime. Indeed, the state of inflation is estimated to 2 for the time period 1987-I to 1990-IV (16 quarters) and becomes 1 for the time period 1991-I to 1996-IV (24 quarters). If we consider the system globaly, the estimated state is (1,2) during the 15 first quarters (1987-I to 1993-III), it switches to the state (2,2) for only one quarter (1990-IV), goes to state (2,1) for 9 quarters (1991-I to 1993-I) and ends in state (1,1) for 15 quarters (1993-II to 1996-IV).

It is interesting to note that the observed average inflation growth rate and volatility are 0.32% and 0.33% during the I-91 to IV-96 period and 1.21% and 0.68% during the I-87 to VI-90 period. The observed average consumption growth rate and volatility are 0.39% and 0.34% during the two periods I-87 to III-90 and II-93 to IV-96 while they are -0.07% and 0.91% during the IV-90 to I-93 period.

Figures 5 and 6 illustrate the changes of regime behavior for the growth rate of personal consumption expeditures per capita and for the growth rate of price index respectively. The regimes are well related to the business cycles during that period. The consumption rate clearly exhibits different volatility behaviors, one for each regime. For the inflation, we note a change in the level as well as a variation of the volatility.

### 3.4 Estimation of the preference parameters

In this section, we explain how the impatience coefficients $\boldsymbol{\beta} = (\beta_{11}, \beta_{12}, \beta_{21}, \beta_{22})$ and the risk aversion coefficients $\boldsymbol{\gamma} = (\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22})$ are estimated.

We assume that the parameters $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$ of the Markov Switching processes are known and are set to their estimated value. The empirical gouvernmental zero-coupon yield $y^{\text{Gouv}}(t,T)$ for the $q$ th quarter is obtained from the Nelson and Siegel approach and is given by Equation (9a).

As argued in Dai, Singleton and Yang (2006), because the state of the economy is unknown at a particular point in time, we define the theoretical zero-coupon bond price as the expected bond price, with the expectation computed over the possible states of the Markov chain. Using $\widehat{\boldsymbol{\theta}}$ and $\widehat{\boldsymbol{\phi}}$, the estimated parameters for the Markov chain, the zero-coupon bond price at time $t$ is thus defined as

$$\bar{P}(t,T,\boldsymbol{\beta},\boldsymbol{\gamma}) = \sum_{\mathbf{s}_t} \hat{\xi}_{\mathbf{s}_t} \times \widetilde{P}\left(t,T,\mathbf{s}_t,\Delta c_t,\Delta\pi_t,\widehat{\boldsymbol{\theta}},\widehat{\boldsymbol{\phi}};\boldsymbol{\beta},\boldsymbol{\gamma}\right)$$

where $\mathbf{s}_t \in \{(1,1),(1,2),(2,1),(2,2)\}$ and $\hat{\xi}_{\mathbf{s}_t}$ is the estimated ex-ante probability of beeing in state $\mathbf{s}_t$ at time $t$ and $\widetilde{P}(\bullet)$ is the approximate zero-coupon bond price given by equation (3). The estimates of the preference parameters are obtained by minimizing the objective function with respect to $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ :

$$Q(t,\boldsymbol{\beta},\boldsymbol{\gamma}) = \sum_{k=8}^{40}\left(-\frac{\ln\bar{P}(t,t+k,\boldsymbol{\beta},\boldsymbol{\gamma})}{k} - y^{\text{Gouv}}(t,t+k)\right)^2. \tag{11a}$$

We use maturities up to ten years. We dropped the two first years (8 quaters) since, as it is well documented in the literature, the Nelson-Siegel model does not approximate short maturities very well (see Elton et al., 2001, for a discussion on this issue).

The objective function is related to a particular quarter. So we obtain some estimates of $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ that change throught time ($\widehat{\boldsymbol{\beta}}_t$ and $\widehat{\boldsymbol{\gamma}}_t$). This calibration procedure allows us to obtain a model which can accurately replicate the level and slope or the risk free term structure. This feature is important and will allow us to accurately determine the relation of the spread with maturity.

The model, in its current form, lacks parsimony and contains many parameters to be estimated. For this reason, we estimate a restricted version with $\beta_{1,1} = \beta_{1,2} = \beta_{2,1} = \beta_{2,2} = \beta$, $\gamma_{1,1} =$

$\gamma_{1,2} = \gamma_1$ and $\gamma_{2,1} = \gamma_{2,2} = \gamma_2$ leaving three parameters to be estimated each quarter instead of eight. The results of the calibration are presented in Figures 7 and 8. It is interesting to observe that most of the estimated $\gamma_1$ and $\gamma_2$ are well below 15 and all estimated $\beta$ are around 1. In fact, the average $\beta$ is equal to 1.0 and the averages $\gamma_1$ and $\gamma_2$ are equal to 3.8 and 5.7 respectively. It is also important to emphasize that the estimated $\gamma$ is higher in the more volatile state of consumption. It should be noted that the estimated subjective discount factor can be larger than one in a growth economy (Kocherlakota, 1990). Note that the one period discount factor is $M_{t,t+1} = \exp\left(\ln \beta_{S^C_{t+1},S^\Pi_{t+1}} - \gamma_{S^C_{t+1},S^\Pi_{t+1}} \Delta c_{t+1} - \Delta\pi_{t+1}\right)$. Therefore, we expect $\ln \beta_{S^C_{t+1},S^\Pi_{t+1}} - \gamma_{S^C_{t+1},S^\Pi_{t+1}} \Delta c_{t+1} - \Delta\pi_{t+1}$ to be negative and this could happen even if $\beta$ is greater than one.

In order to study how good the model fits the data, we report the root mean squared error (RMSE), the average absolute error (AAE) and the average error (AE) in Table 4. Figures 9, 10 and 11 illustrate the evolution of the yield to maturities of 1, 2 and 10 years respectively. It can be noticed that the differences between the yields associated with $\widetilde{P}$ and the empirical yields are larger for a maturity of one year and that the fit is very good for the maturities of two years and more. This was expected since the interest rates with a maturity below two years were not used to calibrate our model. The inclusion of these bonds lead to a better fit for the very small maturities and a deterioration of the fit for the longer maturities.

### 3.5 Estimation of conditional default probability parameters

We describe here the calibration procedure for the conditional default probability parameters

$$\boldsymbol{\alpha} = \left(\alpha_{1,1}, \alpha_{1,2}, \alpha_{2,1}, \alpha_{2,2}, \alpha^c_{1,1}, \alpha^c_{1,2}, \alpha^c_{2,1}, \alpha^c_{2,2}, \alpha^\pi_{1,1}, \alpha^\pi_{1,2}, \alpha^\pi_{2,1}, \alpha^\pi_{2,2}\right)$$

required by our corporate bond pricing model. As shown in Appendix E.2, the theoritical survival probabilities at time $t$ are

$$P_{\mathcal{G}_t}\left[\tau > t+s \,|\tau > t\right] \cong \mathrm{E}_{\mathcal{G}_t}\left[\exp\left(-\sum_{u=t+1}^{t+s}\left(\alpha_{S^C_u,S^\Pi_u} + \alpha^c_{S^C_u,S^\Pi_u}\Delta c_u + \alpha^\pi_{S^C_u,S^\Pi_u}\Delta\pi_u\right)\right)\right].$$

As for the default-risk free zero-coupon bond value we based our estimation strategy on an approximation similar to those used for the risk-free and corporate zero-coupon bond models. We thus assume that $P_{\mathcal{G}_t}[\tau > t+s\,|\tau > t] \cong p\left(t, t+s, S_t^C, S_t^\Pi, \Delta c_t, \Delta\pi_t; \boldsymbol{\alpha}, \boldsymbol{\theta}, \boldsymbol{\phi}\right)$ where

$$p\left(t, t+s, S_t^C, S_t^\Pi, \Delta c_t, \Delta\pi_t; \boldsymbol{\alpha}, \boldsymbol{\theta}, \boldsymbol{\phi}\right) = \exp\left(-\overline{A}_{s,S_t^C,S_t^\Pi} - \overline{B}^c_{s,S_t^C,S_t^\Pi}\Delta c_t - \overline{B}^\pi_{s,S_t^C,S_t^\Pi}\Delta\pi_t\right). \quad (12)$$

The coefficients $\overline{A}_{s,S_t^C,S_t^\Pi}$, $\overline{B}^c_{s,S_t^C,S_t^\Pi}$ and $\overline{B}^\pi_{s,S_t^C,S_t^\Pi}$ are obtained recursively starting with $\overline{A}_{0,S_t^C,S_t^\Pi} = \overline{B}^c_{0,S_t^C,S_t^\Pi} = \overline{B}^\pi_{0,S_t^C,S_t^\Pi} = 0$ and using an approximation based on the Taylor expansion of $\exp(x)$. Their expressions are available in Appendix E.2.

We can observe a term structure of survival probabilities via credit rating transition matrices. We estimate these rating transitions through the generator of the Markov chain underlying the rating migration, as in Lando and Skodeberg (2002) and Christensen, Hansen and Lando (2004). These studies suggest estimating a Markov-process generator rather than the one-year transition matrix. Lando and Skodeberg (2002) have shown that this continuous-time analysis of rating transitions using generator matrices improves the estimates of rare transitions even when they are not observed in the data, a result that cannot be obtained with the discrete-time analysis of Carty and Fons (1993) and Carty (1997). A continuous-time analysis of defaults permits estimates of default probabilities even for cells that have no defaults. The rating transition histories used to estimate the generator are taken from Moody's Corporate Bond Default Database (January, 09, 2002). A precise description of the data used to obtain the transition estimates is given in Appendix E.3.

Using default data from 1987 to 1996, a generator matrix is estimated. The estimated generator matrix and the estimated one-year transition probability matrix are presented in tables 5 and 6. A term structure of survival probabilities is then obtained. To get the parameter estimates for $\boldsymbol{\alpha}$, we minimize the squared errors between the average of both the theoretical and the empirical survival probabilities. Again, as in the case of the theoretical risk-free bond prices, we define the survival probability as the expected survival probability, with the expectation taken over the regime of the Markov chain. More formally, the expected survival probability is defined as :

$$\bar{p}(t, t+u, \boldsymbol{\alpha}) = \sum_{\mathbf{s}_t} \hat{\xi}_{\mathbf{s}_t} \times p\left(t, t+u, \mathbf{s}_t, \Delta c_t, \Delta\pi_t; \boldsymbol{\alpha}, \widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\phi}}\right)$$

where $\mathbf{s}_t \in \{(1,1), (1,2), (2,1), (2,2)\}$ and $\hat{\xi}_{\mathbf{s}_t}$ is the estimated ex-ante probability of beeing in state $\mathbf{s}_t$ at time $t$ and $p(\bullet)$ is given by equation (12). Let $p^{\text{obs}}(t, t+u)$ be the "observed" probability.

To get the estimates for $\boldsymbol{\alpha}$,we minimize the expression below with respect to $\boldsymbol{\alpha}$:

$$R(\boldsymbol{\alpha}) = \frac{1}{40} \sum_{t} \sum_{u=1}^{40} \left( p^{\text{obs}} \left[ \tau > t + u \,|\tau > t \right] - \bar{p}\left(t, t+u, \boldsymbol{\alpha}\right) \right)^2$$

with $t \in \{4, 8, ..., 36, 40\}$. The minimisation of the above function is done numerically using some starting point $\boldsymbol{\alpha}_0$ under the constraint that the one-period conditional default probability is non-negative. The estimated parameters of the conditional default probability are given in Table 7. Figure 12 reproduces the estimated conditional default probability. It is interesting to see that the conditional default probability is time varying and exhibit a regime shift. Particularly, it is higher during the high inflation regime and increases dramatically in the period of economic recession. Moreover, the conditional default probability is negatively correlated with the real consumption growth rate (-0.35 over the period 1987-1996). Hence, when the real consumption increases the default probability decreases. This can be explained by the positive relation between the firm's cash flows and the consumption level. The conditional default probability is negatively correlated with the inflation rate in the low inflation regime (-0.14 during the period 1987-1990) and positively correlated with the inflation rate in the high inflation regime (0.31 over the period 1991-1996). The correlation between the conditional default probability and the risk free interest rate is negative in all states. This correlation is equal to -0.14 for the period 1987-1996. Altman and Brady (2001) find the same result with observed default probabilities. This result is also consistent with the negative correlation between credit spreads and interest rates implied by structural credit models and documented in recent empirical papers (Duffee, 1998; Collin-Dufresne et al., 2001). The positive relation between the conditional default probability and the inflation rate during the high inflation regime explains why the conditional default probability is higher over the period 1987-1990 as shown in Figure 12. This period doesn't coincide exactly with the economic recession as specified by the NBER (III-90 to I-91). The main reason is that the default probability regime is determined in our model by the high inflation regime (I-1987 to IV-1990) that is quite different from the economic recession.

## 4 Default Spread in Baa Corporate Yield Spread

The main objective of this research is to compute empirically the proportion of corporate yield spread explained by the default risk and the default risk premium. We also show how the default spread is decomposed into the default risk and the default risk premium. We have all the estimated parameters to proceed to this decomposition with one exception, the recovery rate. For the moment, the recovery rate $\rho_\tau = \rho$ will be assumed constant through time and fixed at the average recovery rates during the 1987-1996 period, that is 36.67% for industrials. The data comes from Moody's.

Figure 13 presents the credit spread for ten-year Baa bonds. Figure 14 reproduces the evolution of the default spread for ten-year Baa bonds implied by our model. The default spread is the sum of the default risk and the default risk premium. Table 8 presents the average default spread proportions for Baa bonds under the switching regime model. We observe that the average default spread proportion over the period 1987-1996 is 41% for 10 years to maturity Baa zero-coupon bonds. The average default spread proportion depends on the default probability regime. The average default spread proportion is 48.5% in the high default regime and 35.5% in the low default regime. More interesting, as shown in figure 15, the default spread proportion can attain 72.4% in the high default regime and 25% in the low default regime.

Figure 16 presents the credit spread term structure of Baa industrial bonds for the period 1987-1996. It shows how this term structure can be decomposed into three parts: the default risk, the default risk premium and the other unexplained components of credit spread. We observe how the default spread proportion (41%) is decomposed into 40% for default risk and 1% for the default risk premium. Table 9 presents the corresponding premiums and proportion values. The default risk premium prportion is positive and low because the correlation between the conditional default probability and the risk free interest rate is negative and low (-0.14 over the period 1987-1996). However, the default risk premium is time varying and can reach 4% of observed 10 years corporate spreads. These results can be compared to those in the literature.

Longstaff et al (2005) find that default component, measured by default swap premium, represents 68% of five year Baa corporate bond spreads. This proportion is about 71% when they use a reduced form model. The main difference with our results can be attributed to the fact that they use risk neutral default intensity instead of observed default probabilities.

Our results are in concordance on average with those in Elton et al (2001) and Huang and Huang (2003), and other researches that find that default risk represents on average only a small fraction of corporate bond spreads. In the Elton et al (2001) article, the 25% default risk proportion is obtained by assuming that risk free rates and default intensity are independent and a recovery rate equal to 50%. Moreover, they show that the residual risk premium can account for about 40% of Baa corporate bond spreads. These results shoud be interpreted with caution since default probabilities and default spreads exhibit a regime shift. In our model we introduce both, a correlation between risk free rates and default intensity, and a regime shift risk. We think that the regime shift risk explains the main of the difference between our results and those of Elton and al (2001) since the default risk premium in our model accounts for a small fraction of corporate bond spreads.

Finally, Chen et al (2005) show that a large variation in time-varying risk premiums and a default boundary (that is more countercyclical than those corresponding to standard structural models) are essential ingredients for explaining the credit spread level puzzle. As suggested by Sundaresan (2005), their model overstates the need to have a time varying risk premium since it assumes that firm's cash flows and default probabilities are independent of macroeconomic states.

# 5 Conclusion

We have proposed a new approach to estimate the proportion of default spread in corporate yield spreads. Our model adds a macroeconomic risk premium to default risk in a non-structural framework. The macroeconomic risk premium is modeled by introducing risk aversion and impatience parameters in the pricing kernel that are function of discrete regime shifts in consumption and inflation. The parameters of the consumption, inflation and conditional default probability variations over time are also function of the discrete regime shifts. For the moment the recovery rate is not estimated in function of regime shifts. Our model contains two independent Markov chain processes one for the growth rate of inflation and one for the growth rate of consumption.

In a first step, we have derived a recursive formula for the risk-free bond prices in different states using a log-linear approximation. Our estimation period is I-1987 to IV-1996. The estimated regimes are well related to the business cycles during that period. The consumption rate exhibits different volatility behaviors while the inflation rate exhibits a change in level as well a variation in volatility. The estimated risk aversion parameters and impatience parameters are reasonable and coherent with the theoretical predictions. We also obtained a very good approximation of the governmental bond yield curve that we use as benchmark to compute the proportion of default spreads in the corporate yield spreads.

We then extended the basic model to introduce our default risk premium in the price of risky bonds. For the moment the empirical analysis is limited to industrial Baa bonds. Our data comes from Moody's database. Our results indicate that default probabilities and default spreads exhibit a regime shift. Hence, the average default spread proportion depends on the default probability regime. The average default spread proportion is 48.5% in the high default regime and 35.5% in the low default regime. More interesting, the default spread proportion can attain 72.4% in the high default regime and 25% in the low default regime.

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# Appendices

## A The default-risk free zero-coupon bond

### A.1 The exact solution

In this section, we derive the time $t$ value of a default-risk free zero-coupon bond. Applying equation (2) recursively, the time $t$ value of a default-risk free zero-coupon bond that pays one dollar at time $T$ $(t < T)$ is

$$P\left(t, T, S_t^C, S_t^\Pi\right) = \mathrm{E}_{\mathcal{G}_t}\left[M_{t,T}\right] = \mathrm{E}_{\mathcal{G}_t}\left[\exp\left(\sum_{u=t+1}^{T} \ln \beta_{S_u^C, S_u^\Pi} - \sum_{u=t+1}^{T} \gamma_{S_u^C, S_u^\Pi} \Delta c_u - \sum_{u=t+1}^{T} \Delta \pi_u\right)\right]$$

where $M_{t,T} = \prod_{u=t}^{T-1} M_{u,u+1}$ is the discount factor for the time period $]t, T]$. We want to express $\Delta c_u$ and $\Delta \pi_u$ in terms of their respective value at time $t$, some independent error terms and some constant. One can show by induction that for any integers $t$ and $u$ such that $t < u$,

$$\Delta c_u = \sum_{v=t+1}^{u} \left(a_{S_v^C}^c \prod_{w=v+1}^{u} b_{S_w^C}^c\right) + \left(\prod_{w=t+1}^{u} b_{S_w^C}^c\right) \Delta c_t + \sum_{v=t+1}^{u} \left(\prod_{w=v+1}^{u} b_{S_w^C}^c\right) e_v^c,$$
$$\Delta \pi_u = \sum_{v=t+1}^{u} \left(a_{S_v^\Pi}^\pi \prod_{w=v+1}^{u} b_{S_w^\Pi}^\pi\right) + \left(\prod_{w=t+1}^{u} b_{S_w^\Pi}^\pi\right) \Delta \pi_t + \sum_{v=t+1}^{u} \left(\prod_{w=v+1}^{u} b_{S_w^\Pi}^\pi\right) e_v^\pi.$$

From replacing $\Delta c_u$ and $\Delta \pi_u$ in the previous expression for $P(t, T)$, reorganizing the terms and noticing that $\sum_{u=t+1}^{T} \sum_{v=t+1}^{u} a_u b_v = \sum_{v=t+1}^{T} \left(\sum_{u=v}^{T} a_u\right) b_v$, one can show that

$$\begin{aligned} &P\left(t, T, S_t^C, S_t^\Pi\right) \\ &= \mathrm{E}_{\mathcal{G}_t}\left[\exp\left(\begin{array}{c} \sum_{u=t+1}^{T}\left[\ln \beta_{S_u^C, S_u^\Pi} - \sum_{v=t+1}^{u}\left(\gamma_{S_u^C, S_u^\Pi} a_{S_v^C}^c \left(\prod_{w=v+1}^{u} b_{S_w^C}^c\right) + a_{S_v^\Pi}^\pi \left(\prod_{w=v+1}^{u} b_{S_w^\Pi}^\pi\right)\right)\right] \\ - \left[\sum_{u=t+1}^{T} \gamma_{S_u^C, S_u^\Pi} \left(\prod_{w=t+1}^{u} b_{S_w^C}^c\right)\right] \Delta c_t - \left[\sum_{u=t+1}^{T} \left(\prod_{w=t+1}^{u} b_{S_w^\Pi}^\pi\right)\right] \Delta \pi_t \\ - \sum_{v=t+1}^{T}\left[\left[\sum_{u=v}^{T} \gamma_{S_u^C, S_u^\Pi} \left(\prod_{w=v+1}^{u} b_{S_w^C}^c\right)\right] e_v^c + \left[\sum_{u=v}^{T} \left(\prod_{w=v+1}^{u} b_{S_w^\Pi}^\pi\right)\right] e_v^\pi\right] \end{array}\right)\right] \end{aligned}$$

Using embedded conditional expectation rule $\left(\mathrm{E}_{\mathcal{G}_t}[\bullet] = \mathrm{E}_{\mathcal{G}_t}\left[\mathrm{E}_{\mathcal{G}_t}\left[\bullet \middle| S_{t+1}^C, S_{t+1}^\Pi, ..., S_T^C, S_T^\Pi\right]\right]\right)$ and since for a Gaussian random variable $Z$, $\mathrm{E}[\exp Z] = \exp\left(\mathrm{E}[Z] + \frac{1}{2}\mathrm{Var}[Z]\right)$, the time $t$ value of the bond is given by

$$\begin{aligned} &P\left(t, T, S_t^C, S_t^\Pi, \Delta c_t, \Delta \pi_t\right) \\ &= \sum_{k=t+1}^{T} \sum_{i_k=1}^{2} \sum_{\ell_k=1}^{2} \left(\begin{array}{c} \left[\phi_{S_t^C, i_{t+1}}^C \phi_{S_t^\Pi, \ell_{t+1}}^\Pi \left(\prod_{m=t+1}^{T-1} \phi_{i_m, i_{m+1}}^C \phi_{\ell_m, \ell_{m+1}}^\Pi\right)\right] \\ \times \exp\left(A_{i_{t+1}, \ell_{t+1}, ..., i_T, \ell_T} - B_{i_{t+1}, \ell_{t+1}, ..., i_T, \ell_T}^c \Delta c_t - B_{\ell_{t+1}, ..., \ell_T}^\pi \Delta \pi_t\right) \end{array}\right) \end{aligned} \quad (13a)$$

where

$$A_{S_{t+1}^C,S_{t+1}^\Pi,\dots,S_T^C,S_T^\Pi} = \sum_{u=t+1}^{T}\left[\ln\beta_{S_u^C,S_u^\Pi} - \gamma_{S_u^C,S_u^\Pi}\sum_{v=t+1}^{u} a^c_{S_v^C}\left(\prod_{w=v+1}^{u} b^c_{S_w^C}\right) - \sum_{v=t+1}^{u} a^\pi_{S_v^\Pi}\left(\prod_{w=v+1}^{u} b^\pi_{S_w^\Pi}\right)\right]$$

$$+\sum_{v=t+1}^{T}\left[\begin{array}{c} \frac{1}{2}\left[\sum_{u=v}^{T}\gamma_{S_u^C,S_u^\Pi}\left(\prod_{w=v+1}^{u} b^c_{S_w^C}\right)\right]^2\left(\sigma^c_{S_v^C}\right)^2 + \frac{1}{2}\left[\sum_{u=v}^{T}\left(\prod_{w=v+1}^{u} b^\pi_{S_w^\Pi}\right)\right]^2\left(\sigma^\pi_{S_v^\Pi}\right)^2 \\ +\left[\sum_{u=v}^{T}\gamma_{S_u^C,S_u^\Pi}\left(\prod_{w=v+1}^{u} b^c_{S_w^C}\right)\right]\left[\sum_{u=v}^{T}\left(\prod_{w=v+1}^{u} b^\pi_{S_w^\Pi}\right)\right]\rho_{S_v^C,S_v^\Pi}\sigma^c_{S_v^C}\sigma^\pi_{S_v^\Pi}\end{array}\right], \tag{13b}$$

$$B^c_{S_{t+1}^C,S_{t+1}^\Pi,\dots,S_T^C,S_T^\Pi} = \sum_{u=t+1}^{T}\gamma_{S_u^C,S_u^\Pi}\left(\prod_{w=t+1}^{u} b^c_{S_w^C}\right), \tag{13c}$$

$$B^\pi_{S_{t+1}^\Pi,\dots,S_T^\Pi} = \sum_{u=t+1}^{T}\left(\prod_{w=t+1}^{u} b^\pi_{S_w^\Pi}\right). \tag{13d}$$

Note that we define $\sum_{u=t+1}^{t}(\bullet) = 0$ and $\prod_{u=t+1}^{t}(\bullet) = 1$ whenever it happens.

## A.2 The approximated default-risk free zero-coupon bond value

In this section, we derive the functions $\widetilde{A}_{T-t,S_t^C,S_t^\Pi}, \widetilde{B}^c_{T-t,S_t^C,S_t^\Pi}$, and $\widetilde{B}^\pi_{T-t,S_t^C,S_t^\Pi}$ contained in the approximated zero-coupon bond value $\widetilde{P}\left(t,T,S_t^C,S_t^\Pi\right)$ knowing the actual states of consumption and inflation. It is based on two major approximations : (i) the time $t$ value of the zero-coupon bond given the actual states of consumption and inflation, $P\left(t,T,S_t^C,S_t^\Pi\right)$, is well approximated by an exponential function (instead of a sum of exponential functions) and (ii) the function $\exp(x)$ may be replaced by its Taylor expansion around zero truncated after the second term, that is, $\exp(x) \cong 1+x$.

Starting from equation (2), we have

$$1 = \mathrm{E}^P_{\mathcal{G}_t}\left[M_{t,t+1}\frac{P\left(t+1,T,S_{t+1}^C,S_{t+1}^\Pi\right)}{P\left(t,T,S_t^C,S_t^\Pi\right)}\right] \cong \mathrm{E}^P_{\mathcal{G}_t}\left[M_{t,t+1}\frac{\widetilde{P}\left(t+1,T,S_{t+1}^C,S_{t+1}^\Pi\right)}{\widetilde{P}\left(t,T,S_t^C,S_t^\Pi\right)}\right].$$

Substituting $M_{t,t+1}$, $\widetilde{P}\left(t,T,S_t^C,S_t^\Pi\right)$ and $\widetilde{P}\left(t+1,T,S_{t+1}^C,S_{t+1}^\Pi\right)$ using equations (**??**) and (3), applying embedded conditional expectation rule $\mathrm{E}^P_{\mathcal{G}_t}[\bullet] = \mathrm{E}^P_{\mathcal{G}_t}\left[\mathrm{E}^P_{\mathcal{G}_t}\left[\bullet\middle| S_{t+1}^C, S_{t+1}^\Pi\right]\right]$, and using the fact that for a Gaussian random variable $Z$, $\mathrm{E}[\exp Z] = \exp\left(\mathrm{E}[Z] + \frac{1}{2}\mathrm{Var}[Z]\right)$, we get

$$\sum_{i=1}^{2}\sum_{j=1}^{2}\phi^C_{S_t^C,i}\phi^\Pi_{S_t^\Pi,j}\exp\left(\begin{array}{c}\ln\beta_{i,j} + \widetilde{A}_{T-t-1,i,j} - \widetilde{A}_{T-t,S_t^C,S_t^\Pi} + \widetilde{B}^c_{T-t,S_t^C,S_t^\Pi}\Delta c_t + \widetilde{B}^\pi_{T-t,S_t^C,S_t^\Pi}\Delta\pi_t \\ -\left(\widetilde{B}^c_{T-t-1,i,j}+\gamma_{i,j}\right)\left(a^c_i + b^c_i\Delta c_t\right) \\ -\left(\widetilde{B}^\pi_{T-t-1,i,j}+1\right)\left(a^\pi_j + b^\pi_j\Delta\pi_t\right) \\ +\frac{1}{2}\left(\widetilde{B}^c_{T-t-1,i,j}+\gamma_{i,j}\right)^2\left(\sigma^c_i\right)^2 + \frac{1}{2}\left(\widetilde{B}^\pi_{T-t-1,i,j}+1\right)^2\left(\sigma^\pi_j\right)^2 \\ +\left(\widetilde{B}^c_{T-t-1,i,j}+\gamma_{i,j}\right)\left(\widetilde{B}^\pi_{T-t-1,i,j}+1\right)\rho_{i,j}\sigma^c_i\sigma^\pi_j\end{array}\right) \cong 1.$$

Because $\exp(x) \cong 1 + x$ for $x$ in the neighborhood of zero and since $\sum_{i=1}^{2} \sum_{j=1}^{2} \phi^{C}_{S^C_t,i} \phi^{\Pi}_{S^\Pi_t,j} = 1$,

$$0 \cong -\widetilde{A}_{T-t,S^C_t,S^\Pi_t} + \sum_{i=1}^{2} \sum_{j=1}^{2} \phi^{C}_{S^C_t,i} \phi^{\Pi}_{S^\Pi_t,j} \left( \begin{array}{c} \widetilde{A}_{T-t-1,i,j} + \ln \beta_{i,j} \\ -a^c_i \left( \widetilde{B}^c_{T-t-1,i,j} + \gamma_{i,j} \right) - a^\pi_j \left( \widetilde{B}^\pi_{T-t-1,i,j} + 1 \right) \\ +\frac{1}{2} \left( \widetilde{B}^c_{T-t-1,i,j} + \gamma_{i,j} \right)^2 \left( \sigma^c_{S^C_{t+1}} \right)^2 + \frac{1}{2} \left( \widetilde{B}^\pi_{T-t-1,i,j} + 1 \right)^2 \left( \sigma^\pi_j \right)^2 \\ + \left( \widetilde{B}^c_{T-t-1,i,j} + \gamma_{i,j} \right) \left( \widetilde{B}^\pi_{T-t-1,i,j} + 1 \right) \rho_{i,j} \sigma^c_i \sigma^\pi_j \end{array} \right)$$
$$+ \left( \widetilde{B}^c_{T-t,S^C_t,S^\Pi_t} - \sum_{i=1}^{2} \sum_{j=1}^{2} \phi^{C}_{S^C_t,i} \phi^{\Pi}_{S^\Pi_t,j} b^c_i \left( \widetilde{B}^c_{T-t-1,i,j} + \gamma_{i,j} \right) \right) \Delta c_t$$
$$+ \left( \widetilde{B}^\pi_{T-t,S^C_t,S^\Pi_t} - \sum_{i=1}^{2} \sum_{j=1}^{2} \phi^{C}_{S^C_t,i} \phi^{\Pi}_{S^\Pi_t,j} b^\pi_j \left( \widetilde{B}^\pi_{T-t-1,i,j} + 1 \right) \right) \Delta \pi_t$$

Therefore, we set the coefficients in front of $\Delta c_t$ and $\Delta \pi_t$ and the remaining term equal to zero to obtain the following:

$$\widetilde{A}_{T-t,S^C_t,S^\Pi_t} = \sum_{i=1}^{2} \sum_{j=1}^{2} \phi^{C}_{S^C_t,i} \phi^{\Pi}_{S^\Pi_t,j} \left( \begin{array}{c} \widetilde{A}_{T-t-1,i,j} + \ln \beta_{i,j} \\ -a^c_i \left( \widetilde{B}^c_{T-t-1,i,j} + \gamma_{i,j} \right) - a^\pi_j \left( \widetilde{B}^\pi_{T-t-1,i,j} + 1 \right) \\ +\frac{1}{2} \left( \widetilde{B}^c_{T-t-1,i,j} + \gamma_{i,j} \right)^2 \left( \sigma^c_i \right)^2 + \frac{1}{2} \left( \widetilde{B}^\pi_{T-t-1,i,j} + 1 \right)^2 \left( \sigma^\pi_j \right)^2 \\ + \left( \widetilde{B}^c_{T-t-1,i,j} + \gamma_{i,j} \right) \left( \widetilde{B}^\pi_{T-t-1,i,j} + 1 \right) \rho_{i,j} \sigma^c_i \sigma^\pi_j \end{array} \right),$$
$$\widetilde{B}^c_{T-t,S^C_t,S^\Pi_t} = \sum_{i=1}^{2} \sum_{j=1}^{2} \phi^{C}_{S^C_t,i} \phi^{\Pi}_{S^\Pi_t,j} b^c_i \left( \widetilde{B}^c_{T-t-1,i,j} + \gamma_{i,j} \right),$$
$$\widetilde{B}^\pi_{T-t,S^C_t,S^\Pi_t} = \sum_{i=1}^{2} \sum_{j=1}^{2} \phi^{C}_{S^C_t,i} \phi^{\Pi}_{S^\Pi_t,j} b^\pi_j \left( \widetilde{B}^\pi_{T-t-1,i,j} + 1 \right).$$

# B The approximated risky zero-coupon bond value

In this section, we derive the functions $A^*_{T-t,S^C_t,S^\Pi_t}$, $B^{*c}_{T-t,S^C_t,S^\Pi_t}$, and $B^{*\pi}_{T-t,S^C_t,S^\Pi_t}$ contained in the approximated risky zero-coupon bond value $V^*\left(t, T, S^C_t, S^\Pi_t\right)$ knowing the actual states of consumption and inflation. It is based on two major approximations : (i) the time $t$ value of the zero-coupon bond given the actual states of consumption and inflation, $V\left(t, T, S^C_t, S^\Pi_t\right)$, is well approximated by an exponential function and (ii) the function $\exp(x)$ may be replaced by its Taylor expansion around zero truncated after the second term, that is, $\exp(x) \cong 1 + x$.

Starting from equation (7), we have

$$
\begin{aligned}
1 &\cong \mathrm{E}_{\mathcal{G}_t}^{P}\left[\exp\begin{pmatrix} \left(\ln\beta_{S_{t+1}^C,S_{t+1}^\Pi} - L\alpha_{S_{t+1}^C,S_{t+1}^\Pi}\right) \\ -\left(\gamma_{S_{t+1}^C,S_{t+1}^\Pi} + L\alpha^c_{S_{t+1}^C,S_{t+1}^\Pi}\right)\Delta c_{t+1} \\ -\left(1+L\alpha^\pi_{S_{t+1}^C,S_{t+1}^\Pi}\right)\Delta\pi_{t+1}\end{pmatrix}\frac{V\left(t+1,T,S_{t+1}^C,S_{t+1}^\Pi\right)}{V\left(t,T,S_t^C,S_t^\Pi\right)}\right] \\
&\cong \mathrm{E}_{\mathcal{G}_t}^{P}\left[\exp\begin{pmatrix} \left(\ln\beta_{S_{t+1}^C,S_{t+1}^\Pi} - L\alpha_{S_{t+1}^C,S_{t+1}^\Pi}\right) \\ -\left(\gamma_{S_{t+1}^C,S_{t+1}^\Pi} + L\alpha^c_{S_{t+1}^C,S_{t+1}^\Pi}\right)\Delta c_{t+1} \\ -\left(1+L\alpha^\pi_{S_{t+1}^C,S_{t+1}^\Pi}\right)\Delta\pi_{t+1}\end{pmatrix}\frac{V^*\left(t+1,T,S_{t+1}^C,S_{t+1}^\Pi\right)}{V^*\left(t,T,S_t^C,S_t^\Pi\right)}\right].
\end{aligned}
$$

Substituting $V^*\left(t,T,S_t^C,S_t^\Pi\right)$ and $V^*\left(t+1,T,S_{t+1}^C,S_{t+1}^\Pi\right)$ using equation (10a), applying embedded conditional expectation rule $\mathrm{E}_{\mathcal{G}_t}^P[\bullet] = \mathrm{E}_{\mathcal{G}_t}^P\left[\mathrm{E}_{\mathcal{G}_t}^P\left[\bullet|\, S_{t+1}^C, S_{t+1}^\Pi\right]\right]$, and using the fact that for a Gaussian random variable $Z$, $\mathrm{E}[\exp Z] = \exp\left(\mathrm{E}[Z] + \frac{1}{2}\mathrm{Var}[Z]\right)$, we get

$$
\sum_{i=1}^{2}\sum_{j=1}^{2}\phi^C_{S_t^C,i}\phi^\Pi_{S_t^\Pi,j}\exp\begin{pmatrix}
\ln\beta_{i,j} - L\alpha_{i,j} + A^*_{T-t-1,i,j} - A^*_{T-t,S_t^C,S_t^\Pi} \\
-\left(\gamma_{i,j} + L\alpha^c_{i,j} + B^{*c}_{T-t-1,i,j}\right)a_i^c - \left(1+L\alpha^\pi_{i,j} + B^{*\pi}_{T-t-1,i,j}\right)a_j^\pi \\
+\frac{1}{2}\left(\gamma_{i,j} + L\alpha^c_{i,j} + B^{*c}_{T-t-1,i,j}\right)^2\left(\sigma_i^c\right)^2 + \frac{1}{2}\left(1+L\alpha^\pi_{i,j} + B^{*\pi}_{T-t-1,i,j}\right)^2\left(\sigma_j^\pi\right)^2 \\
+\left(\gamma_{i,j} + L\alpha^c_{i,j} + B^{*c}_{T-t-1,i,j}\right)\left(1+L\alpha^\pi_{i,j} + B^{*\pi}_{T-t-1,i,j}\right)\rho_{i,j}\sigma_i^c\sigma_j^\pi \\
+\left(B^{*c}_{T-t,S_t^C,S_t^\Pi} - \left(\gamma_{i,j} + L\alpha^c_{i,j} + B^{*c}_{T-t-1,i,j}\right)b_i^c\right)\Delta c_t \\
+\left(B^{*\pi}_{T-t,S_t^C,S_t^\Pi} - \left(1+L\alpha^\pi_{i,j} + B^{*\pi}_{T-t-1,i,j}\right)b_j^\pi\right)\Delta\pi_t
\end{pmatrix} \cong 1
$$

Because $\exp(x) \cong 1+x$ for $x$ in the neighborhood of zero and since $\sum_{i=1}^2\sum_{j=1}^2\phi^C_{S_t^C,i}\phi^\Pi_{S_t^\Pi,j} = 1$,

$$
\begin{aligned}
0 \cong\; & -A^*_{T-t,S_t^C,S_t^\Pi} \\
&+\sum_{i=1}^{2}\sum_{j=1}^{2}\phi^C_{S_t^C,i}\phi^\Pi_{S_t^\Pi,j}\begin{pmatrix}
\ln\beta_{i,j} - L\alpha_{i,j} + A^*_{T-t-1,i,j} \\
-\left(\gamma_{i,j} + L\alpha^c_{i,j} + B^{*c}_{T-t-1,i,j}\right)a_i^c - \left(1+L\alpha^\pi_{i,j} + B^{*\pi}_{T-t-1,i,j}\right)a_j^\pi \\
+\frac{1}{2}\left(\gamma_{i,j} + L\alpha^c_{i,j} + B^{*c}_{T-t-1,i,j}\right)^2\left(\sigma_i^c\right)^2 + \frac{1}{2}\left(1+L\alpha^\pi_{i,j} + B^{*\pi}_{T-t-1,i,j}\right)^2\left(\sigma_j^\pi\right)^2 \\
+\left(\gamma_{i,j} + L\alpha^c_{i,j} + B^{*c}_{T-t-1,i,j}\right)\left(1+L\alpha^\pi_{i,j} + B^{*\pi}_{T-t-1,i,j}\right)\rho_{i,j}\sigma_i^c\sigma_j^\pi
\end{pmatrix} \\
&+\Delta c_t\left(B^{*c}_{T-t,S_t^C,S_t^\Pi} - \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^C_{S_t^C,i}\phi^\Pi_{S_t^\Pi,j}\left(\gamma_{i,j} + L\alpha^c_{i,j} + B^{*c}_{T-t-1,i,j}\right)b_i^c\right) \\
&+\Delta\pi_t\left(B^{*\pi}_{T-t,S_t^C,S_t^\Pi} - \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^C_{S_t^C,i}\phi^\Pi_{S_t^\Pi,j}\left(1+L\alpha^\pi_{i,j} + B^{*\pi}_{T-t-1,i,j}\right)b_j^\pi\right)
\end{aligned}
$$

Since this relation must be true for any $\Delta c_t$ and $\Delta\pi_t$, we set the coefficients in front of $\Delta c_t$ and $\Delta\pi_t$ and the remanding term equal to zero to obtain

$$
\begin{aligned}
A^{*}_{T-t,S^C_t,S^\Pi_t} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^{C}_{S^C_t,i}\phi^{\Pi}_{S^\Pi_t,j}\left(\begin{array}{c}
\ln\beta_{i,j}-L\alpha_{i,j}+A^{*}_{T-t-1,i,j}\\
-\left(\gamma_{i,j}+L\alpha^{c}_{i,j}+B^{*c}_{T-t-1,i,j}\right)a^{c}_{i}-\left(1+L\alpha^{\pi}_{i,j}+B^{*\pi}_{T-t-1,i,j}\right)a^{\pi}_{j}\\
+\frac{1}{2}\left(\gamma_{i,j}+L\alpha^{c}_{i,j}+B^{*c}_{T-t-1,i,j}\right)^{2}\left(\sigma^{c}_{i}\right)^{2}\\
+\frac{1}{2}\left(1+L\alpha^{\pi}_{i,j}+B^{*\pi}_{T-t-1,i,j}\right)^{2}\left(\sigma^{\pi}_{j}\right)^{2}\\
+\left(\gamma_{i,j}+L\alpha^{c}_{i,j}+B^{*c}_{T-t-1,i,j}\right)\left(1+L\alpha^{\pi}_{i,j}+B^{*\pi}_{T-t-1,i,j}\right)\rho_{i,j}\sigma^{c}_{i}\sigma^{\pi}_{j}
\end{array}\right),\\
B^{*c}_{T-t,S^C_t,S^\Pi_t} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^{C}_{S^C_t,i}\phi^{\Pi}_{S^\Pi_t,j}b^{c}_{i}\left(\gamma_{i,j}+L\alpha^{c}_{i,j}+B^{*c}_{T-t-1,i,j}\right),\\
B^{*\pi}_{T-t,S^C_t,S^\Pi_t} &= \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^{C}_{S^C_t,i}\phi^{\Pi}_{S^\Pi_t,j}b^{\pi}_{j}\left(1+L\alpha^{\pi}_{i,j}+B^{*\pi}_{T-t-1,i,j}\right).
\end{aligned}
$$

# C Decomposition of the Default Spread

In this section, we study the value of the default spread. We decompose the default spread into two parts: the default risk related to the default and the loss given default and the default risk premium related to risk aversion measured by the correlation between the default intensity and the nominal pricing kernel.

Recall that the nominal pricing kernel is

$$M_{t,t+1}=\beta_{S^C_{t+1},S^\Pi_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma_{S^C_{t+1},S^\Pi_{t+1}}}\frac{\Pi_t}{\Pi_{t+1}}=\exp\left(\ln\beta_{S^C_{t+1},S^\Pi_{t+1}}-\gamma_{S^C_{t+1},S^\Pi_{t+1}}\Delta c_{t+1}-\Delta\pi_{t+1}\right).$$

According to Equation (2), the value of a security is $V(t,T)=\mathrm{E}_{\mathcal{G}_t}\left[M_{t,t+1}X_{t+1}\right]$. Starting from the approximation (5) and using induction, the value of the risky zero-coupon bond is approximated by

$$V(t,T)\cong\mathrm{E}_{\mathcal{G}_t}\left[\exp\left(\sum_{u=t+1}^{T}\ln M_{u-1,u}\right)\exp\left(-L\sum_{u=t+1}^{T}h_u\right)\right]\mathbf{1}_{\tau>t}$$

Based on the identity $\mathrm{E}[XY]=\mathrm{E}[X]\,\mathrm{E}[Y]+\mathrm{Cov}[X,Y]$, we obtain the following relation:

$$V(t,T)\cong P(t,T)\,\mathrm{E}_{\mathcal{G}_t}\left[\exp\left(-L\sum_{u=t+1}^{T}h_u\right)\right]+\mathrm{Cov}_{\mathcal{G}_t}\left[\prod_{u=t+1}^{T}M_{u-1,u},\exp\left(-L\sum_{u=t+1}^{T}h_u\right)\right] \tag{14}$$

where $P(t,T)=\mathrm{E}_{\mathcal{G}_t}\left[\exp\left(\sum_{u=t+1}^{T}\ln M_{u-1,u}\right)\right]$ is the time $t$ value of the risk free zero-coupon bond. As we will see, the covariance term is related to the default risk premium.

The conditional expectation term $\mathrm{E}_{\mathcal{G}_t}\left[\exp\left(-L\sum_{u=t+1}^{T}h_u\right)\right]$ may be evaluated using the same methodology as in Appendix B in which the impatience coefficients are set equal to one $\left(\beta_{i,j}=1\right)$, the risk aversion coefficients are nul $\left(\gamma_{i,j}=0\right)$ and the coefficient 1 in front of the term $\Delta\pi_{t+1}$ is set to 0:

$$\mathrm{E}_{\mathcal{G}_t}\left[\exp\left(-L\sum_{u=t+1}^{T}h_u\right)\right]=\exp\left(\widehat{A}_{T-t,S^C_t,S^\Pi_t}-\widehat{B}^{c}_{T-t,S^C_t,S^\Pi_t}\Delta c_t-\widehat{B}^{\pi}_{T-t,S^C_t,S^\Pi_t}\Delta\pi_t\right)$$

where $\widehat{A}_{0,S_t^C,S_t^\Pi} = \widehat{B}^c_{0,S_t^C,S_t^\Pi} = \widehat{B}^\pi_{0,S_t^C,S_t^\Pi} = 0$ and

$$\widehat{A}_{T-t,S_t^C,S_t^\Pi} = \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^C_{S_t^C,i}\phi^\Pi_{S_t^\Pi,j}\left(\begin{array}{c} -L\alpha_{i,j} + \widehat{A}_{T-t-1,i,j} \\ -\left(L\alpha^c_{i,j} + \widehat{B}^c_{T-t-1,i,j}\right)a^c_i - \left(L\alpha^\pi_{i,j} + \widehat{B}^\pi_{T-t-1,i,j}\right)a^\pi_j \\ +\frac{1}{2}\left(L\alpha^c_{i,j} + \widehat{B}^c_{T-t-1,i,j}\right)^2\left(\sigma^c_i\right)^2 + \frac{1}{2}\left(L\alpha^\pi_{i,j} + \widehat{B}^\pi_{T-t-1,i,j}\right)^2\left(\sigma^\pi_j\right)^2 \\ +\left(L\alpha^c_{i,j} + \widehat{B}^c_{T-t-1,i,j}\right)\left(L\alpha^\pi_{i,j} + \widehat{B}^\pi_{T-t-1,i,j}\right)\rho_{i,j}\sigma^c_i\sigma^\pi_j \end{array}\right),$$

$$\widehat{B}^c_{T-t,S_t^C,S_t^\Pi} = \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^C_{S_t^C,i}\phi^\Pi_{S_t^\Pi,j}b^c_i\left(L\alpha^c_{i,j} + \widehat{B}^c_{T-t-1,i,j}\right),$$

$$\widehat{B}^\pi_{T-t,S_t^C,S_t^\Pi} = \sum_{i=1}^{2}\sum_{j=1}^{2}\phi^C_{S_t^C,i}\phi^\Pi_{S_t^\Pi,j}b^\pi_j\left(L\alpha^\pi_{i,j} + \widehat{B}^\pi_{T-t-1,i,j}\right).$$

Therefore, using the approximated value of the risk free bond given at Equation (3), the time $t$ value of the risky bond in absence of the default risk premium is

$$\begin{aligned} V^{**}(t,T) &\cong P(t,T)\,\mathrm{E}_{\mathcal{G}_t}\left[\exp\left(-L\sum_{u=t+1}^{T}h_u\right)\right] \\ &\cong \exp\left(A^{**}_{T-t,S_t^C,S_t^\Pi} - B^{**c}_{T-t,S_t^C,S_t^\Pi}\Delta c_t - B^{**\pi}_{T-t,S_t^C,S_t^\Pi}\Delta\pi_t\right) \end{aligned}$$

where

$$\begin{aligned} A^{**}_{T-t,S_t^C,S_t^\Pi} &= \widetilde{A}_{T-t,S_t^C,S_t^\Pi} + \widehat{A}_{T-t,S_t^C,S_t^\Pi}, \\ B^{**c}_{T-t,S_t^C,S_t^\Pi} &= \widetilde{B}^c_{T-t,S_t^C,S_t^\Pi} + \widehat{B}^c_{T-t,S_t^C,S_t^\Pi} \\ \text{and } B^{**\pi}_{T-t,S_t^C,S_t^\Pi} &= \widetilde{B}^\pi_{T-t,S_t^C,S_t^\Pi} + \widehat{B}^\pi_{T-t,S_t^C,S_t^\Pi}. \end{aligned}$$

In this setting the default risk $DS^*(t,T)$ is approximated by

$$\begin{aligned} &DS^*(t,T) \\ &\cong \frac{-\ln V^{**}(t,T)}{T-t} + \frac{\ln \widetilde{P}(t,T)}{T-t} \\ &= \frac{\widetilde{A}_{T-t,S_t^C,S_t^\Pi} - A^{**}_{T-t,S_t^C,S_t^\Pi} + \left(B^{**c}_{T-t,S_t^C,S_t^\Pi} - \widetilde{B}^c_{T-t,S_t^C,S_t^\Pi}\right)\Delta c_t + \left(B^{**\pi}_{T-t,S_t^C,S_t^\Pi} - \widetilde{B}^\pi_{T-t,S_t^C,S_t^\Pi}\right)\Delta\pi_t}{T-t} \\ &= \frac{-\widehat{A}_{T-t,S_t^C,S_t^\Pi} + \widehat{B}^c_{T-t,S_t^C,S_t^\Pi}\Delta c_t + \widehat{B}^\pi_{T-t,S_t^C,S_t^\Pi}\Delta\pi_t}{T-t} \end{aligned}$$

# D The empirical yield curves (Dionne et al., 2005)

## D.1 The Nelson-Siegel (1987) Model

The empirical corporate yield spreads are obtained using Nelson and Siegel's approach which parameterize the instantaneous forward rate as

$$f_{NS}(t,T) = a_t + b_t\exp\left(-\frac{T-t}{\eta_t}\right) + c_t\frac{T-t}{\eta_t}\exp\left(-\frac{T-t}{\eta_t}\right)$$

where $a_t$, $b_t$, $c_t$ and $\eta_t$ are constants to be estimated for every month and rating class. The time $t$ value of a zero-coupon bond paying one dollar at maturity can then be written as

$$P_{NS}(t,T) = \exp\left\{-\zeta_t (T-t) - \omega_t \eta_t \left(1 - \exp\left(-\frac{T-t}{\eta_t}\right)\right) - \psi_t (T-t) \exp\left(-\frac{T-t}{\eta_t}\right)\right\},$$

where $\zeta_t = a_t$, $\psi_t = b_t + c_t$ and $\omega_t = -c_t$.

For a given date $t$, we observe $n$ bond prices $V_{obs}(t,T_1), ..., V_{obs}(t,T_n)$. Using Nelson-Siegel parametrization and the fact that a coupon bond can be expressed as a portfolio of zero-coupon bonds, we can write the $i$th bond price as :

$$V_{NS}(t,T_i) = C_i \sum_{k=1}^{K_i} P_{NS}(t,T_{i,k}) + P_{NS}(t,T_{i,K_i}) \tag{15}$$

where $C_i$ is the coupon rate of the $i^{th}$ bond, $T_i$ is its maturity date, $K_i$ is its number of remaining coupons and $T_{i,1}, ..., T_{i,K_i}$ are the coupon dates with $T_{i,K_i} = T_i$. Note that $V_{NS}(t,T_i)$ is a function of the known quantities $C_i$, $K_i$, $T_{i,1}, ..., T_{i,K_i} = T_i$ and of the unknown quantities $\zeta_t$, $\omega_t$, $\psi_t$ and $\eta_t$. For each date $t$ in our sample, we choose $\zeta_t$, $\omega_t$, $\psi_t$ and $\eta_t$ by minimizing the objective function

$$g(\zeta_t, \omega_t, \psi_t, \eta_t) = \sum_{i=1}^{n} (V_{NS}(t,T_i) - V_{obs}(t,T_i))^2$$

using the Levenberg-Marquardt algorithm (Marquardt, 1963). To minimize the chances of converging to a local rather than a global minimum, a grid search of $20^4 = 160\ 000$ points is performed to find a suitable starting point for the numerical minimization.

## D.2 Treatment of Accrued Interest (from Dionne et al., 2005)

First, when the date of the next coupon payment is the same as the transaction date and if it is not an odd coupon payment, the accrued interest in the Warga (1998) database is equal to zero before February 1991 and equal to the accrued interest of one day after February 1991. Second, when the date of the next coupon payment is the day following the transaction date and if it is not an odd coupon payment, the accrued interest in the database is equal to the coupon amount minus the accrued interest of one day before February 1991 and equal to zero after February 1991. Finally, for the other bonds, the accrued interest in the database is equal to the theoretical accrued interest (defined below) before February 1991 and equal to the theoretical accrued interest plus the accrued interest of one day after February 1991.

In order to get a similar accrued interest for the entire period, we calculated the theoretical accrued interest which is the same as that in the database for the first period (01-1987 to 02-1991) and we corrected the accrued interest for the second period (03-1991 to 12-1996). The theoretical accrued interest (AI) is given by this formula:

$$\text{AI} = \frac{nC}{N}$$

where $n$ is the number of interest-bearing days, $N$ is the number of days between two successive coupons[4] and $C$ is the semiannual coupon amount.

In the database there are two methods of calculating the parameters $n$ and $N$. For government bonds it is the actual/actual method. For corporate bonds it is the 30/360 method. Let $(d_1, m_1, y_1), (d_2, m_2, y_2)$ and $(d_3, m_3, y_3)$ represent, respectively, the date from which accrued interest is calculated, the settlement date, and the relevant interest payment date with $d_i$ the day's number from 1 to 31, $m_i$ the month number from 1

[4] Or the emission date and the first coupon date if the transaction date falls before the first coupon payment date.

and 12 and $y_i$ the year number. The parameters $n$ and $N$ are given by the next formula for the actual/actual method:

$$\begin{aligned} n &= (d_2, m_2, y_2) - (d_1, m_1, y_1) \\ N &= (d_3, m_3, y_3) - (d_1, m_1, y_1) \end{aligned}$$

and by the next formula for the 30/360 method:

$$\begin{aligned} &n = \left(\tilde{d}_2 - \tilde{d}_1\right) + 30\,(m_2 - m_1) + 360\,(y_2 - y_1) \\ &N = 180 \end{aligned}$$

with

$$\tilde{d}_1 = \begin{cases} 30 & : \quad m_1 \neq 2 \text{ and } d_1 = 31 \\ 30 & : \quad m_1 = 2 \text{ and } d_1 \geq 28 \\ d_1 & : \quad \text{otherwise} \end{cases}$$

and

$$\tilde{d}_2 = \begin{cases} 30 & : \quad m_2 \neq 2 \text{ and } d_2 = 31 \\ 30 & : \quad m_1 = m_2 = 2,\ d_2 \geq 28 \text{ and } d_1 \geq 28 \\ d_2 & : \quad \text{otherwise} \end{cases} .$$

# E Parameter estimation

## E.1 The log-likelihood function of the observed sample

This section contains the proofs of some equations of the Section 3.2.

### E.1.1 Proof of Equation (10b)

$$\begin{aligned} f_{\mathbf{Y}_t|\mathbf{V}_{t-1}}\left(\mathbf{y}_t \,|\mathbf{v}_{t-1}\,; \boldsymbol{\theta}, \boldsymbol{\phi}\right) &= \frac{f_{\mathbf{Y}_t,\mathbf{V}_{t-1}}\left(\mathbf{y}_t, \mathbf{v}_{t-1}; \boldsymbol{\theta}, \boldsymbol{\phi}\right)}{f_{\mathbf{V}_{t-1}}\left(\mathbf{v}_{t-1}; \boldsymbol{\theta}, \boldsymbol{\phi}\right)} \\ &= \frac{\sum_{\mathbf{s}_t \in \{(1,1),(1,2),(2,1),(2,2)\}} f_{\mathbf{Y}_t,\mathbf{S}_t,\mathbf{V}_{t-1}}\left(\mathbf{y}_t, \mathbf{s}_t, \mathbf{v}_{t-1}; \boldsymbol{\theta}, \boldsymbol{\phi}\right)}{f_{\mathbf{V}_{t-1}}\left(\mathbf{v}_{t-1}; \boldsymbol{\theta}, \boldsymbol{\phi}\right)} \\ &= \sum_{\mathbf{s}_t \in \{(1,1),(1,2),(2,1),(2,2)\}} f_{\mathbf{Y}_t|\mathbf{S}_t,\mathbf{V}_{t-1}}\left(\mathbf{y}_t \,|\mathbf{s}_t, \mathbf{v}_{t-1}\,; \boldsymbol{\theta}\right) f_{\mathbf{S}_t|\mathbf{V}_{t-1}} \\ &\quad \left(\mathbf{s}_t \,|\mathbf{v}_{t-1}\,; \boldsymbol{\theta}, \boldsymbol{\phi}\right) \\ &= \boldsymbol{\eta}'_{t;\boldsymbol{\theta}} \boldsymbol{\xi}_{t|t-1\,;\boldsymbol{\theta},\boldsymbol{\phi}}. \end{aligned}$$

### E.1.2 Proof of Equation (10d)

$$
\begin{aligned}
& f_{\mathbf{S}_t|\mathbf{V}_{t-1}}\left(\mathbf{s}_t\left|\mathbf{v}_{t-1}\right.;\boldsymbol{\theta},\boldsymbol{\phi}\right) \\
&= \frac{f_{\mathbf{S}_t,\mathbf{V}_{t-1}}\left(\mathbf{s}_t,\mathbf{v}_{t-1};\boldsymbol{\theta},\boldsymbol{\phi}\right)}{f_{\mathbf{V}_{t-1}}\left(\mathbf{v}_{t-1};\boldsymbol{\theta},\boldsymbol{\phi}\right)} \\
&= \frac{\sum_{\mathbf{s}_{t-1}\in\{(1,1),(1,2),(2,1),(2,2)\}} f_{\mathbf{S}_t,\mathbf{S}_{t-1},\mathbf{V}_{t-1}}\left(\mathbf{s}_t,\mathbf{s}_{t-1},\mathbf{v}_{t-1};\boldsymbol{\theta},\boldsymbol{\phi}\right)}{f_{\mathbf{V}_{t-1}}\left(\mathbf{v}_{t-1};\boldsymbol{\theta},\boldsymbol{\phi}\right)} \\
&= \sum_{\mathbf{s}_{t-1}\in\{(1,1),(1,2),(2,1),(2,2)\}} f_{\mathbf{S}_t|\mathbf{S}_{t-1},\mathbf{V}_{t-1}}\left(\mathbf{s}_t\left|\mathbf{s}_{t-1},\mathbf{v}_{t-1}\right.;\boldsymbol{\theta},\boldsymbol{\phi}\right) f_{\mathbf{S}_{t-1}\mathbf{V}_{t-1}} \\
& \left(\mathbf{s}_{t-1}\left|\mathbf{v}_{t-1}\right.;\boldsymbol{\theta},\boldsymbol{\phi}\right) \\
&= \sum_{\mathbf{s}_{t-1}\in\{(1,1),(1,2),(2,1),(2,2)\}} f_{\mathbf{S}_t|\mathbf{S}_{t-1}}\left(\mathbf{s}_t\left|\mathbf{s}_{t-1},\mathbf{v}_{t-1}\right.;\boldsymbol{\phi}\right) f_{\mathbf{S}_{t-1}|\mathbf{V}_{t-1}}\left(\mathbf{s}_{t-1}\left|\mathbf{v}_{t-1}\right.;\boldsymbol{\theta},\boldsymbol{\phi}\right) \\
&= \left(\mathbf{P}_\phi\big|_{\mathbf{s}_t}\right)'\boldsymbol{\zeta}_{t-1}
\end{aligned}
$$

where $\mathbf{P}_\phi\big|_{\mathbf{s}_t}$ is the column of the transition matrix $\mathbf{P}_\phi$ which corresponds to the state $\mathbf{s}_t$ and $\boldsymbol{\zeta}_{t-1}$ is a $4\times 1$ vector filled with $f_{\mathbf{S}_{t-1}|\mathbf{V}_{t-1}}\left(\mathbf{s}_{t-1}\left|\mathbf{v}_{t-1}\right.;\boldsymbol{\theta},\boldsymbol{\phi}\right)$, $\mathbf{s}_{t-1}=\{(1,1),(1,2),(2,1),(2,2)\}$. The entries of $\boldsymbol{\zeta}_{t-1}$ are computed as follows :

$$
\begin{aligned}
& f_{\mathbf{S}_{t-1}|\mathbf{V}_{t-1}}\left(\mathbf{s}_{t-1}\left|\mathbf{v}_{t-1}\right.;\boldsymbol{\theta},\boldsymbol{\phi}\right) \\
&= \frac{f_{\mathbf{S}_{t-1},\mathbf{V}_{t-1}}\left(\mathbf{s}_{t-1},\mathbf{v}_{t-1};\boldsymbol{\theta},\boldsymbol{\phi}\right)}{f_{\mathbf{V}_{t-1}}\left(\mathbf{v}_{t-1};\boldsymbol{\theta},\boldsymbol{\phi}\right)} \\
&= \frac{f_{\mathbf{S}_{t-1},\mathbf{Y}_{t-1},\mathbf{V}_{t-2}}\left(\mathbf{s}_{t-1},\mathbf{y}_{t-1},\mathbf{v}_{t-2};\boldsymbol{\theta},\boldsymbol{\phi}\right)}{f_{\mathbf{Y}_{t-1},\mathbf{V}_{t-2}}\left(\mathbf{y}_{t-1},\mathbf{v}_{t-2};\boldsymbol{\theta},\boldsymbol{\phi}\right)} \\
&= \frac{f_{\mathbf{Y}_{t-1}|\mathbf{S}_{t-1},\mathbf{V}_{t-2}}\left(\mathbf{y}_{t-1}\left|\mathbf{s}_{t-1},\mathbf{v}_{t-2}\right.;\boldsymbol{\theta},\boldsymbol{\phi}\right) f_{\mathbf{S}_{t-1}|\mathbf{V}_{t-2}}\left(\mathbf{s}_{t-1}\left|\mathbf{v}_{t-2}\right.;\boldsymbol{\theta},\boldsymbol{\phi}\right)}{f_{\mathbf{Y}_{t-1}|\mathbf{V}_{t-2}}\left(\mathbf{y}_{t-1}\left|\mathbf{v}_{t-2}\right.;\boldsymbol{\theta},\boldsymbol{\phi}\right)} \\
&= \frac{\boldsymbol{\eta}_{t-1;\boldsymbol{\theta}}\big|_{\mathbf{s}_{t-1}}\ \boldsymbol{\xi}_{t-1|t-2;\boldsymbol{\theta},\boldsymbol{\phi}}\Big|_{\mathbf{s}_{t-1}}}{\boldsymbol{\eta}'_{t-1;\boldsymbol{\theta}}\boldsymbol{\xi}_{t-1|t-2;\boldsymbol{\theta},\boldsymbol{\phi}}}
\end{aligned}
$$

where $\boldsymbol{\eta}_{t-1;\boldsymbol{\theta}}\big|_{\mathbf{s}_{t-1}}$ and $\boldsymbol{\xi}_{t-1|t-2;\boldsymbol{\theta},\boldsymbol{\phi}}\Big|_{\mathbf{s}_{t-1}}$ are the component of $\boldsymbol{\eta}_{t-1;\boldsymbol{\theta}}$ and $\boldsymbol{\xi}_{t-1|t-2;\boldsymbol{\theta},\boldsymbol{\phi}}$ respectively that correspond to the state $\mathbf{s}_{t-1}$.

## E.2 Estimation of the default risk parameters

Let first get an expression for the survival probabilities at time $t$, that is, the probability $P_{\mathcal{G}_t}\left[\tau > t+s\left|\tau>t\right.\right]$ that the default occurs in more than $s$ periods from time $t$ knowing that the firm has not defaulted at time $t$. Recall that $h_u=\left(1-e^{-\lambda(\mathbf{X}_u)}\right)=P_{\mathcal{G}_u}\left[\tau=u\right|\ \tau>u-1]$. Since this probability is usually very small, the approximation $e^{-h_u} \cong 1 - h_u$ used below is acceptable.

$$\begin{aligned}
&P_{\mathcal{G}_t}\left[\tau > t+s \,|\tau > t\right] \\
&= \mathrm{E}_{\mathcal{G}_t}\left[\exp\left(-\sum_{u=t+1}^{t+s} \lambda\left(\mathbf{X}_u\right)\right)\right] = \mathrm{E}_{\mathcal{G}_t}\left[\prod_{u=t+1}^{t+s}\left(1-h_u\right)\right] \cong \mathrm{E}_{\mathcal{G}_t}\left[\exp\left(-\sum_{u=t+1}^{t+s} h_u\right)\right] \\
&\cong \mathrm{E}_{\mathcal{G}_t}\left[\exp\left(-\sum_{u=t+1}^{t+s}\left(\alpha_{S_u^C,S_u^\Pi} + \alpha^c_{S_u^C,S_u^\Pi}\Delta c_u + \alpha^\pi_{S_u^C,S_u^\Pi}\Delta \pi_u\right)\right)\right] \text{ from approximation (6).}
\end{aligned}$$

As for the default-risk free zero-coupon bond value, the general expression for the last expectation is expressed as a sum of $4^s$ exponential functions. Since it becomes quickly numerically unmanageable, we based our estimation strategy on an approximation which assumes that this sum of exponential functions is well approximated by a single exponential, that is, we postulate that $P_{\mathcal{G}_t}\left[\tau > t+s \,|\tau > t\right] \cong p\left(s, S_t^C, S_t^\Pi\right)$ where

$$p\left(t, t+s, S_t^C, S_t^\Pi, \Delta c_t, \Delta \pi_t, \boldsymbol{\alpha}, \boldsymbol{\theta}, \boldsymbol{\phi}\right) = \exp\left(-\overline{A}_{s,S_t^C,S_t^\Pi} - \overline{B}^c_{s,S_t^C,S_t^\Pi}\Delta c_t - \overline{B}^\pi_{s,S_t^C,S_t^\Pi}\Delta \pi_t\right).$$

The coefficients $\overline{A}_{s,S_t^C,S_t^\Pi}$, $\overline{B}^c_{s,S_t^C,S_t^\Pi}$ and $\overline{B}^\pi_{s,S_t^C,S_t^\Pi}$ are obtained recursively starting with $\overline{A}_{0,S_t^C,S_t^\Pi} = \overline{B}^c_{0,S_t^C,S_t^\Pi} = \overline{B}^\pi_{0,S_t^C,S_t^\Pi} = 0$. Indeed, since

$$\begin{aligned}
P_{\mathcal{G}_t}\left[\tau > t+s \,|\tau > t\right] &\cong \mathrm{E}_{\mathcal{G}_t}\left[\exp\left(-h_{t+1}\right)\mathrm{E}_{\mathcal{G}_{t+1}}\left[\exp\left(-\sum_{u=t+2}^{t+s} h_u\right)\right]\right] \\
&\cong \mathrm{E}_{\mathcal{G}_t}\left[\exp\left(-h_{t+1}\right)P_{\mathcal{G}_{t+1}}\left[\tau > t+s \,|\tau > t+1\right]\right]
\end{aligned}$$

where $\sum_{u=t+2}^{t+1} h_u$ is set to zero whenever it happens, then

$$\begin{aligned}
1 &\cong \mathrm{E}_{\mathcal{G}_t}\left[\exp\left(-h_{t+1}\right)\frac{P_{\mathcal{G}_{t+1}}\left[\tau > t+s \,|\tau > t+1\right]}{P_{\mathcal{G}_t}\left[\tau > t+s \,|\tau > t\right]}\right] \\
&\cong \mathrm{E}_{\mathcal{G}_t}\left[\exp\left(-h_{t+1}\right)\frac{p\left(t+1, t+s, S_{t+1}^C, S_{t+1}^\Pi, \Delta c_{t+1}, \Delta \pi_{t+1}; \boldsymbol{\alpha}, \boldsymbol{\theta}, \boldsymbol{\phi}\right)}{p\left(t, t+s, S_t^C, S_t^\Pi, \Delta c_t, \Delta \pi_t; \boldsymbol{\alpha}, \boldsymbol{\theta}, \boldsymbol{\phi}\right)}\right] \\
&\cong \mathrm{E}_{\mathcal{G}_t}\left[\exp\left(\begin{array}{c}
-\alpha_{S_{t+1}^C,S_{t+1}^\Pi} - \overline{A}_{s-1,S_{t+1}^C,S_{t+1}^\Pi} + \overline{A}_{s,S_t^C,S_t^\Pi} \\
+\overline{B}^c_{s,S_t^C,S_t^\Pi}\Delta c_t - \left(\alpha^c_{S_{t+1}^C,S_{t+1}^\Pi} + \overline{B}^c_{s-1,S_{t+1}^C,S_{t+1}^\Pi}\right)\Delta c_{t+1} \\
+\overline{B}^\pi_{s,S_t^C,S_t^\Pi}\Delta \pi_t - \left(\alpha^\pi_{S_{t+1}^C,S_{t+1}^\Pi} + \overline{B}^\pi_{s-1,S_{t+1}^C,S_{t+1}^\Pi}\right)\Delta \pi_{t+1}
\end{array}\right)\right]
\end{aligned}$$

where the last line is obtained by replacing $h_{t+1}$ by the approximation (6). Replacing $\Delta c_{t+1}$ and $\Delta \pi_{t+1}$ using equations (1a) and (1b), applying the embedded conditional expectation rule $\mathrm{E}_{\mathcal{G}_t}\left[\bullet\right] = \mathrm{E}_{\mathcal{G}_t}\left[\mathrm{E}_{\mathcal{G}_t}\left[\bullet\right| S_{t+1}^C, S_{t+1}^C\right]\right]$, using $\mathrm{E}\left[\exp\left(a + bZ\right)\right] = \exp\left(a + \frac{1}{2}b^2\right)$ for a standard normal random variable $Z$, and finally, replacing $\exp\left(x\right) \cong 1 + x$, we get

$$0 \cong \sum_{i=1}^{2}\sum_{j=1}^{2}\left(\begin{array}{c}
-\alpha_{i,j} - \overline{A}_{s-1,i,j} + \overline{A}_{s,S_t^C,S_t^\Pi} - \left(\alpha^c_{i,j} + \overline{B}^c_{s-1,i,j}\right)a^c_i - \left(\alpha^\pi_{i,j} + \overline{B}^\pi_{s-1,i,j}\right)a^\pi_j \\
+\frac{1}{2}\left(\alpha^c_{i,j} + \overline{B}^c_{s-1,i,j}\right)\left(\sigma^c_i\right)^2 + \frac{1}{2}\left(\alpha^\pi_{i,j} + \overline{B}^\pi_{s-1,i,j}\right)\left(\sigma^\pi_j\right)^2 \\
+\left(\alpha^c_{i,j} + \overline{B}^c_{s-1,i,j}\right)\left(\alpha^\pi_{i,j} + \overline{B}^\pi_{s-1,i,j}\right)\rho_{i,j}\sigma^c_i\sigma^\pi_j \\
+\left(\overline{B}^c_{s,S_t^C,S_t^\Pi} - \left(\alpha^c_{i,j} + \overline{B}^c_{s-1,i,j}\right)b^c_i\right)\Delta c_t + \left(\overline{B}^\pi_{s,S_t^C,S_t^\Pi} - \left(\alpha^\pi_{i,j} + \overline{B}^\pi_{s-1,i,j}\right)b^\pi_j\right)\Delta \pi_t
\end{array}\right)\phi^C_{S_t^C,i}\phi^\Pi_{S_t^\Pi,j}.$$

Because $\sum_{i=1}^{2}\sum_{j=1}^{2}\phi^{C}_{S_t^C,i}\phi^{\Pi}_{S_t^{\Pi},j}=1$ then

$$\overline{A}_{s,S_t^C,S_t^{\Pi}} = \sum_{i=1}^{2}\sum_{j=1}^{2}\left(\begin{array}{c} \overline{A}_{s-1,i,j}+\alpha_{i,j}+\left(\alpha^{c}_{i,j}+\overline{B}^{c}_{s-1,i,j}\right)a^{c}_{i}+\left(\alpha^{\pi}_{i,j}+\overline{B}^{\pi}_{s-1,i,j}\right)a^{\pi}_{j} \\ -\frac{1}{2}\left(\alpha^{c}_{i,j}+\overline{B}^{c}_{s-1,i,j}\right)\left(\sigma^{c}_{i}\right)^{2}-\frac{1}{2}\left(\alpha^{\pi}_{i,j}+\overline{B}^{\pi}_{s-1,i,j}\right)\left(\sigma^{\pi}_{j}\right)^{2} \\ -\left(\alpha^{c}_{i,j}+\overline{B}^{c}_{s-1,i,j}\right)\left(\alpha^{\pi}_{i,j}+\overline{B}^{\pi}_{s-1,i,j}\right)\rho_{i,j}\sigma^{c}_{i}\sigma^{\pi}_{j} \end{array}\right)\phi^{C}_{S_t^C,i}\phi^{\Pi}_{S_t^{\Pi},j},$$

$$\overline{B}^{c}_{s,S_t^C,S_t^{\Pi}} = \sum_{i=1}^{2}\sum_{j=1}^{2}\left(\alpha^{c}_{i,j}+\overline{B}^{c}_{s-1,i,j}\right)b^{c}_{i}\phi^{C}_{S_t^C,i}\phi^{\Pi}_{S_t^{\Pi},j},$$

$$\overline{B}^{\pi}_{s,S_t^C,S_t^{\Pi}} = \sum_{i=1}^{2}\sum_{j=1}^{2}\left(\alpha^{\pi}_{i,j}+\overline{B}^{\pi}_{s-1,i,j}\right)b^{\pi}_{j}\phi^{C}_{S_t^C,i}\phi^{\Pi}_{S_t^{\Pi},j}.$$

## E.3 Data for default probability estimation

We considered only issuers domiciled in United States and having at least one senior unsecured estimated rating. We started with 5,719 issuers (in all industry groups) with 46,305 registered debt issues and 23,666 ratings observations. For each issuer we checked the number of default dates in the Master Default Table (Moody's, January, 09, 2002). We obtained 1,041 default dates for 943 issuers in the period 1970-2001. Some issuers (91) had more than one default date. In the rating transition histories, there are 728 withdrawn ratings that are not the last observation of the issuer. Theses irrelevant withdrawals were eliminated and so we obtained 22,938 ratings observations. The most important and difficult task was to get a proper definition of default. In order to compare our results with recent studies, we treated default dates as did Christensen *et al.* (2004). First, all the non withdrawn-rating observations up to the date of default have typically been unchanged. However, the ratings that occur within a week before the default date were eliminated. Rating changes observed after the date of default were eliminated unless the new rating reached the B3 level or higher and the new ratings were related to debt issued after the date of default. In theses cases we treated theses ratings as related to a new issuer. It is important to emphasize that the first rating date of the new issuer is the latest date between the date of the first issue after default and the first date we observe an issuer rating higher than or equal to B3. The same treatment is applied for the case of two and three default dates. Finally, few issuers have a registered default date before the first rating observation in the Senior Unsecured Estimated Rating Table (Moody's, January, 09, 2002). In theses cases, we considered that there was no default. With this procedure we got 5821 issuers with 965 default dates. We aggregated all rating notches and so we got the nine usual ratings Aaa, Aa, A, Baa, Ba, B, Caa-C, Default and NR (Not Rated) with 15,564 rating observations. The final data set corresponds to that without entries and right censoring in Dionne et al. (2005) which is more in line with the standard data set used by Moodys'.

**Table 1**
**Risk Free Yields and Corporate Baa Bond Spreads**

This table reports the average corporate bond spreads from government bonds for industrial Baa corporate bonds and maturities from one to ten years. Spot rates were computed for the 1987-1996 period, using the Nelson and Siegel model. Treasuries spot rates are annualized. Corporate bond spreads are calculated as the difference between the corporate spot rates and treasury spot rates for a given maturity.

| Maturity | Treasuries | Baa |
|---|---|---|
| 1.00 | 6.174 | 1.403 |
| 2.00 | 6.454 | 1.180 |
| 3.00 | 6.709 | 1.206 |
| 4.00 | 6.920 | 1.229 |
| 5.00 | 7.090 | 1.237 |
| 6.00 | 7.226 | 1.234 |
| 7.00 | 7.337 | 1.224 |
| 8.00 | 7.426 | 1.210 |
| 9.00 | 7.500 | 1.193 |
| 10.00 | 7.562 | 1.174 |

**Table 2**
**Parameter Estimates for the Regime Switching Model**

| | $a_1^c$ | $a_2^c$ | $b_1^c$ | $b_2^c$ | $\sigma_1^c$ | $\sigma_2^c$ |
|---|---|---|---|---|---|---|
| Point estimate | 0.00286 | 0.00277 | **0.16172** | 0.32978 | 0.00360 | 0.00910 |
| Standard deviation | 0.00068 | 0.00109 | 0.12809 | 0.10973 | 0.00043 | 0.00090 |
| P-value | 0.00% | 1.10% | 20.68% | 0.27% | 0.00% | 0.00% |
| | | | | | | |
| | $a_1^\pi$ | $a_2^\pi$ | $b_1^\pi$ | $b_2^\pi$ | $\sigma_1^\pi$ | $\sigma_2^\pi$ |
| Point estimate | 0.00397 | 0.00627 | **0.06904** | 0.59119 | 0.00396 | 0.00736 |
| Standard deviation | 0.00062 | 0.00152 | 0.09976 | 0.08191 | 0.00035 | 0.00065 |
| P-value | 0.00% | 0.00% | 48.89% | 0.00% | 0.00% | 0.00% |
| | | | | | | |
| | $\rho_{1,1}$ | $\rho_{1,2}$ | $\rho_{2,1}$ | $\rho_{2,2}$ | | |
| Point estimate | **-0.13308** | -0.37932 | **-0.12396** | -0.58733 | | |
| Standard deviation | 0.23721 | 0.17541 | 0.19834 | 0.11809 | | |
| P-value | 57.48% | 3.06% | 53.20% | 0.00% | | |
| | | | | | | |
| | $\phi_{11}^C$ | $\phi_{22}^C$ | $\phi_{11}^\Pi$ | $\phi_{22}^\Pi$ | | |
| Point estimate | 0.87495 | 0.88528 | 0.96932 | 0.95610 | | |
| Standard deviation | 0.06494 | 0.07384 | 0.02566 | 0.03237 | | |
| P-value | 0.00% | 0.00% | 0.00% | 0.00% | | |

**Table 3**
**Approximate Tests on Parameter Equality**
**for the Regime Switching Model**

| Consumption | | Inflation | |
|---|---|---|---|
| | **p-values** | | **p-values** |
| $a_1^c = a_2^c$ | **94.44%** | $a_1^\pi = a_2^\pi$ | **15.15%** |
| $b_1^c = b_2^c$ | **35.48%** | $b_1^\pi = b_2^\pi$ | 0.01% |
| $\sigma_1^c = \sigma_2^c$ | 0.00% | $\sigma_1^\pi = \sigma_2^\pi$ | 0.00% |
| $\rho_{11} = \rho_{21}$ | **97.96%** | $\rho_{11} = \rho_{21}$ | **41.43%** |
| $\rho_{12} = \rho_{22}$ | **34.84%** | $\rho_{12} = \rho_{22}$ | **5.60%** |

**Table 4**
**Fit of the Risk-free Zero-coupon Bond Pricing Model**
**with Respect to the Nelson and Siegel Yields**

| T (quarters) | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
|---|---|---|---|---|---|---|---|---|---|---|
| RMSE (T) | 1.003% | 0.417% | 0.262% | 0.158% | 0.075% | 0.056% | 0.109% | 0.165% | 0.216% | 0.260% |
| AAE (T) | 0.744% | 0.286% | 0.175% | 0.118% | 0.057% | 0.040% | 0.069% | 0.112% | 0.150% | 0.183% |
| AE (T) | -0.199% | 0.124% | 0.147% | 0.102% | 0.044% | -0.010% | -0.056% | -0.095% | -0.126% | -0.152% |
| $\overline{\tilde{y}(T)}$ | 5.988% | 6.596% | 6.878% | 7.043% | 7.150% | 7.226% | 7.282% | 7.325% | 7.360% | 7.388% |
| $\overline{y(T)}$ | 6.187% | 6.472% | 6.731% | 6.941% | 7.106% | 7.236% | 7.338% | 7.420% | 7.486% | 7.540% |

$\text{RMSE}(T) = \sqrt{\sum_{t=1}^{40} (\varepsilon(t,T))^2 / 40}$ are the root-mean-squared errors with $\varepsilon(t,T) = \tilde{y}(t,t+T) - y_{NS}(t,t+T)$. $\tilde{y}(t,t+T) = -\frac{1}{T^*} \ln \tilde{P}(t,t+T)$ is the yield to maturity of the theoretical bond price and $y_{NS}(t,t+T)$ is the estimated zero-coupon yield obtained from the Nelson-Siegel procedure. $\text{AAE}(T) = \sum_{t=1}^{40} |\varepsilon(t,T)| / 40$ is the absolute average error while $\text{AE}(T) = \sum_{k=1}^{40} \varepsilon(t,T)/40$ is the average error. $\overline{\tilde{y}(T)} = \sum_{t=1}^{40} \tilde{y}(t,T)/40$ and $\overline{y(T)} = \sum_{t=1}^{40} y_{NS}(t,T)/40$ are, respectively, the average theoretical yields and average Nelson and Siegel yields.

**Table 5**
**Estimated Generator for the I-1987 to IV-1996 Period**

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | -0.0757 | 0.0729 | 0.0000 | 0.0000 | 0.0028 | 0.0000 | 0.0000 | 0.0000 |
| AA | 0.0103 | -0.1146 | 0.1019 | 0.0008 | 0.0000 | 0.0016 | 0.0000 | 0.0000 |
| A | 0.0000 | 0.0142 | -0.0761 | 0.0555 | 0.0047 | 0.0017 | 0.0000 | 0.0000 |
| BBB | 0.0004 | 0.0000 | 0.0593 | -0.1247 | 0.0553 | 0.0064 | 0.0012 | 0.0020 |
| BB | 0.0005 | 0.0005 | 0.0031 | 0.0672 | -0.1907 | 0.1037 | 0.0010 | 0.0146 |
| B | 0.0009 | 0.0000 | 0.0009 | 0.0027 | 0.0641 | -0.2661 | 0.0614 | 0.1362 |
| CCC | 0.0000 | 0.0000 | 0.0164 | 0.0000 | 0.0327 | 0.0818 | -1.1783 | 1.0474 |
| D | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

**Table 6**
**Estimated One-year Transition Probability Matrix**

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | 0.9274 | 0.0663 | 0.0034 | 0.0002 | 0.0025 | 0.0002 | 0.0000 | 0.0000 |
| AA | 0.0094 | 0.8927 | 0.0927 | 0.0033 | 0.0003 | 0.0014 | 0.0000 | 0.0001 |
| A | 0.0001 | 0.0129 | 0.9289 | 0.0504 | 0.0056 | 0.0018 | 0.0001 | 0.0002 |
| BBB | 0.0004 | 0.0004 | 0.0538 | 0.8859 | 0.0477 | 0.0077 | 0.0008 | 0.0032 |
| BB | 0.0005 | 0.0005 | 0.0046 | 0.0577 | 0.8307 | 0.0829 | 0.0025 | 0.0206 |
| B | 0.0008 | 0.0001 | 0.0013 | 0.0040 | 0.0518 | 0.7705 | 0.0310 | 0.1407 |
| CCC | 0.0000 | 0.0001 | 0.0093 | 0.0011 | 0.0188 | 0.0422 | 0.3089 | 0.6195 |
| D | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |

**Table 7**
**Parameter Estimates for the Conditional Default Probabilities**

| Parameters/Estimates | Regime (1,1) | Regime (1,2) | Regime (2,1) | Regime (2,2) |
|---|---|---|---|---|
| $\alpha$ | 0.00010 | 0.00670 | 0.00094 | 0.00201 |
| $\alpha^c$ | -0.00758 | -0.37672 | -0.06275 | 0.04117 |
| $\alpha^\pi$ | 0.00141 | -0.29211 | -0.03488 | 0.31734 |

**Table 8**
**Default Spread Proportions for Baa Bonds over the Period 1987-1996**
**(Recovery Rates are 36.67% for all states)**

| Maturity (Years) | T=1 | T=2 | T=3 | T=4 | T=5 | T=6 | T=7 | T=8 | T=9 | T=10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Default Spreads | 0.0037 | 0.0040 | 0.0042 | 0.0043 | 0.0044 | 0.0045 | 0.0045 | 0.0046 | 0.0046 | 0.0046 |
| Credit Spreads | 0.0139 | 0.0116 | 0.0119 | 0.0122 | 0.0122 | 0.0122 | 0.0122 | 0.0121 | 0.0119 | 0.0118 |
| Average Proportion (%) | 35.40 | 36.72 | 36.98 | 36.99 | 37.28 | 37.73 | 38.30 | 38.98 | 39.77 | 40.71 |

**Table 9**
**Default Spread Decomposition for Baa Bonds over the Period 1987-1996**
**(Recovery Rates are 36.67% for all states)**

| Maturity (Years) | T=1 | T=2 | T=3 | T=4 | T=5 | T=6 | T=7 | T=8 | T=9 | T=10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Credit Spreads | 0.0139 | 0.0116 | 0.0119 | 0.0122 | 0.0122 | 0.0122 | 0.0122 | 0.0121 | 0.0119 | 0.0118 |
| Default Risk | 0.0036 | 0.0039 | 0.0041 | 0.0042 | 0.0043 | 0.0043 | 0.0044 | 0.0044 | 0.0045 | 0.0045 |
| Average Proportion (%) | 34.76 | 35.85 | 36.09 | 36.10 | 36.38 | 36.82 | 37.38 | 38.05 | 38.84 | 39.76 |
| Default Risk Premium | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| Average Proportion (%) | 0.64 | 0.87 | 0.89 | 0.89 | 0.90 | 0.91 | 0.92 | 0.93 | 0.94 | 0.95 |
| Default Spreads | 0.0037 | 0.0040 | 0.0042 | 0.0043 | 0.0044 | 0.0045 | 0.0045 | 0.0046 | 0.0046 | 0.0046 |
| Average Proportion (%) | 35.40 | 36.72 | 36.98 | 36.99 | 37.28 | 37.73 | 38.30 | 38.98 | 39.77 | 40.71 |

**Figure 1**

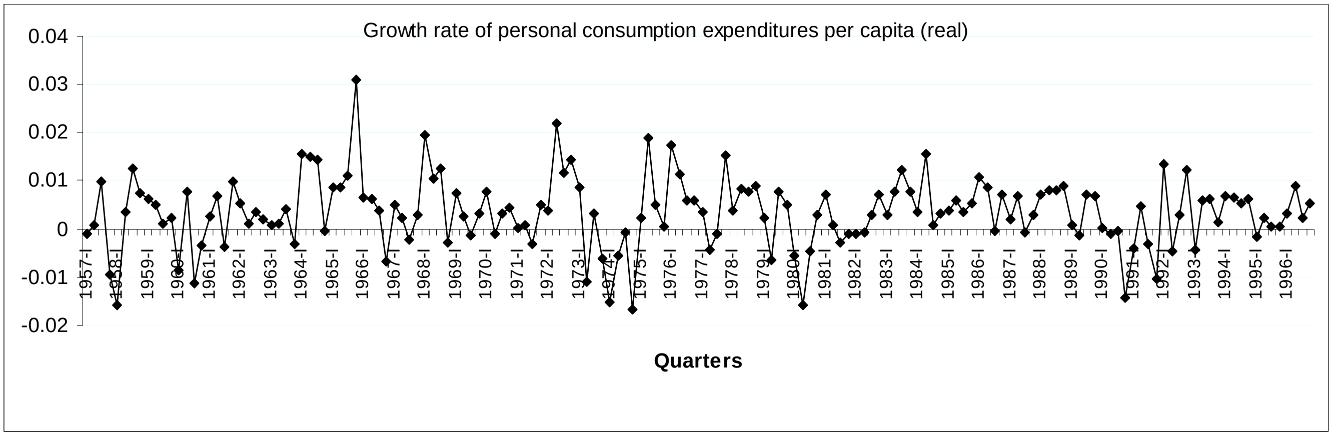



**Figure 2**

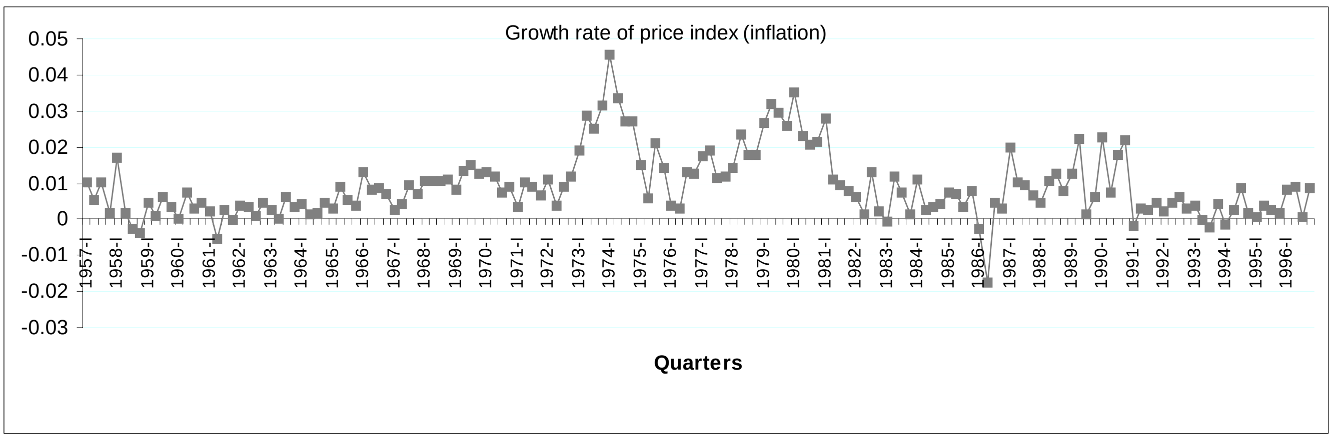
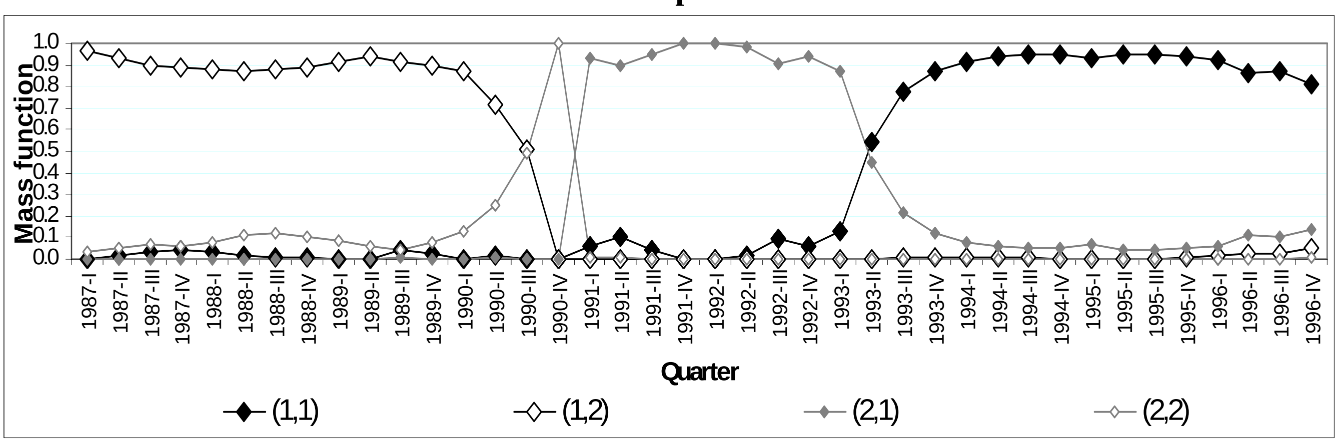



**Figure 3**
**Smoothed Probabilities of Consumption and Inflation States**




Note: State (1,1) corresponds to less volatile consumption rates and lower level and less volatile inflation rates. State (1,2) is associated to less volatile consumption rates and higher level and more volatile inflation rates. State (2,1) corresponds to more volatile consumption rates and lower level and less volatile inflation rates. Finally, state (2,2) is associated to more volatile consumption rates and higher level and more volatile inflation rates.

**Figure 4**
**Smoothed Estimates of Consumption and Inflation States**

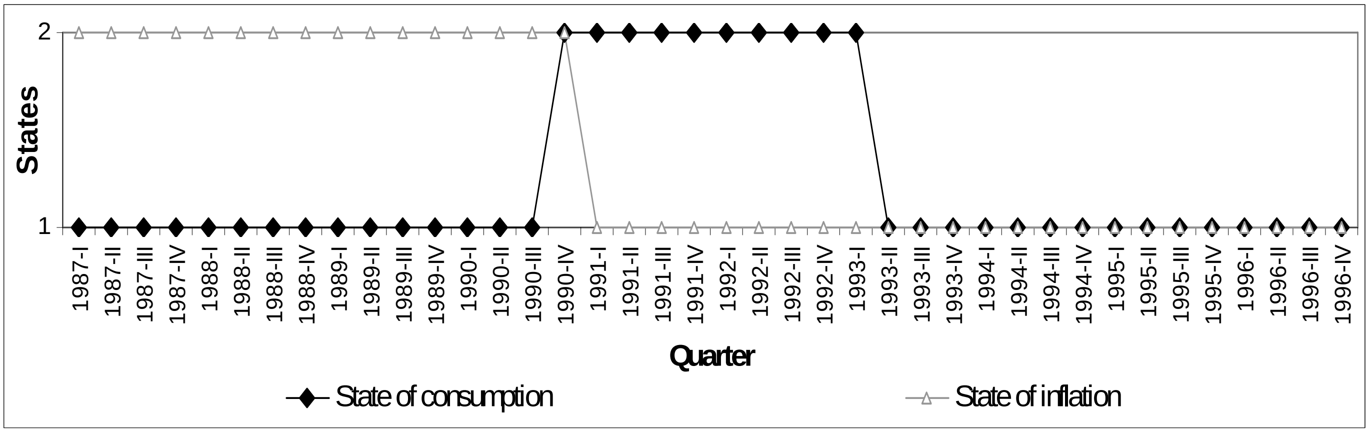
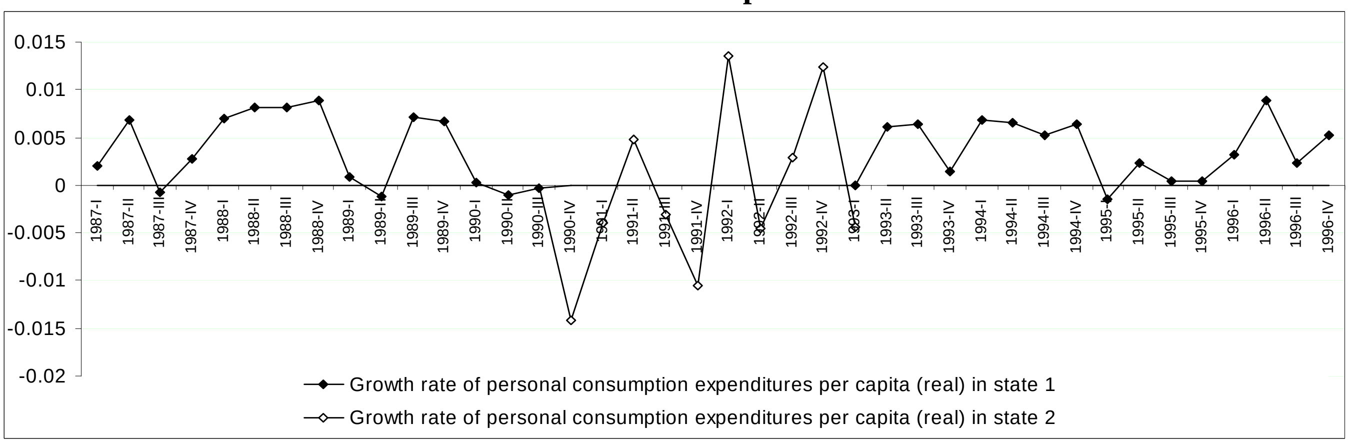



**Figure 5**
**Growth Rate of Consumption**




Note: According to the Business Cycle Dating Committee of the National Bureau of Economic Research, there was a recession during the 1990-III to 1991-I periods.

**Figure 6**
**Growth Rate of Inflation**

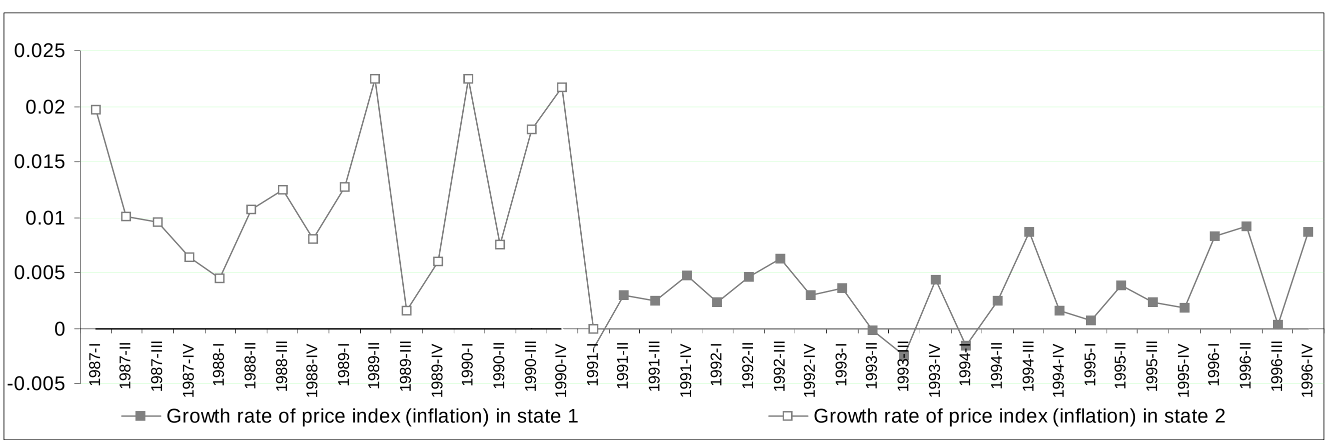


**Figure 7**
**Estimates of Risk Aversion Parameters**

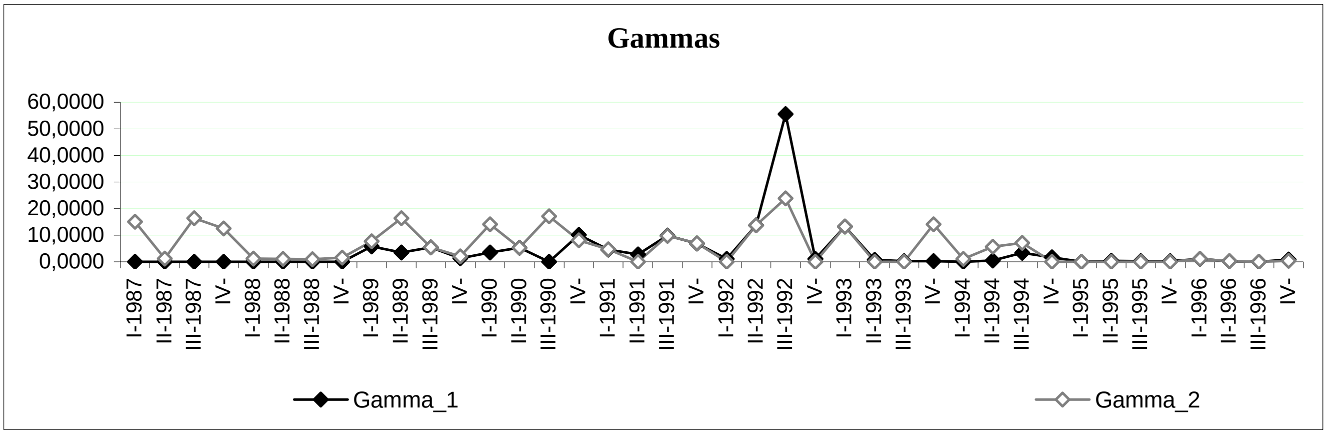



**Figure 8**
**Estimates of Impatience Parameters**

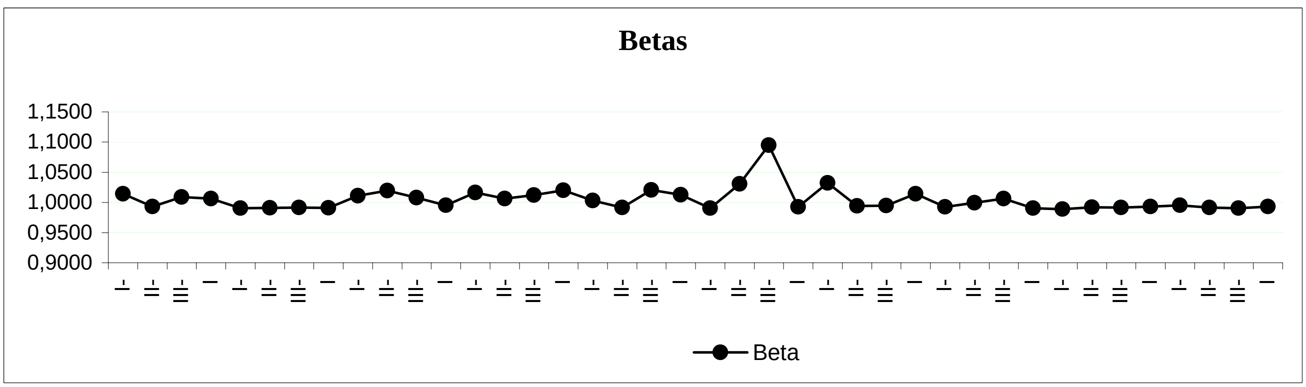



**Figure 9**
**Zero Coupon Yield for a Time to Maturity of One Year**

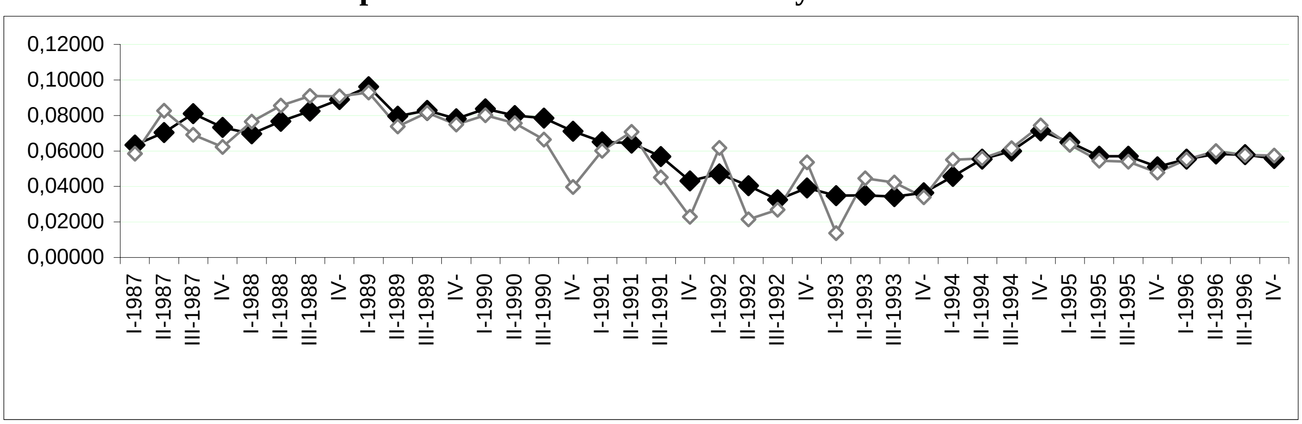


**Figure 10**
**Zero Coupon Yield for a Time to Maturity of Two Years**

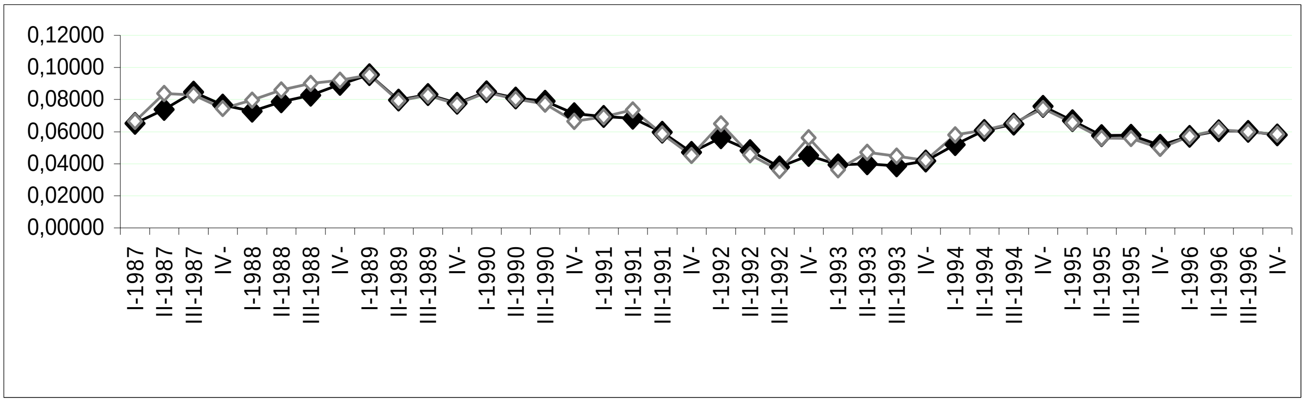



**Figure 11**
**Zero Coupon Yield for a Time to Maturity of Ten Years**

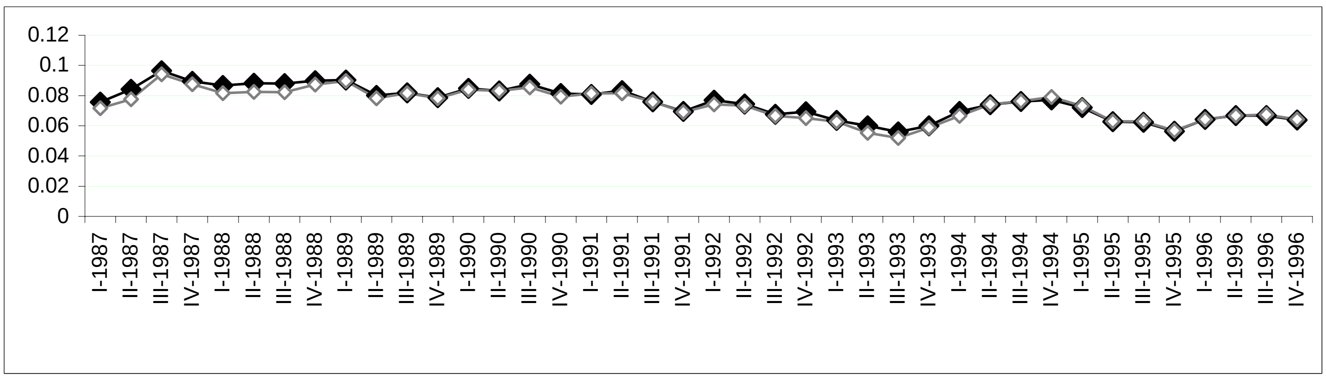



**Figure 12**
**Conditional Default Probability**

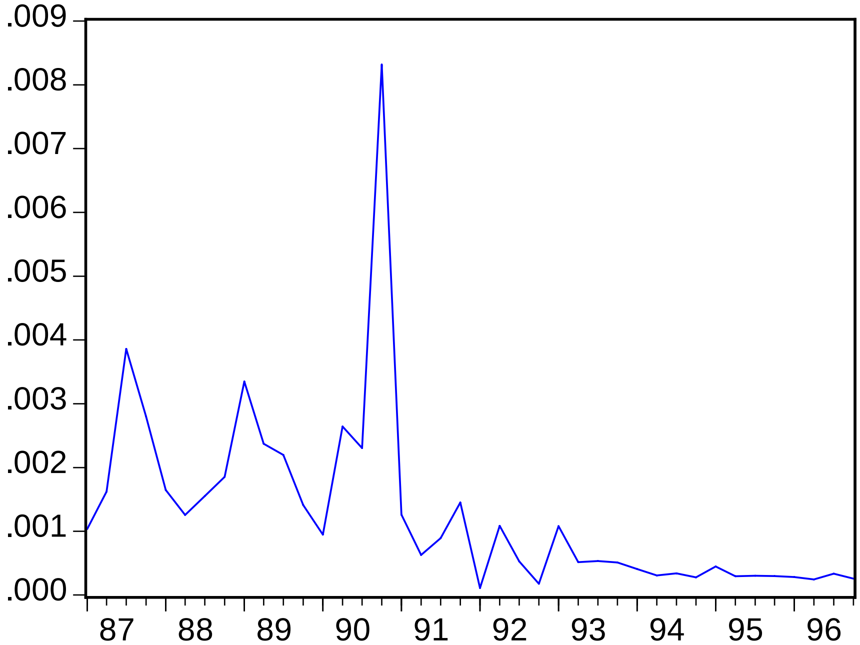


**Figure 13**
**Corporate Spreads for 10 Years to Maturity**

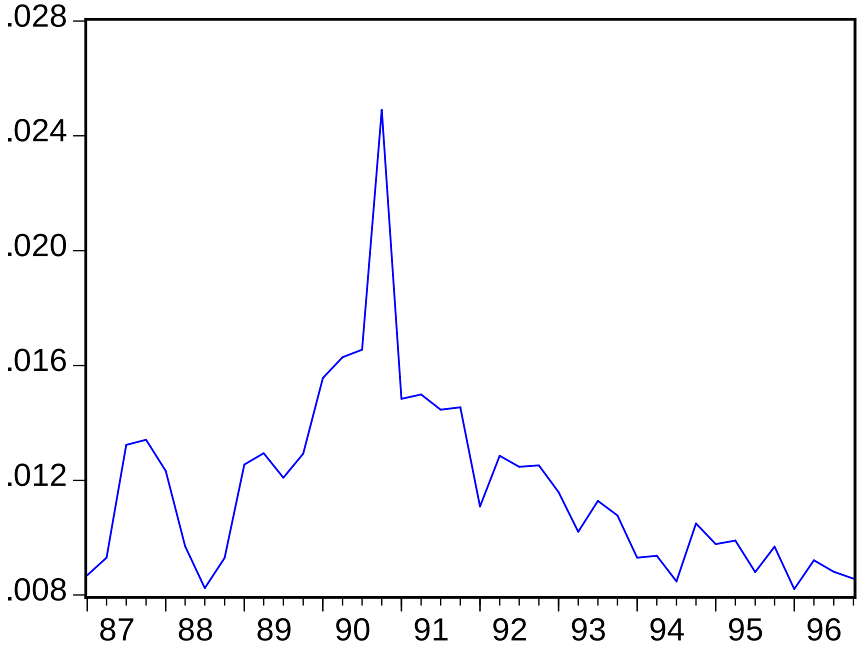



**Figure 14**
**Default Spreads for 10 Years to Maturity**

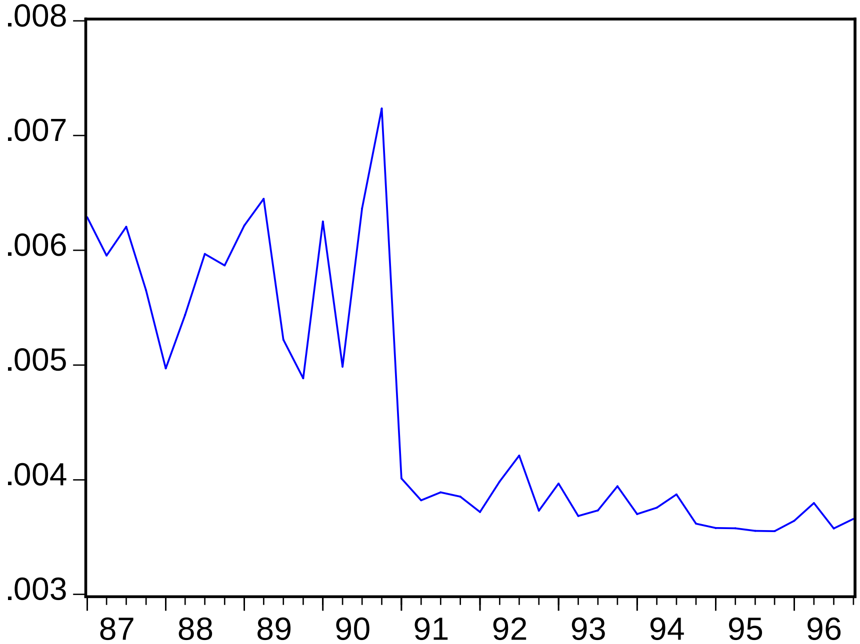


**Figure 15**
**Default Spread Proportion for 10 Years to Maturity**

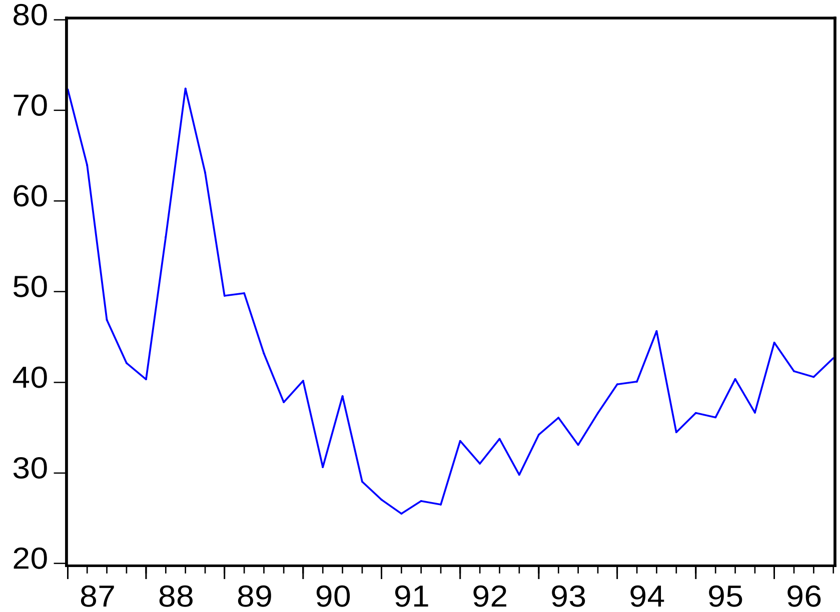
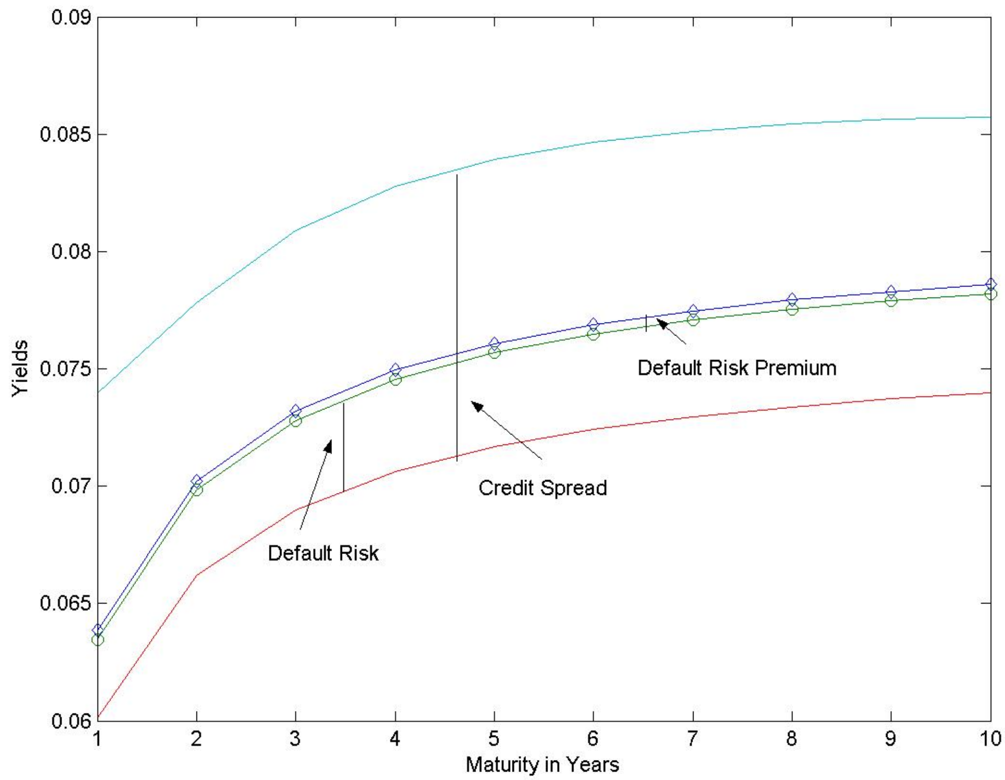



**Figure 16**
**Credit Spread Decomposition of Baa Corporate Spreads**



# Regime Shift in Corporate Bond Spreads

Georges Dionne, Khemais Hammami, and Jean-Guy Simonato*

April 26, 2007

**Abstract**

This paper proposes and estimates a comprehensive model for U.S. corporate credit spreads volatilities. The model includes the level and the GARCH effects in addition to regime shifts. We bring evidence that the level and the GARCH components are essential in modeling the conditional volatility of corporate spreads. The level elasticity parameter is found to be in accordance with the square-root specification used in reduced form models. The exception are the long term non-investment grade bonds where the elasticity parameter is higher than 0.5. The conditional volatility is more persistent for non-investment grade spreads. Negative and positive unexpected corporate spreads have the same effect on the conditional volatility except for A short term corporate spreads, for which positive shocks lead to an increase in the conditional volatility while negative shocks have no influence on the conditional volatility.

Corporate spreads are found to exhibit regime shifts between a high volatility regime and a low volatility regime. The high volatility regime has lower (higher) average spreads for investment (non-investment) grade spreads. In the high volatility regime, corporate spreads are more stationary than in the low volatility regime. For all corporate spreads, the elasticity parameter is higher in the high volatility regime. Non-investment grade spread regimes are related to economic and default cycles. In contrast, there is a weak relation between investment grade credit spread regimes and economic and default cycles. Performance of different model specifications in forecasting volatility and matching unconditional moments are also compared.

Keywords: Corporate bond spread, level effect, leverage effect, regime shift, default cycle, GARCH.

JEL classification: C12, C13, C15, C22.

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# 1 Introduction

Corporate spreads are fundamental ingredients in the valuation of credit risk instruments and the management of credit investment strategies. The present paper proposes a comprehensive model for corporate spread volatilities which includes, simultaneously, the level and the GARCH effects in addition to regime shifts.

The dynamic behavior of corporate spreads has become increasingly important over the last few years for three reasons. First, the market of credit risk instruments has become much more liquid and popular. Financial institutions contribute to the growth of this market by using credit risk instruments to manage their credit exposure without breaking up relationships with their clients. In addition, credit risk instruments have helped financial institutions to avoid potential losses during the U.S. recent economic recession of 2000-2001 where defaults reached the highest level in history. Appropriate pricing and hedging of these credit instruments require an adequate description of the dynamic behavior of the risk-free interest rate, the default intensity, the recovery rate, the liquidity premium, and the credit spread.

Second, recent regulatory developments recommend that banks take into account, in addition to interest rate risk, the credit-spread risk when assessing their capital requirements for corporate bond portfolios. Therefore, it is important to correctly specify the conditional distribution of the term structure of corporate spreads. A major concern with this framework is the risk of procyclical capital requirements, particularly if corporate spreads exhibit regime shifts. In this case, assuming that corporate spreads are driven from the same distribution leads to insufficient economic capital at the peak of the cycle, since recent corporate spreads are much lower than future spreads. Generally, banks estimate VaR with one year of historical spreads (Jorion, 1997). So, the accumulation of economic capital, after a peak of the cycle, will be too slow because the expansion period is more than one year. The issue of procyclicality emphasizes the need to assess whether credit spreads exhibit regime shift or not and to have good estimates of the conditional drift and volatility of corporate spreads.

Third, reduced form credit risk models imply that corporate spreads are affine functions of some risk factors such as interest rate factors, systematic risk factors, and sometimes idiosyncratic risk factors. So, theses models can easily take into account the macroeconomic effect on corporate spreads as documented in many theoretical and empirical researches (Bernanke and Gertler, 1995; Bernanke et al, 1996; Figlewski et al, 2006; Allen and Saunders, 2003). The main drawback of this approach is the assumption that all the parameters of the underlying state variables and the relationship between the state variables and the observed corporate spreads are constant over time. This is inconsistent with the literature, which documents that the stochastic behavior of risk-free interest rate factors and others systematic state variables varies over time because they exhibit a regime shift (Bansal and Zhou, 2002 ; Ang and Bekaert, 2002). Therefore, the conditional distribution of corporate spreads should depend on both the realized risk factors and on their future regimes.

Our paper brings four important contributions to the literature. First, to our knowledge, this is the first paper that includes regime shifts in the corporate spread dynamics, a feature that has been demonstrated to be important in fitting short term interest rates and others financial variables. Second, we study the link between the level and the volatility of corporate spreads. Third, we examine the role of the GARCH effect on corporate spreads volatility. Moreover, we test if negative unexpected corporate spreads (good news) have the same effect as positive unexpected corporate spreads (bad news) on the conditional volatility of corporate spreads. Finally, we look at the mean-reversion of corporate spreads under different specifications of the conditional volatility.

We find that the level and the GARCH components are essential in modeling the conditional volatility of corporate spreads. The marginal contribution of the GARCH effect is stronger than the contribution of the level effect. The level elasticity parameter is generally in accordance with the square-root specification used in reduced form models except for long term non-investment grade bonds where it is statistically higher than 0.5. The conditional volatility is more persistent for non-investment grade spreads. The leverage effect test shows that negative and positive unexpected corporate spreads have the same effect on the conditional volatility except for A short term corporate spreads, for which positive shocks lead to an increase in the conditional volatility while negative shocks have no influence on the conditional volatility. The conditional volatility specification affects the conclusion about the mean-reversion of corporate spreads. In contrast to results of Neal et al (2000), all the corporate spreads are mean reverting when the GARCH component is incorporated in the conditional volatility.

We find also that corporate spreads exhibit regime shifts between a high volatility regime and a low volatility regime. The high volatility regime has lower (higher) average spreads for investment (non investment) grade spreads. In the high volatility regime, corporate spreads are more stationary than in the low volatility regime. For all corporate spreads, the elasticity parameter is higher in the high volatility regime. Non-investment grade spread regimes are related to economic and default cycles. In contrast, there is a weak relation between investment grade credit spread regimes and economic and default cycles.

The remainder of this paper is organized as follows. In the next section we present a literature review on corporate spread components and discuss why corporate spreads should exhibit a regime shift. Section 3 presents an econometric model for corporate spreads. In this section we also discuss the estimation method. Section 4 describes the corporate spreads data. Then, we analyze the results of our empirical models in section 5. Section 6 discusses the implications of the empirical results. Finally section 7 concludes.

## 2 Literature review

The main question is whether corporate spreads exhibit pro-cyclicality or not. It is well documented that corporate spreads, particularly for non-investment grade bonds, are higher in economic recessions than in expansion periods. This can be viewed as a cyclical pattern in the sense that higher and lower corporate spreads are driven from the same conditional distribution. In contrast, regime shifts imply that the conditional distribution of corporate spreads in an economic recession is different from the conditional distribution during an expansion period. In the regime shift framework, the conditional distribution of the corporate spreads depends on the realization of the regime modeled by a discrete state variable.

The evidence of the macroeconomic effect on corporate spreads motivates comparing the level of the corporate spreads with respect to business cycles (See Allen and Saunders, 2003, for an excellent review). However, Gorton and He (2003), using a theoretical model, show that credit cycles may have their own dynamic distinct from business cycles. So, it is important to specify corporate spreads cycles and compare them with the economic and the default cycle. Corporate spread cycles can be different from the default cycles, although default probability is an important component of credit spreads, because other types of risk, such as the liquidity risk, are also important components of observed corporate spreads.

Corporate spreads are not explained only by default risk related to the default and the recovery risks. Other components of corporate spreads are important such as liquidity risk, systematic risk, and contagion risk. We expect that corporate spreads exhibit a regime shift because macroeconomic fluctuations affect corporate spreads components.

The observed default rates increase during recessions and fall during expansions. Default rates of non-investment grade issuers attained more than 10% in the two recent recessions (1990-1991 and 2000-2001). During the recent expansion (1992-1996) it was about 2.66%. The relation between default probability and macroeconomic conditions has been documented in recent empirical papers. Nickell, Perraudin and Varotto (2000) find evidence of macroeconomic and industry effects on rating transitions and defaults. Altman and Brady (2001) show also an apparent relationship between default probability and macroeconomic conditions. Intuitively, during expansion current income of firms is high and investors forecasts are optimistic. In theses conditions the firm asset value increases leading corporate spreads and default rates to decrease.

The macroeconomic conditions affect also the recovery rate and thus the expected loss. Altman and Kishore (1996) find that recovery rates are time-varying. Dalianes (1999) show that recovery rates are negatively correlated with risk-free interest rates. Gupton, Gates and Carty (2000) and Crouhy, Galai and Mark (2001) find a cyclical effect in the loss given default time series. Finally, in an extensive study of recovery rates, Altman and Brady (2001) show that the weighted average recovery rates for all securities are lower in recessions. They find also a non-linear negative correlation between recovery rates and default probabilities that can accentuate the cyclical dynamic of corporate spreads.

Using a structural model, Ericsson and Renault (2005) report that liquidity premium is an increasing function of both the firm's assets volatility and leverage. Moreover, they find a positive correlation between credit and liquidity risks. Longstaff, Mithal and Neis (2005) examine the proportion of the liquidity premium in the corporate spreads. They use a reduced form model to modelize the default risk premium and a regression analysis for the non-default component.

They document a weak evidence of differential state tax effect and a substantial effect of different measures of individual corporate bond liquidity such as new issuance amounts.

The relationship between corporate liquidity risk and macroeconomic conditions is not well studied. However, the positive correlation between liquidity premium and both the leverage and the new issuance amounts imply that liquidity premium is higher in an economic boom. Shleifer and Vishny (1992), Korajczyk and Levy (2003), and Guo, Miao and Morellec (2005), find that firms are able to borrow more funds in a boom. In addition, they conclude that the debt capacity of the firm in a boom can be up to 25% larger than the debt capacity of that same firm in a contraction. In contrast, the positive correlation between liquidity risk and default risk imply that the liquidity risk is lower in an economic boom than in a recession. The net effect is not clear and unfortunately there is insufficient research on this issue.

The investors' risk aversion explains a proportion of the credit spreads. The risk aversion is time-varying and depends on the economic and the market conditions. Dionne et al (2006) find that the risk aversion and the impatience parameters, calibrated to the risk-free term structure using a consumption-based model, depend on the real consumption regimes. Chen et al. (2005) show that large variations in time-varying risk premiums are essential for explaining both the credit spread level and equity premium puzzles. Moreover, it is documented that the Fama and French (1993) systematic factors used to explain the equity premium are cyclical and depend on the state of the economy.

Finally, Collin-Dufresne, Goldstein, and Helwege (2003) find that the contagion risk explains a fraction of corporate bond spreads. Contagion risk is measured by default correlations that depend on the credit cycle. Crouhy, Galai, and Mark (2001) show that default correlations among US non-financial public firms are higher when default rates are higher. Moreover, Das, Freed, Geng, and Kapadia (2001) estimate a switching regression regime model in order to identify the periods of default correlations regimes. They find that high default regime correlations do not coincide with economic cycle dates specified by the NBER. They are conforming to high default rates. This result reinforces that credit cycles and business cycles can be different. In the next section, we present the model specification and the estimation method to be used.

# 3 Models specification

This section introduces the various empirical models considered in this paper for the corporate spreads dynamic and outlines the estimation method. First, we develop a simple discrete model with level effect under a single regime. This model, called LEVEL model, is a discrete form of the continuous diffusion model. Then, we extend this model by introducing a stochastic volatility component in order to consider explicitly both the clustering and the leverage effects in the conditional volatility. Finally, we introduce the switching regime model by allowing both the conditional drift and volatility to exhibit a regime shift. For each model, we emphasize its implications on the dynamic of the conditional drift and volatility.

## 3.1 Single regime LEVEL model

Under the single regime framework, the LEVEL model characterizes the corporate spreads $y_t$ by the following equation:

$$\Delta y_t = u\left(y_{t-1}, \theta_u, \Phi_{t-1}\right) + \sqrt{v\left(y_{t-1}, \theta_v, \Phi_{t-1}\right)} e_t \tag{1}$$

where $u\left(y_{t-1}, \theta_u, \Phi_{t-1}\right)$ and $v\left(y_{t-1}, \theta_u, \Phi_{t-1}\right)$ are the conditional drift and volatility of the corporate spreads given the available information $\Phi_{t-1}$. The parameters $\theta_u$ and $\theta_v$ are unknown. They will be estimated given the specification of the conditional drift, the conditional volatility and the distribution of the error term $e_t$.

The conditional drift is assumed to be a linear function of the recent corporate spread. This specification allows us to test if corporate spreads are mean-reverting. The dynamic of the conditional drift is given by the following equation:

$$u\left(y_{t-1}, \theta_u, \Phi_{t-1}\right) = a + b y_{t-1} \tag{2}$$

Corporate spreads are mean reverting if $b \prec 0$. The mean-reverting corporate spread in this case is equal to $-a/b$. The constant of the conditional drift is the adjustment speed of the corporate spreads to their mean reverting level.

The conditional volatility specification has important implications on the pricing of credit derivatives, the management of credit-spread risk and the forecasting of future volatilities. In the single regime LEVEL model we allow the conditional volatility to depends on the level of the corporate spreads. This is known in the empirical finance literature as the level effect and well documented for many financial variables such as the risk-free rates.

$$v\left(y_{t-1}, \theta_v, \Phi_{t-1}\right) = \sigma^2 y_{t-1}^{2\gamma} \tag{3}$$

The specification in equation (3) introduces for the level effect by having the conditional volatility depending on the recent realization of the corporate spreads. When the corporate spread is higher the volatility will be higher. The level effect of the corporate spreads on its volatility depends on the level elasticity parameter ($\gamma$). The conditional volatility is stationary when the level elasticity is less than one.

### 3.2 Single regime stochastic volatility model

The level effect is not sufficient to generate the clustering effect observed in the volatility of financial markets. One way to take into account this phenomenon is to introduce a GARCH effect in the volatility dynamic. Moreover, unexpected changes in corporate spreads can affect differently the conditional volatility depending on their signs. In equity returns and risk-free rates, it is well documented that negative shocks increase volatility more than positive shocks. This is known as the leverage effect. In the corporate bond spreads, this effect is not tested. So, we allow in the stochastic volatility model negative and positive unexpected corporate spread changes to affect differently the conditional volatility.

In order to account for the level, the GARCH and the leverage effects, we modelize the conditional volatility of corporate spreads with an asymmetric level GARCH model:

$$v\left(y_{t-1}, \theta_v, \Phi_{t-1}\right) \equiv v_t = y_{t-1}^2 h_t \tag{4}$$

$$h_t = w + \alpha\left(1 - \eta_t\right) e_{t-1}^2 + \beta h_{t-1} + \delta \eta_t e_{t-1}^2$$

where $\eta_t$ is an indicator function that takes 1 when the unexpected change in corporate spreads is negative and 0 when it is positive.

The GARCH component generates the clustering and the leverage effects. The negative shock increases (decreases) the volatility more than a positive shock when $\delta$ is higher than $\alpha$. The parameters $w$, $\alpha+\delta$, and $\beta$ have to be positive to ensure that the conditional volatility is positive. The conditional volatility is mean-reverting if $0.5\left(\alpha+\delta\right)+\beta$ is less than one. In this case, the mean-reverting volatility is given by $w/\left(1-0.5\alpha-0.5\delta-\beta\right)$.

In our specification, the level and the GARCH effects are multiplicative. This is different from recent models of Bali (2000, 2006) and Gray (1996) where the level effect is incorporated in the GARCH equation as an explicative variable. Our specification maintains the traditional interpretation of the GARCH model for the scaled residuals.

### 3.3 Switching regime model

The LEVEL model assumes a single regime for the corporate spreads in the entire sample period. However, as it is argued, corporate spreads can exhibit more than one regime depending on default, recovery, and liquidity cycles. Regime shift is introduced by allowing the parameters of both the conditional drift ($\theta_u$) and volatility ($\theta_v$) to be regime dependent.

We assume that there are two regimes for corporate spreads. They are defined as in Hamilton (1989 and 1994) by a Markov state process, assumed to be not observable. The transition probabilities between these two regimes, denoted by $s_t=\{1,2\}$, are $p=P\left[s_t=1/s_{t-1}=1,\Phi_{t-1}\right]$ and $q=P\left[s_t=2/s_{t-1}=2,\Phi_{t-1}\right]$. In this paper we assume that the transition probabilities are constant. However, we can easily extend the model to time-varying transition probabilities.

The switching regime model characterizes the dynamic of the corporate spread in the regime $s_t=\{1,2\}$ as follow:

$$\Delta y_t=u\left(y_{t-1},\theta_u,\Phi_{t-1},s_t\right)+\sqrt{v\left(y_{t-1},\theta_v,\Phi_{t-1},s_t\right)}e_t \tag{5}$$

The conditional drift depends on the regime $s_t=\{1,2\}$. For each regime it is specified as a linear function of the recent realization of the corporate spread:

$$u\left(y_{t-1},\theta_u,\Phi_{t-1},s_t\right)=a_{s_t}+b_{s_t}y_{t-1} \tag{6}$$

In this specification the conditional drift, conditional on past corporate spread changes, $E\left[\Delta y_t/\Phi_{t-1}\right]$ given by equation (7), is a function of the latest corporate spread and the ex-ante probability of being in a given regime $s_t$, $p_{i,t}=P\left[s_t=i/\Phi_{t-1}\right]$. In particular, the conditional drift is a weighted average of the drifts conditional on the regime with the weights being the ex-ante probabilities. Since ex-ante probabilities depend on the recent corporate spread, the conditional drift is a non-linear function of the previous corporate spread. This non-linearity is another advantage of switching regime models over single regime models.

$$E\left[\Delta y_t/\Phi_{t-1}\right]=\sum_{s_t=1}^{2}p_{s_t,t}u_{s_t,t}=\sum_{s_t=1}^{2}p_{s_t,t}\left[a_{s_t}+b_{s_t}y_{t-1}\right] \tag{7}$$

The conditional volatility $v\left(y_{t-1},\theta_v,\Phi_{t-1},s_t\right)$ in each regime is specified by:

$$v\left(y_{t-1},\theta_v,\Phi_{t-1},s_t\right)=v_{s_t,t}=\sigma_{s_t}^2 y_{t-1}^{2\gamma_{s_t}} \tag{8}$$

The switching regime model encompasses two models. The first model is the constant variance switching regime model specified by setting a restriction on the level elasticity ($\gamma_{s_t}=0$). The second model allows for a switching regime level effect specified by relaxing the previous assumption on the level elasticity ($\gamma_{s_t}$). So, one can test whether the level effect and the level elasticity depend on the regime of corporate spreads. In this model, all the components of the conditional volatility are stochastic since for each regime the conditional volatility depends on the corporate spread level.

The conditional volatility in the regime shift model depends on the conditional drift and volatility of each regime and the ex-ante probabilities of these regimes. The following equations describe the evolution of the conditional volatility and the unexpected change of corporate spreads:

$$v_t=E\left[\Delta y_t^2/\Phi_{t-1}\right]-E\left[\Delta y_t/\Phi_{t-1}\right]^2=\sum_{s_t=1}^{2}p_{s_t,t}v_{s_t,t}+p_{1,t}\left(1-p_{1,t}\right)\left(u_{1,t}-u_{2,t}\right)^2 \tag{9}$$

$$e_t=\Delta y_t-E\left[\Delta y_t/\Phi_{t-1}\right]=\Delta y_t-\sum_{s_t=1}^{2}p_{s_t,t}u_{s_t,t}=\sum_{s_t=1}^{2}p_{s_t,t}e_{s_t,t} \tag{10}$$

Even when the volatility of each regime $v_{s_t,t}$ is constant, the conditional volatility $v_t=v\left[\Delta y_t/\Phi_{t-1}\right]$ is stochastic and has two different sources. The first is the average of the regime conditional variances, and the second adds a jump component to the variance due to the switching effect of moving from one regime to another. The jump size depends on the difference between the conditional drift in one regime to another and the ex-ante probabilities at time $t$. Moreover, the conditional volatility is a quadratic function of the corporate spread. Once we specify the corporate spreads models, we have to estimate these empirical models. In the next section we present the estimation procedure.

### 3.4 Estimation method

In this section we discuss the estimation of the corporate spread change models. First, we present the estimation procedure for the single regime model. Then, we present the estimation of the switching regime model.

Under the single regime model, we assume that the scaled error $e_t$ is normally distributed with mean zero and variance one. The conditional distribution of the corporate spread change $\Delta y_t$ is a normal distribution with mean $u_t = a + by_{t-1}$ and variance $v_t = y_{t-1}^{2\gamma} h_t$. The parameters of the single regime model $\theta = \{\theta_u, \theta_v\}$ can conveniently be estimated by the Maximum Likelihood method (ML). The maximum likelihood estimate of $\theta$ is obtained by maximizing the log likelihood function for the entire sample:

$$l(\theta) = -\frac{T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln(v_t) - \frac{1}{2}\sum_{t=1}^{T}\frac{(\Delta y_t - u_t)^2}{v_t} \tag{11}$$

Since the maximization of the log likelihood function cannot be done analytically for all single regime models, we use the iterative optimization procedure of Berndt et al (1974), which is by far the most popular procedure to estimate GARCH models. The log likelihood function is not convex on the parameters to be estimated. In order to get a global optimum, we start the optimization procedure with different values and we retain the highest optimum.

In the level effect model, we estimate the level elasticity parameter ($\gamma$). It is well known that this parameter is highly correlated with the constant of the conditional volatility (Bali, 2000; Gray, 1996). That is why we estimate two alternative models. The first model does not include the level effect ($\gamma = 0$). The second incorporates the level effect with an elasticity parameter $\gamma$ equal to 0.5. This model is very popular in reduced form credit models. It is known as the Cox, Ingersoll and Ross (1985) model. A comparison of these models allows us to emphasize the marginal contribution of the level effect on corporate spread volatility.

The parameters of the switching regime model can be estimated using the maximum likelihood method. In addition to estimating the parameters and the transition probabilities, we have to estimate the ex-ante probabilities since the regime Markov process $s_t$ is not observed. We assume that the corporate spread change in each regime $s_t = \{1, 2\}$ is normally distributed with mean $u_{s_t,t}$ and variance $v_{s_t,t}$ given by equations (6) and (8) respectively. The conditional distribution of corporate spread changes is a mixture normal distribution:

$$f\left(\Delta y_t/\Phi_{t-1},\theta\right)=\sum_{i=1}^{2} f\left(\Delta y_t/s_t=i,\Phi_{t-1},\theta\right)P\left[s_t=i/,\Phi_{t-1},\theta\right] \tag{12}$$

where

$$f\left(\Delta y_t/s_t=i,\Phi_{t-1},\theta\right)=\frac{1}{\sqrt{2\pi v_{i,t}}}\exp\left(-\frac{1}{2}\frac{\left(\Delta y_t-u_{i,t}\right)^2}{v_{i,t}}\right) \tag{13}$$

In order to calculate the conditional distribution of corporate spread changes, we need to quantify the ex-ante probability for the entire sample. Hamilton (1989, 1994) shows that the ex-ante probability is the optimal forecast of the Markov process $s_t$. To simplify notation, let $p_{i,t} = P\left[s_t=i/\Phi_{t-1},\theta\right]$ and $f_{i,t} = f\left(\Delta y_t/s_t=i,\Phi_{t-1},\theta\right)$. The ex-ante probability of the first regime is given by:

$$p_{1,t}=\frac{(1-q)\,f_{2,t-1}\left(1-p_{1,t-1}\right)+pf_{1,t-1}p_{1,t-1}}{f_{2,t-1}\left(1-p_{1,t-1}\right)+f_{1,t-1}p_{1,t-1}} \tag{14}$$

The previous equation needs a starting value $p_{1,1}$ to work. There are three alternatives. First, one can treat this starting value as a separate parameter to be estimated. Second, one can choose randomly a parameter between zero and one. Finally, the starting value $p_{1,1}$can be set to the ergodic probability of the first regime equal to $1-q/2-p-q$. In our framework, we retain the last alternative for two reasons. First, we don't need to estimate the starting value $p_{1,1}$. Second, Hamilton (1989, 1994) emphasizes that the ergodic probability is problematic just when we use the EM algorithm to estimate switching parameters.

The maximum likelihood estimate for the switching regime parameter $\theta$ is found by maximizing the log likelihood function for the entire sample using again an iterative optimization procedure with different starting values. The log likelihood function is given by the following equation:

$$l\left(\theta\right)=\sum_{t=1}^{T}\ln\left[f\left(\Delta y_t/\Phi_{t-1},\theta\right)\right] \tag{15}$$

In estimating switching regime models two other conditional probabilities are of interest. The first is the inference probability defined as the probability that a particular regime occurs at time $t$ given actual and past information $P\left[s_t=i/\Phi_t,\theta\right]$. The second, called smoothed probability, is the probability that a particular regime occurs given all informations in the sample $P\left[s_t=i/\Phi_T,\theta\right]$.

The smoothed probability is usually used in the switching regime literature to classify regimes in the sample period. If the switching regime model is well specified, the smoothed probabilities have to be near one. Kim (1993) develops an algorithm to obtain the smoothed probabilities as function of both ex-ante and inference probabilities. Let $\zeta_{t/m}$ be the vector of $P\left[s_t = i/\Phi_m, \theta\right]$ and $f_t$ the vector of $f\left(\Delta y_t/s_t = i, \Phi_{t-1}, \theta\right)$. The inference, the ex-ante and the smoothed probabilities for all observations in the sample can be calculated by this algorithm:

$$\zeta_{t/t} = \frac{\zeta_{t/t-1} \odot f_t}{\zeta'_{t/t-1} \odot f_t} \tag{16}$$

$$\zeta_{t/t-1} = P'\zeta_{t-1/t-1} \tag{17}$$

$$\zeta_{t/T} = \zeta_{t/t} \odot \left[P\left(\zeta_{t+1/T} \div \zeta_{t+1/t}\right)\right] \tag{18}$$

where $\odot$ indicates element by element multiplication and $\div$ indicates element by element division. In the next section we present the data used to estimate the corporate spread models.

# 4 Data description

To measure the zero-coupon corporate spread term structure, we need the risk-free and the corporate zero-coupon yields for different maturities. The corporate spreads for a given credit rating and maturity are calculated as the difference between the corporate zero-coupon yield for this rating and the risk-free yield for the chosen maturity. The advantage of using zero-coupon yields is that it allows us to avoid any possible bias introduced by comparing yields on bonds with different coupon rates.

The data are obtained from the Fair Market Yield Curves of the Bloomberg system. The data consist of annualized zero-coupon yield curves of U.S. government and corporate bonds based on weekly observations for the period 30/12/1994 to 15/12/2006 (625 weekly observations). This period contains the economic recession of 2001 and the high default cycle (2000-2001) where defaults exhibit their highest level in the corporate default history. In addition, this period contains the corporate liquidity crisis of 1998 due to Russian default (August of 1998) and the bond crush (October of 1998). So, our regime shift model is a potential candidate for corporate spreads. We restrict our sample to industrial corporate yields since for others industries zero-coupon yields are not available for non-investment grade bonds (lower than BBB Moody's credit rating). Credit ratings are from A to B which allows us to compare the dynamics of corporate spreads across ratings and maturities. Industrial Aaa and Aa rated curves are eliminated because of insufficient data.

Table 1 reports summary statistics on treasury yields and corporate spreads. Figure 1 plots the term structure of risk-free and corporate zero-coupon yields over the period 1995-2006. Figure 2 plots the corporate spreads term structure of industrial A, Baa, Ba and B bonds.

Like risk-free rates, the unconditional means of corporate spreads are increasing with the maturity for all ratings. This result is in contrast with the Merton (1974) model prediction for non-investment corporate bonds since it implies that their corporate spread term structure is downward slopping. Helwege and Turner (1999) showed that the non-investment grade term structure is upward slopping. For any maturity, the corporate spreads increase with the credit risk measured by the credit rating. The term structure of the unconditional volatility is upward sloping for investment grade bonds and downward sloping for non-investment grade and risk-free rates.

In addition to the two first moments of corporate spreads, we report the third and the fourth moments, skewness and kurtosis. Investment-grade bonds and risk-free rates have a kurtosis lower than for the normal distribution. In contrast, non investment-grade bonds have an excess kurtosis. Corporate spread kurtosis seems to increase with maturity for risk-free rates. For corporate bonds, it is not clear whether the relation between the maturity and the kurtosis is positive or negative. The skewness of risk-free rates is negative except for long term maturities. Corporate bonds exhibit a positive skewness. Since the higher moments of risk-free yields and corporate spreads are different from those of a normal distribution, we perform the Jarques-Berra normality test. For all variables the statistics of the normality test is higher than its critical value 5.99, confirming evidence of non normality in the distribution of corporate spreads and risk-free rates.

The correlation between the change in corporate spreads and risk-free rates and their recent realizations gives us an idea about the potential of mean-reverting. The last column of table 1 shows that the correlation is negative for all yields, as expected. It seems that risk-free rates and non-investment corporate spreads are not mean-reverting since the correlation is low. The correlation for these bonds increases with maturity. For investment grade corporate spreads, the correlation is higher and corporate spreads are more mean-reverting than non-investment grade bonds. In addition, the correlation decreases with maturity.

Figure 3 plots the time series of short term, long term and the slope of government and corporate term structures. The shaded area represents the economic recession of 2001 as specified by the NBER. Figure 4 contains plots of the short term, long term and the slope of corporate spreads. It is interesting to observe that the term structure slope of risk-free and corporate yields decreases just before the recession of 2001 and increases dramatically after the recession. In particular, the term structure slopes of risk-free and A corporate yields become negative just before the recession. The term structure slopes of all corporate spreads increase before the recession and decrease after the recession.

Three months and ten years corporate yields increase before the recession and decease after the recession. The low level of short term risk-free yields during the period 2002-2003 explains why corporate spreads in this period are very high although there are no defaults during this period. So, during the period 1995-2006 high levels of corporate spreads are not only explained by default rates and economic recession. Corporate spreads have increased also in 1998 when the Russian default and the bond crush occurred.

Figure 5 plots the squared residuals obtained from OLS regression of corporate spreads changes and recent corporate spreads as a proxy of volatilities. One can clearly observe that the conditional volatility of risk-free rates and corporate spreads are time-varying. Moreover, it seems that the conditional volatility of corporate spreads and risk-free rates exhibit a regime shift since there are episodes where volatilities are lower than other episodes. Moreover, conditional volatilities seem to be more persistent for non-investment corporate spreads. In the next section, we report the empirical results of retained models for short term and long term corporate spreads.

# 5 Empirical results

In this section we present the estimation results for short term and long term corporate spreads. For each empirical model, we emphasize its strength and weakness. Moreover, we compare different nested models to access the marginal contribution of each feature in the dynamic of corporate spreads. Finally, we compare the relative performance of these models in terms of volatility forecasting and matching the sample moments.

## 5.1 Single regime LEVEL model

We begin by considering the case of the constant volatility model, called the VASICEK model. In this model the conditional volatility is constant and the elasticity parameter is equal to zero. So, we can test if the volatility of credit spreads is time-varying. Then, we allow the conditional volatility to depend on the corporate spread level.

Panel A in table 2 reports the estimated parameters of the VASICEK model for short term spreads. In panel B we estimate the LEVEL model with an elasticity level equal to 0.5, the CIR model. This version is a discrete form of the popular CIR (1985) model. In panel C we estimate the elasticity level parameter of corporate spreads, the LEVEL model. Table 6 presents the estimation results for long term corporate spreads.

For each model, we present the parameter estimates, the value of the log likelihood function, and the AIC and SIC information criteria. Moreover, we report the Ljung-Box and the ARCH tests for the squared residuals and the Jarques-Berra normality test for the standardized residuals. These tests allow us to emphasize the contribution of each feature in the corporate spread models.

The conditional mean parameters of corporate spreads in the VASICEK model are significantly different from zero for investment grade bonds. In addition, the coefficients $b$ are negative, so short term corporate spreads of investment grade bonds are mean-reverting with any level of confidence. For non-investment grade bonds, the conditional mean parameters of short term corporate spreads are not significantly different from zero but the coefficients $b$ are still negative. This implies no mean reversion in non-investment grade short term corporate spreads. In contrast, non-investment grade long term corporate spreads are stationary at a confidence level of 95%.

The long run mean of corporate spreads given by the model increases with credit risk and maturity. For short term corporate spreads, they are respectively 0.62, 0.89, 1.87, and 2.73 percent for A, Baa, Ba and B corporate bonds. The constant volatility model, as shown in table 1, underestimate the sample mean of short term corporate spreads (0.63, 0.92, 2.07, and 3.29 percent for A, Baa, Ba and B corporate bonds). For long term corporate spreads, the constant volatility model fits well the average of corporate spreads.

The estimates of the conditional volatility increase with credit risk. The long run volatility implied by the model is similar to the sample volatility for investment grade bonds and overestimate the volatility of non-investment grade bonds, particularly for short term spreads.

The constant volatility model does a poor job in modeling the conditional volatility. First, there is a serial correlation in the squared residuals. The Ljung-Box statistic rejects the absence of serial correlation for any confidence level. Second, the ARCH test implies that the constant conditional volatility assumption is restrictive. Finally, the standardized residuals are not normally distributed since the Jarques-Berra normality test rejects the Gaussian distribution with a p-value of 0.1%. Theses results imply that the volatility of corporate spreads is time-varying. There are three ways to introduce stochastic volatility: level effect, GARCH effect, and regime shift.

The CIR model assumes that the level elasticity parameter is equal to 0.5. So, the conditional volatility is time-varying. The panels B of tables 2 and 6 show that the CIR model outperforms the constant volatility for all corporate spreads since the value of the log likelihood function increases and the AIC and SBC information criteria decrease. Moreover, the diagnostic tests of the standardized residuals show a substantial improvement in the CIR model.

The CIR model does also a poor job in modeling the conditional volatility. The level effect is not sufficient to account for the clustering effect observed in the conditional volatility of corporate spreads. The Ljung-Box statistic rejects the absence of serial correlation for any confidence level. The ARCH test implies that the squared residuals are also correlated. The standardized residuals are not normally distributed since the Jarques-Berra normality test rejects the Gaussian distribution with a p-value of 0.1%.

In panel C of tables 2 and 6, we estimate the level elasticity parameters. For all corporate spreads, the estimate of the level elasticity is statistically different from zero for any level of confidence. It is interesting to note that these estimates are near 0.5 for short term corporate spreads. For long term corporate spreads, the elasticity parameter is near 0.5 for investment grade bonds and higher than 0.5 for non-investment grade bonds.

The Likelihood Ratio Test (LRT) of the CIR model versus the LEVEL model is unable to reject the CIR model with a p-value of 22% for short term spreads and investment grade long term spreads. The LRT statistic, for long term non-investment grade spreads is larger than 16 while the critical value is equal to 3.84. So, the elasticity parameter of long term non-investment grade bonds is statistically higher than 0.5. The LEVEL model has the same limits as the CIR model. In order to capture the clustering effect in the corporate spreads volatility, we estimate a GARCH (1,1) model with level effect.

## 5.2 Stochastic volatility model

Table 3 reports the parameter estimates of the GARCH-CEV model for short term corporate spreads. Long term corporate spreads results are presented in table 7. In panel A, the level elasticity parameter is assumed to be equal to zero, so there is no level effect (GARCH model). In panel B, we incorporate the level effect with a level elasticity equal to 0.5 (CIR-GARCH model). In the last panels of tables 3 and 7, we report the results of the GARCH-CEV model. Theses results allow us to test if the level effect remains significant when we introduce a clustering effect in the corporate spread volatility.

Parameter estimates of the GARCH models illustrate a set of interesting findings. First, modeling explicitly the serial correlation in the conditional volatility, as shown in the LEVEL model, leads to a superior fit. The LRT statistics, when the elasticity parameter is equal to zero, can no longer reject the GARCH effect for all corporate spreads at any confidence level. The same result holds if the elasticity parameter is equal to 0.5 as implied by the CIR model or when it is estimated by the GARCH-CEV model. In addition, the p-value of the Ljung-Box test is greater than 52% for all elasticity parameter cases and corporate spreads, implying an absence of serial correlation in the standardized residuals. The ARCH tests also have a p-value greater than 52%. However, the standardized residuals in the GARCH models are not normally distributed.

Second, the level effect in the conditional volatility of corporate spreads remains strong, despite the incorporation of the GARCH effects. Moreover, for short term corporate spreads, the LRT statistics cannot reject the GARCH-CIR model for a confidence level of 95%. The p-values of this test are higher than 26.35%. For long term corporate spreads, the elasticity parameters are not statistically different from 0.5 for investment grade bonds. Long term non-investment corporate spreads have an elasticity level parameter higher than 0.5 with a p-value lower than 0.27%. This implies that both the level effect and the GARCH effect are essential in modeling the conditional volatility of corporate spreads.

Third, the estimates of the elasticity parameters are higher in the GARH-CEV model than in the LEVEL model except for Ba short term corporate spreads where the elasticity parameter is lower.

Fourth, the marginal contribution of the GARCH effect turns out to be stronger than the marginal contribution of the level effect for all corporate spreads. The marginal contribution of the level effect in terms of log likelihood function is respectively 11.63, 15.11, 38.75 and 45.34 for A, Baa, Ba and B short term corporate spreads. For the GARCH effect, the contribution is respectively 19.25, 18.50, 45.40 and 71.01 for A, Baa, Ba and B bonds. The same result holds for long term corporate spreads. The marginal contribution of the level effect in terms of log likelihood function is respectively 18.10, 8.27, 46.12 and 39.93 for A, Baa, Ba and B long term spreads. The GARCH marginal contribution, in this case, is 75.38, 118.03, 81.35 and 116.22. In addition, we see that both the level and the GARCH marginal contribution are positively related to default risk.

In the literature of GARCH models, a lot of attention was given to the persistence of the conditional volatility. This is important for volatility forecasting, derivatives pricing, and risk management. In other words, it is important to test the mean-reversion of the conditional volatility. In our GARCH model, the conditional volatility is mean-reverting if $\alpha + \beta$ is less than one. In this case, the mean-reverting volatility level is given by $w/\left(1 - \alpha - \beta\right)$.

The estimates of persistence parameters in tables 3 and 7 show that the persistence of unexpected corporate spreads depends on the presence or not of the level effect in the specification of the conditional volatility. The persistence of the conditional volatility decreases when the level effect is incorporated. So, the high persistence of non-investment corporate spreads observed in panel A is, on part, due to the level effect. For example, the value of $\alpha + \beta$ is equal respectively to 0.95, 0.67, 0.81 and 0.95 for Aa, Baa, Ba and B short term spreads. To test the mean-reversion of the conditional volatility, we use the Lagrange Multiplier Test (LMT) since this test, unlike the LRT test, can be done from the GARCH model without estimating the Integrated GARCH model for each case of elasticity parameter and corporate spreads. The mean-reversion of the conditional volatility is not rejected by the LMT statistics for a confidence level of 95% except for non-investment grade short term spreads and B long term spreads when the elasticity parameter is equal to zero.

Finally, it is important to emphasize the fact that the conditional volatility specification significantly affects the conclusion on the drift dynamics, particularly the mean-reversion of non-investment grade short term corporate spreads. In the GARCH models, in contrast to results of Neal et al (2000), all the corporate spreads are mean-reverting with a confidence level of 95%.

### 5.3 Asymmetric stochastic volatility model

In GARCH models, positive and negative unexpected credit spreads have the same impact on the conditional volatility of corporate spreads. In the Asymmetric GARCH model, we extend the GARCH model by allowing negative and positive shocks to impact differently the conditional volatility of corporate spreads (GJR-GARCH-CEV model). Table 4 reports the estimated parameters of Asymmetric GARCH model for short term corporate spreads. The results of long term credit spreads are reported in table 8. Similar to the previous models, we have three specifications for the level effect (panel A, B and C).

The GJR-GARCH model, tables 4 and 8 panel A, suggests that negative and positive unexpected credit spreads have different impacts on the conditional volatility ($\delta \neq \alpha$). The LRT statistics for all corporate spreads are higher than the critical value 3.84 for a confidence level of 95%. The conditional volatility of corporate spreads increases more when positive unexpected corporate spreads occur ($\delta \prec \alpha$).

The marginal contribution of the level effect in the conditional volatility of short term and long term corporate spreads remains statistically significant except for A-rated spreads. Moreover, the estimate of the level elasticity is not statically different from 0.5 for Baa, Ba and B bonds. For A-rated bonds, the parameter is not significant (p-value equal to 15.75% for short spreads and 12.83% for long term spreads).

In the asymmetric GARCH model with level effect (LEVEL-GJR-GARCH) negative and positive unexpected spreads have the same impact on the conditional volatility except for A-rated bonds where positive unexpected spreads increase the conditional volatility and negative unexpected spreads have no impact on the conditional volatility ( $\delta = 0$ for any confidence level). In contrast to the leverage effect observed in equity returns and treasury rates, we do not find a significant leverage effect in spreads volatilities of Baa, Ba and B corporate bonds.

## 5.4 Switching regime model

In this section, we present the estimation results of the switching regime model of short term and long term corporate spreads. Table 5 represents the results for short term spreads and table 9 presents the results for long term spreads. As in the previous models, we consider three cases concerning the level effect (panel A, B and C).

In the first model (VASICEK-SR), the volatility is constant within each regime. So, the only source of time-varying volatility is the difference between the regimes conditional drifts. In other words, if the two regimes have the same conditional drift, the conditional volatility will be constant.

For short term spreads, regimes are essentially different in volatility for investment grade spreads and in both volatility and long run mean for non-investment grade bonds. The first regime is a high volatility regime for all short term corporate spreads. The high volatility regime has a lower average spreads for investment grade bonds and a higher average spreads for non-investment grade bonds.

The regimes of non-investment grade short term spreads are more persistent than those of investment grade spreads. Moreover, there is a high persistence in both regimes of non-investment grade spreads since the transition probabilities are less than 0.1. For investment grade spreads, the low volatility regime is more persistent than the high volatility regime. This suggests that episodes of high volatility investment grade spreads have short horizons.

Finally, in the high volatility regime, the corporate spreads tend to converge to their unconditional mean more quickly than in the case of low volatility regime since the adjustment coefficients $b$ are higher in this regime. Investment grade spreads are stationary in both regimes at a confidence level of 90%. At the 95% confidence level, only the high volatility regime is stationary. For non-investment grade spreads, they are not stationary in both regimes at a confidence level of 95%.

For long term spreads, regimes are different in both the volatility and the level of corporate spreads. In the vein of short term spreads, the high volatility regime has a lower average spread for investment grade bonds and a higher average spread for non-investment grade bonds.

There is a high persistence in both regimes of long term corporate spreads since the transition probabilities are less than 0.2. The high volatility regime is more persistent than the low volatility regime for all corporate spreads. In addition, in the high volatility regime, the corporate spreads tend to converge to their unconditional mean more quickly than in the case of the low volatility regime since the adjustment coefficients $b$ are higher in this regime. The p-values of the corporate spread adjustment rate ($b$) suggest that corporate spreads are stationary only in the high volatility periods.

The VASICEK-SR model does a substantial improvement over the constant volatility model in terms of modeling the conditional volatility. First, there is no serial correlation in the squared residuals. The Ljung-Box statistic has a p-value higher than 5% for all corporate spreads. Second, the ARCH test reinforces the absence of serial correlation. The p-value of the ARCH test is also higher than 5%. The Jarques-Berra normality test rejects the Gaussian distribution of the expected residuals $e_t$ with a p-value of 0.1%. However, it is important to emphasize that, for the VASICEK-SR, the expected residuals are not normally distributed but they have a mixture normal distribution. So, the Jarques-Berra normality test does not imply necessarily that the Gaussian distribution is rejected. Finally, the VAICEK-SR model has a log likelihood function higher than all single-regime models. In particular the LRT statistics of the single regime model is higher than 116. Unfortunately, this statistics cannot be compared to its standard critical value because the transition probabilities are nuisance parameters.

In the regime shift literature, the dates where each regime is likely to occur are commonly reported. In addition, it is of interest to know whether these regimes coincide with economic and default cycles. Figure 7 plots, for each credit rating, the high volatility smoothed and ex-ante probabilities, the conditional volatility and the unexpected changes of short term corporate spreads obtained from the VASICEK-SR model. Figure 8 plots the same variables for long term corporate spreads. Shaded area represents the economic recession of 2001. It is clear that the non-investment grade short term spreads are in the high volatility and level regime during the economic recession of 2001. For investment grade short term spreads, both regimes of corporate spreads occur in the economic recession. This result is coherent with the corporate spreads dynamic in figure 4.

Credit-spreads regimes of non-investment grade bonds are related to the default cycle 2000-2001 reported in figure 6 and to the decline of short risk-free rates in 2002-2003. For investment grade spreads there is a weak relation between the occurrence of a particular regime and the economic and the default cycles. Dufresne and al (2001) show that large variations of investment grade bonds are not explained by economic and financial factors but by a common variable related to the market conditions. Long term corporate spreads seem to be more related to economic conditions than short term spreads. During the economic recession, investment grade spreads are in the low volatility regime and non-investment spreads are in the high volatility regime.

The main drawback of the VASICEK-SR model is that the regime conditional volatility is constant. As a consequence, the conditional volatility is highly correlated with the ex-ante probabilities. Figures 7 and 8 show that ex-ante probabilities and conditional volatility have similar dynamic. In panels B and C of tables 5 and 9, we extend the VASICEK-SR model by introducing level effect (CIR-SR and LEVEL-SR models). Hence, we can test the marginal contribution of the level effect in the regime shift model. In addition, it is of interest to test whether the level elasticity parameter depends on the corporate spreads regime.

For all corporate spreads, the LRT statistics[1] is higher than 6.73 and rejects the absence of the level effect (VASICEK-SR model). We also test whether the CIR-SR model is appropriate for the conditional volatility of corporate spreads. The LRT test rejects the CIR-SR model in the case of short term A and Ba bonds and long term non-investment grade bonds. In these cases, there is one regime where the level elasticity is statistically not different from 0.5 and in the other regime the level elasticity parameter is not significant. For all corporate spreads, the level effect is stronger in the high volatility regime. Moreover, the conditional volatility is less correlated to the ex-ante probabilities in the CIR-SR and the LEVEL-SR models. They depend also on the level of corporate spreads.

Finally, we compare the Regime Classification Measure (RCM) defined by Ang and Bekaert (2002) for the three switching regime models. This statistics measure how successfully the switching regime model distinguishes between regimes using estimated parameters and the data. This measure is between 0 and 1 and is given by:

$$RCM = 4\frac{\sum_{t=1}^{T} P\left[s_t = 1/, \Phi_T, \theta\right]\left(1 - P\left[s_t = 1/, \Phi_T, \theta\right]\right)}{T} \tag{19}$$

The switching regime model does a perfect classification when the RCM is equal to zero. When it is near one, this implies that no information about regimes is revealed. Table 10 presents the

[1] The LRT statistics still holds for theses tests because all parameters are identified.

estimates of the RCM. The VASICEK-SR model outperforms the CIR-SR and the LEVEL-SR model in terms of regime classification except for short term Ba spreads and long term B spreads. For short term Ba spreads, the CIR-SR model does a perfect regime classification. For long term B spreads, the LEVEL-SR model has the lowest RCM. However, the CIR-SR and the LEVEL-SR models are more appropriate for corporate spreads as shown in the previous section using the LRT test. Their regime classification performances are acceptable since the RCM is far from one. In the next section, we compare the performance of all the models examined in this paper in terms of volatility forecasting (Gray, 1996; Ang and Bekaert, 2002) and fitting the sample moments (Ang and Bekaert, 2002).

### 5.5 Performance Comparison of Corporate Spreads Models

In this section, we compare the performance of corporate spreads models in forecasting the conditional volatility and matching unconditional moments. Ang and Bekaert (2002) emphasize that single regime models or nested models can perform better in terms of volatility forecasting and sample moments fitting even when they are not the appropriate process.

The forecasting volatility performance of a given model is measured by three statistics: the forecasting $R^2$, the Mean Absolute Errors (MAE) and the Root Mean Squared Errors (RMSE) between the actual volatility $e_t^2$ and the expected volatility $v_t$. These statistics are defined by:

$$R^2 = 1 - \frac{\sum_{t=1}^{T} \left[e_t^2 - v_t\right]^2}{\sum_{t=1}^{T} e_t^4} \tag{20}$$

$$RMSE = \sqrt{\frac{\sum_{t=1}^{T} \left[e_t^2 - v_t\right]^2}{T}} \tag{21}$$

$$MAE = \frac{\sum_{t=1}^{T} \left|e_t^2 - v_t\right|}{T} \tag{22}$$

When the model does a perfect forecasting of volatility, the forecasting $R^2$ is equal to one while the RMSE and the MAE are equal to zero. Table 11 reports the results of the in sample forecasting performance for short and long term corporate spreads.

The regime shift model for all corporate spreads does a good job in forecasting the conditional volatility, particularly when the level effect is introduced. Moreover, the regime shift model performs better for long term corporate spreads. The stochastic volatility models are the best in forecasting short term corporate spreads volatilities. However, when the estimated parameters of the stochastic volatility model imply high persistence volatility, the $R^2$ is negative. In this case, the forecasted volatility error is larger than the actual volatility. The forecasting performance decreases also when the elasticity-parameter is very high, near or higher than one. Finally, the CIR model performs better than others models when the elasticity-parameter is near or higher than one and the implied volatility is persistent.

In the rest of this section we look at the unconditional moments performance. For each model, we estimate the mean, the standard deviation, the skewness, and the kurtosis of simulated corporate spreads. Then, we compare them to the moments of the observed spreads. In order to compare the global performance of all the models, we use the unconditional moments statistic defined by Ang and Bekaert (2002) for each model:

$$H = (g - \widetilde{g})' \Sigma_{\widetilde{g}} (g - \widetilde{g}) \tag{23}$$

where $g$ is the vector of the sample moments, $\widetilde{g}$ is the vector of unconditional moments obtained from Monte Carlo simulations and $\Sigma_{\widetilde{g}}$ is the covariance matrix of estimated unconditional moments. Ang and Bekaert (2002) highlight that a high correlation between the estimated moments leads to poorly weighting matrices. They suggest another statistic $H^*$ using just the diagonal of $\Sigma_{\widetilde{g}}$.

We simulate one million of sample paths using the estimated parameters of each model and the initial observed corporate spreads. The statistics $H$ and $H^*$ depend of the fitting errors of the sample unconditional moments and the uncertainty of each estimate through the covariance matrix. So, we don't use the estimated parameters as true parameters. The results are reported in table 12 for short term corporate spreads and table 13 for long term corporate spreads.

Regime shift models give the best fit of the sample moments for A and Ba short term corporate spreads and Ba long term corporate spreads. GARCH models outperform others models in the case of short term Baa corporate spreads. For others credit spreads, the asymmetric GARCH models are the best in fitting the sample moments. Moreover, when the elasticity level parameter is higher than one and the conditional volatility is persistent, the sample moments of simulated spreads are not defined. This is the case of the LEVEL-GARCH and the LEVEL-GJR-GARCH models of B long term corporate spreads. When the conditional drift or volatility are not mean-reverting we obtain a poor fit for the unconditional moments. Finally, the VASICEK models generally tend to do a poor job in fitting the unconditional moments because they imply a low skewness. In the next section, we discuss the implications of our empirical results.

# 6 Discussion of regime shift implications

We find that corporate spreads exhibit regime shift and their volatilities are time-varying. Theses results have important implications on the pricing of credit derivatives and hedging credit spreads risk.

Since the Basel II, banks have to take into account the credit spreads risk of corporate bond portfolios in addition to interest-rate risk. In order to estimate the VaR of a single asset or a portfolio of assets, the covariance matrix and higher moments of assets returns have a substantial effect. Ignoring the stylized fact of regime shift in corporate spreads leads to an underestimation (overestimation) of the VaR for a corporate bond portfolio just before the occurrence of the high (low) volatility regime. Billio and Pelizzon (2000) show for the case of equities that the switching regime beta model performs better than the Risk Metrics mixture normal and the GARCH models. Moreover, Guidolin and Timmermann (2003) find that the regime switching model, applied to stocks returns, produces a systematically higher expected shortfall than Gaussian and GARCH models. They show also that the relative performance of the regime switching model is better at long horizons.

Regime shifts in corporate spreads have also important implications on the pricing of credit risk derivatives. As in the case of the VaR, the volatility dynamic of the underlying asset can affect the price of the credit derivative. For example, the price of a call option on corporate spreads will be under-priced (overpriced) using a single regime model in a period of a high (low) volatility. Bollen (1998) shows that the Black-Sholes model applied to equities generates significant errors when a regime switching process governs the underlying asset returns. In addition, Bollen (1998) shows that the regime shift model is able to generate implied volatility smiles.

Finally, regime shift in corporate spreads can have implications on the decomposition of corporate spreads. It is interesting to test if the proportions of corporate spread components depend on corporate spreads and default regimes. For example, one can test if the credit event risk estimated by Berndt et al (2004) and Driessen (2005) depends on the corporate spreads regime. We can set up a discrete reduced form model with regime shifts and allow the relation between the risk neutral and the observed default intensity to be regime dependent. Intuitively, the credit event risk should be higher in expansion periods because during this state the observed default probability is low.

# 7 Conclusion

Corporate spreads represent a key variable in the valuation of credit risk instruments and the management of credit investment strategies. Hence, it is important to correctly specify the conditional distribution of the term structure of corporate spreads. In this paper, we build up and estimate a comprehensive model for corporate spreads volatilities that includes, simultaneously, the level and the GARCH effects in addition to regime shifts. We also test if negative and positive unexpected corporate spreads have the same effect on the conditional volatility (leverage effect).

We find that the level and the GARCH components are essential in modeling the conditional volatility of corporate spreads. The marginal contribution of the GARCH effect is stronger than the contribution of the level effect. The level elasticity parameter is generally in accordance with the square-root specification used in reduced form models except for long term non-investment grade bonds where it is statistically higher than 0.5. The conditional volatility is more persistent for non-investment grade spreads. The leverage effect test shows that negative and positive unexpected corporate spreads have the same effect on the conditional volatility except for A short term corporate spreads, for which positive shocks lead to an increase in the conditional volatility while negative shocks have no influence on the conditional volatility.

The conditional volatility specification affects the conclusion about the mean-reversion of corporate spreads. In contrast to results of Neal et al (2000), all the corporate spreads are mean-reverting when the GARCH component is incorporated in the conditional volatility.

Corporate spreads are found to exhibit regime shifts between a high volatility regime and a low volatility regime. The high volatility regime has lower (higher) average spreads for investment (non-investment) grade spreads. In the high volatility regime, corporate spreads are more stationary than in the low volatility regime. For all corporate spreads, the elasticity parameter is higher in the high volatility regime. Non-investment grade spread regimes are related to economic and default cycles. In contrast, there is a weak relation between investment grade credit spread regimes and economic and default cycles.

We compare the predictive power of nested and non-nested models estimated in this article in capturing the conditional volatility and in fitting the unconditional moments of corporate spreads. Concerning the forecasting of conditional volatilities, the regime shift model with level effects does a good job, particularly for long term corporate spreads. The stochastic volatility models are the best in forecasting short term corporate spreads volatilities. However, when the estimated parameters of the stochastic volatility model imply high persistence volatility, the forecasted volatility error is larger than the actual volatility. The forecasting performance decreases also when the elasticity-parameter is very high, near or higher than one.

Regime shift models give the best fit of the sample moments for A and Ba short term spreads and Ba long term corporate spreads. GARCH models outperform other models in the case of short term Baa corporate spreads. For other credit spreads, the asymmetric GARCH models are the best in fitting the sample moments. The absence of mean-reversion in the conditional drift or volatility leads to poor fitting of the unconditional moments. Finally, the Gaussian constant volatility model does a poor job in fitting the unconditional moments because they imply a low skewness.

In future work, it would be interesting to test if introducing regime shifts in corporate spreads term structure has an important impact on the estimation of the VaR of corporate bond portfolio and on the pricing of credit derivatives. For the VaR of corporate bond portfolio, one can compare the estimated VaR obtained by a switching regime model and the historical method used in the industry. One can also test reduced form models by estimating the latent factors of reduced form models and test if they exhibit a regime shift. Lastly, it is important to test if the relative importance of credit spread components varies with credit spreads and default cycles.

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### Table 1: Summary Statistics on Treasury Yields and Corporate Spreads

This table reports the average corporate bond spreads from government bonds for industrial A, Baa, Ba and B corporate bonds during the period 1995-2006. Spot rates of corporate and risk-free bonds are obtained from Bloomberg Fair Market Yield Curves.

| Maturity | Mean | SD | Min | Max | Skewness | Kurtosis | JB-Statistic | $\rho(\Delta y_{t-1}, y_{t-1})$ |
|---|---|---|---|---|---|---|---|---|
| Treasury Yields | | | | | | | | |
| 3 | 3.918 | 1.731 | 0.815 | 6.394 | -0.591 | 1.814 | 73.242 | -0.039 |
| 6 | 4.062 | 1.755 | 0.827 | 6.598 | -0.593 | 1.849 | 71.297 | -0.051 |
| 12 | 4.215 | 1.799 | 0.868 | 7.480 | -0.587 | 1.857 | 70.028 | -0.064 |
| 24 | 4.472 | 1.644 | 1.021 | 7.760 | -0.485 | 2.031 | 49.190 | -0.078 |
| 60 | 4.967 | 1.252 | 2.109 | 7.810 | -0.179 | 2.039 | 27.688 | -0.091 |
| 84 | 5.236 | 1.086 | 2.719 | 7.850 | 0.001 | 2.054 | 23.656 | -0.098 |
| 120 | 5.546 | 0.947 | 3.463 | 7.980 | 0.213 | 2.074 | 27.349 | -0.102 |
| A-Rated Corporate Spreads | | | | | | | | |
| 3 | 0.632 | 0.181 | 0.241 | 1.068 | 0.264 | 2.366 | 17.975 | -0.186 |
| 6 | 0.660 | 0.206 | 0.224 | 1.165 | 0.224 | 2.264 | 19.597 | -0.147 |
| 12 | 0.682 | 0.273 | 0.243 | 1.665 | 1.095 | 3.774 | 139.470 | -0.163 |
| 24 | 0.696 | 0.267 | 0.257 | 1.373 | 0.661 | 2.257 | 59.893 | -0.113 |
| 60 | 0.845 | 0.281 | 0.294 | 1.540 | 0.683 | 2.388 | 58.356 | -0.107 |
| 84 | 0.897 | 0.333 | 0.461 | 1.776 | 0.865 | 2.619 | 81.515 | -0.081 |
| 120 | 0.834 | 0.324 | 0.409 | 1.882 | 1.252 | 4.009 | 188.400 | -0.083 |
| Baa-Rated Corporate Spreads | | | | | | | | |
| 3 | 0.917 | 0.289 | 0.420 | 1.631 | 0.621 | 2.226 | 55.897 | -0.122 |
| 6 | 0.970 | 0.296 | 0.474 | 1.637 | 0.504 | 1.871 | 59.885 | -0.099 |
| 12 | 1.007 | 0.409 | 0.347 | 2.095 | 0.785 | 2.716 | 66.133 | -0.105 |
| 24 | 1.052 | 0.423 | 0.402 | 2.041 | 0.393 | 1.876 | 49.223 | -0.076 |
| 60 | 1.234 | 0.425 | 0.565 | 2.108 | 0.353 | 1.897 | 44.975 | -0.074 |
| 84 | 1.318 | 0.471 | 0.649 | 2.351 | 0.493 | 1.942 | 54.648 | -0.064 |
| 120 | 1.274 | 0.429 | 0.592 | 2.443 | 0.491 | 2.577 | 29.818 | -0.083 |
| Ba-Rated Corporate Spreads | | | | | | | | |
| 3 | 2.070 | 1.222 | 0.724 | 5.981 | 1.125 | 3.493 | 137.340 | -0.051 |
| 6 | 2.167 | 1.256 | 0.734 | 6.103 | 1.096 | 3.398 | 128.490 | -0.042 |
| 12 | 2.232 | 1.346 | 0.762 | 6.564 | 1.124 | 3.515 | 137.680 | -0.050 |
| 24 | 2.334 | 1.297 | 0.768 | 6.415 | 0.994 | 3.133 | 102.750 | -0.044 |
| 60 | 2.684 | 1.060 | 1.160 | 6.032 | 0.928 | 3.087 | 89.353 | -0.056 |
| 84 | 2.801 | 0.908 | 1.256 | 5.580 | 0.798 | 2.880 | 66.423 | -0.064 |
| 120 | 3.032 | 0.795 | 1.616 | 5.314 | 0.631 | 2.735 | 43.299 | -0.079 |
| B-Rated Corporate Spreads | | | | | | | | |
| 3 | 3.296 | 1.536 | 0.993 | 7.914 | 0.792 | 3.135 | 65.371 | -0.047 |
| 6 | 3.436 | 1.602 | 1.034 | 8.045 | 0.715 | 2.979 | 52.971 | -0.042 |
| 12 | 3.569 | 1.696 | 1.202 | 8.641 | 0.832 | 3.185 | 72.590 | -0.045 |
| 24 | 3.772 | 1.594 | 1.453 | 8.283 | 0.776 | 2.978 | 62.439 | -0.046 |
| 60 | 4.239 | 1.312 | 2.475 | 8.121 | 0.940 | 3.115 | 91.935 | -0.057 |
| 84 | 4.308 | 1.247 | 2.345 | 8.165 | 0.986 | 3.055 | 100.860 | -0.064 |
| 120 | 4.389 | 1.186 | 2.266 | 8.185 | 0.892 | 3.223 | 83.725 | -0.074 |

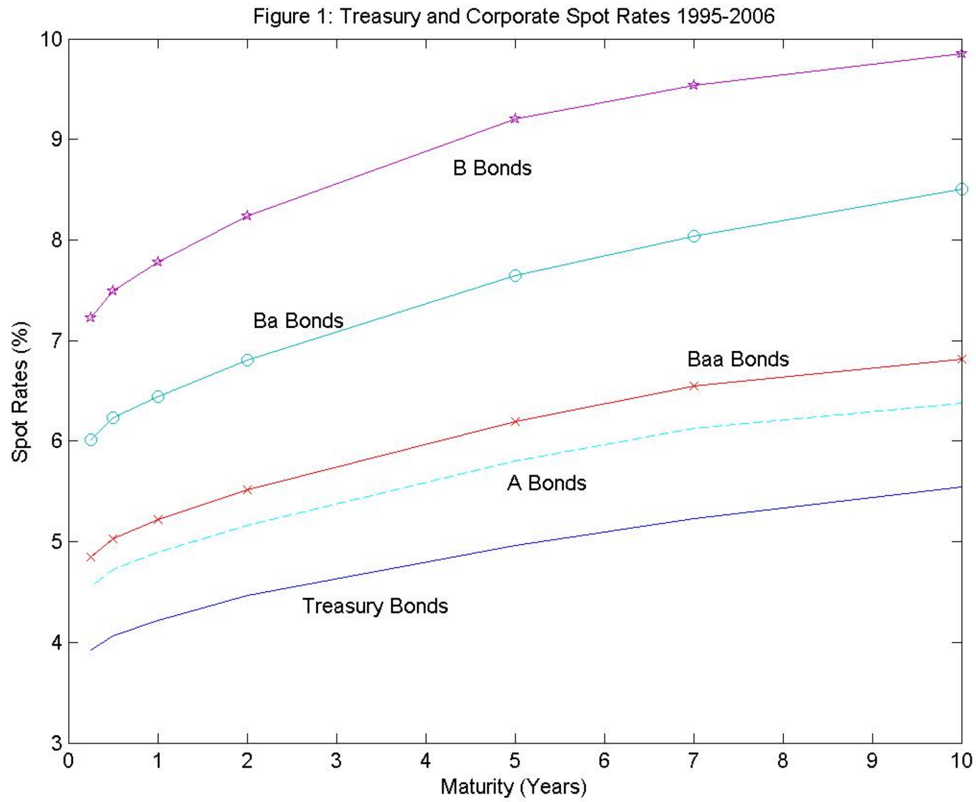

Figure 1: Treasury and Corporate Spot Rates 1995-2006
B Bonds
Ba Bonds
Baa Bonds
A Bonds
Treasury Bonds
Spot Rates (%)
Maturity (Years)


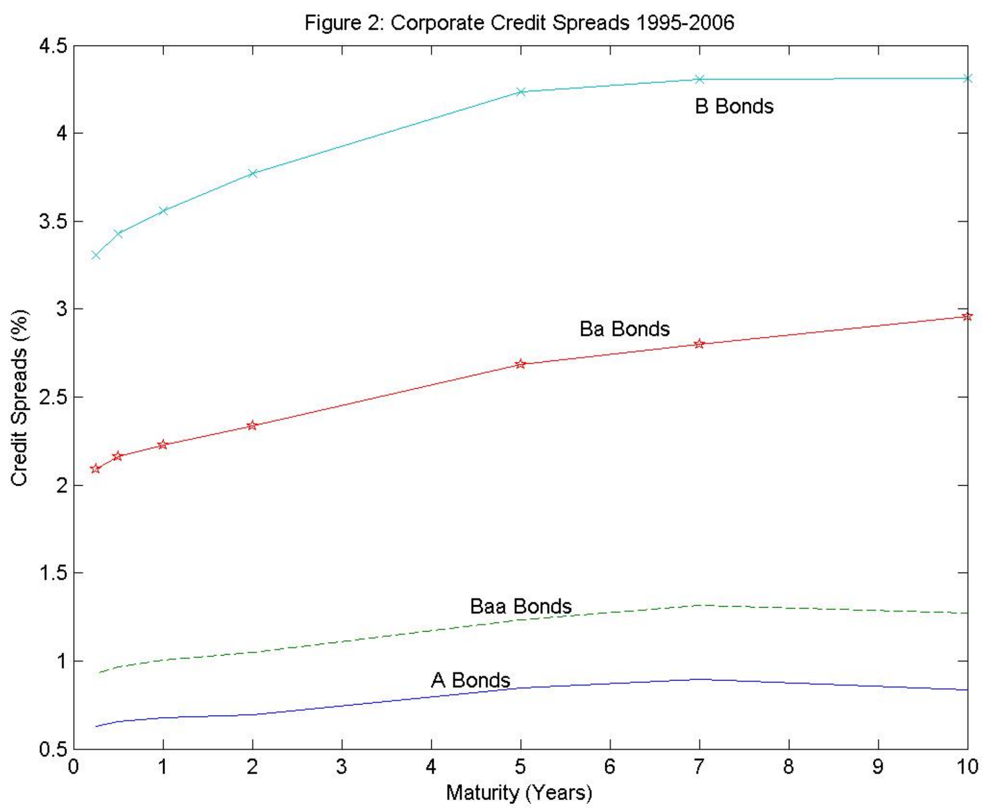

Figure 2: Corporate Credit Spreads 1995-2006
B Bonds
Ba Bonds
Baa Bonds
A Bonds
Credit Spreads (%)
Maturity (Years)

Figure 3: Risk-Free and Corporate Yields Dynamics 1995-2006

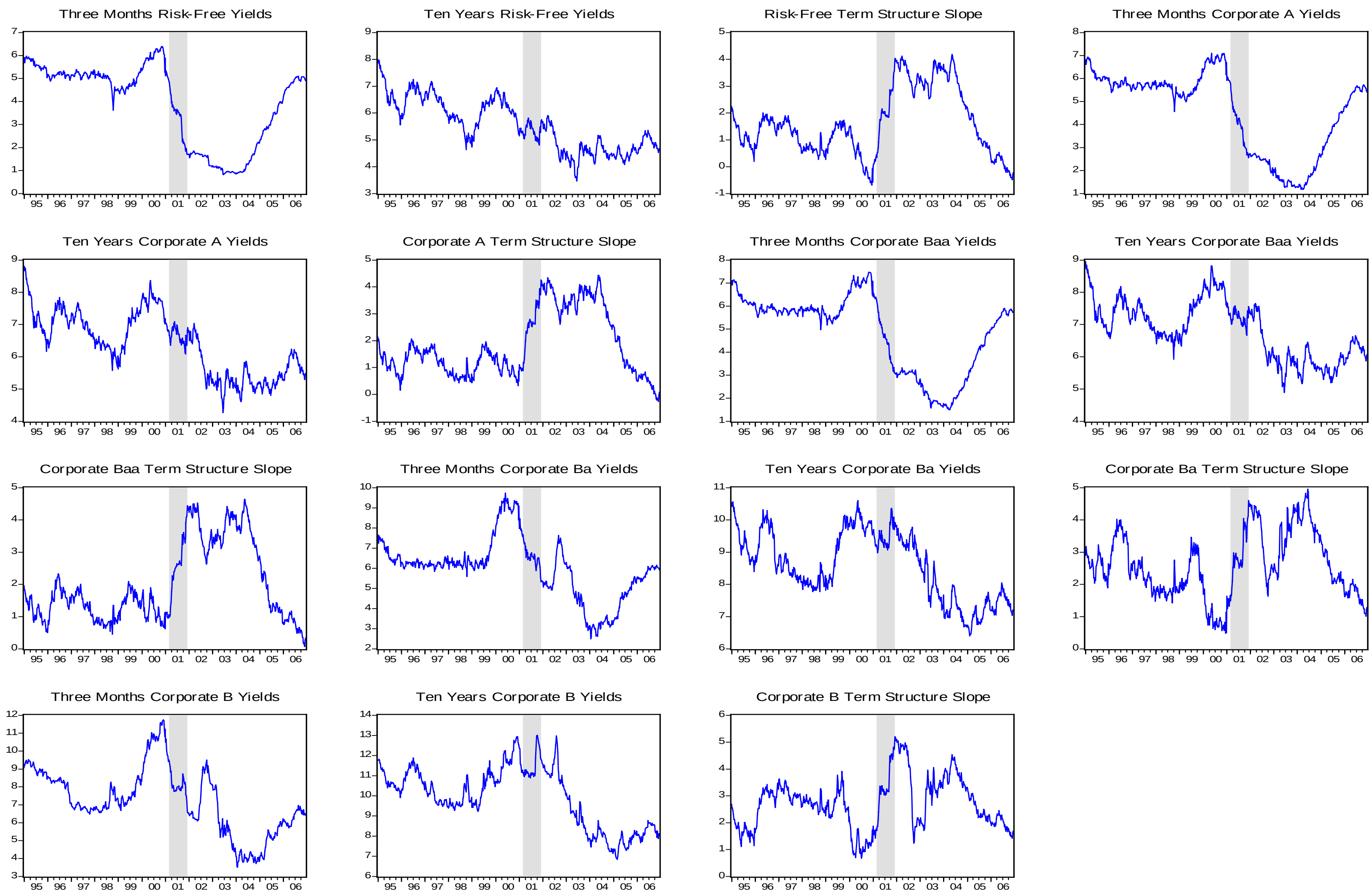

**Figure 4: Corporate Spreads Dynamics 1995-2006**

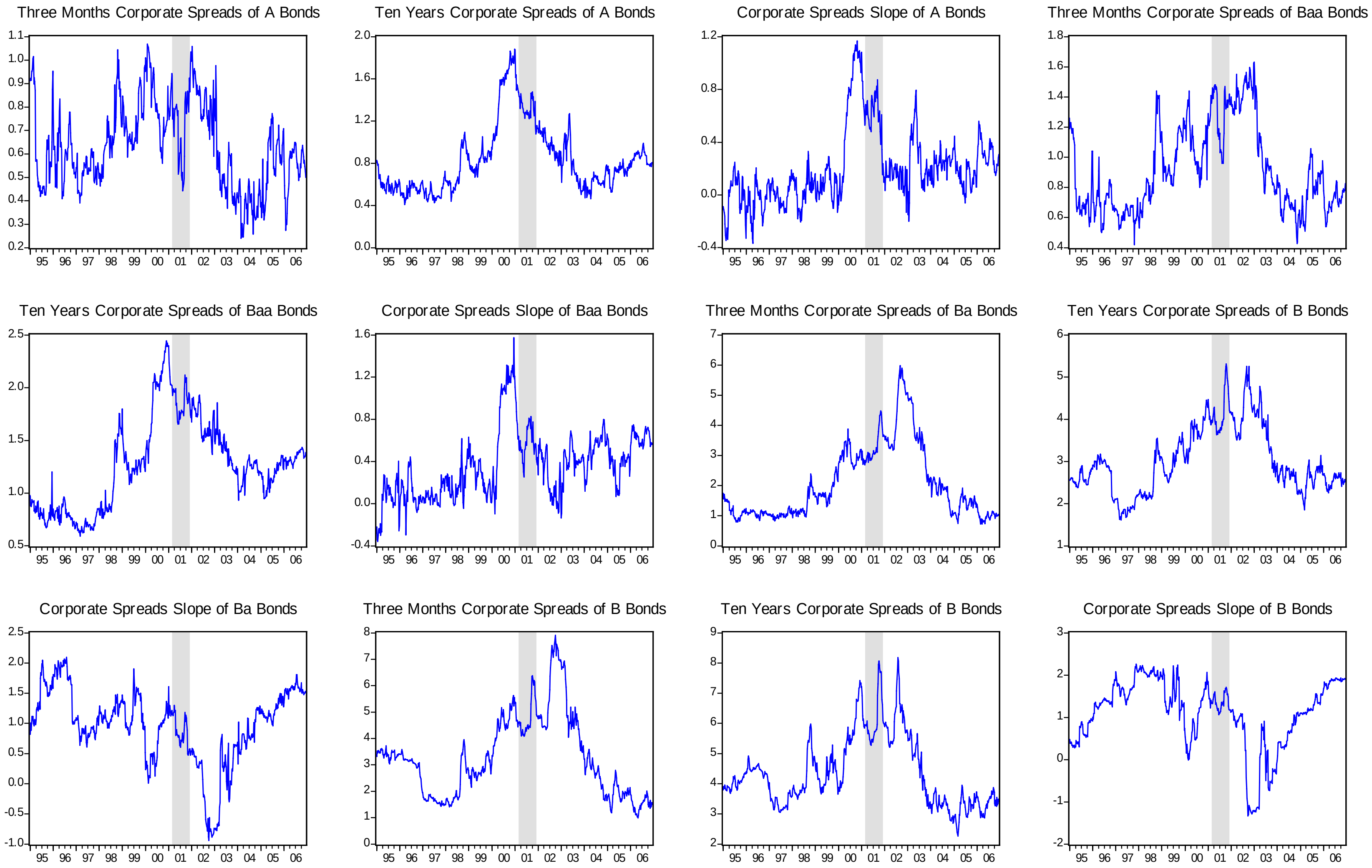

Figure 5: Risk-Free Rates and Corporate Spreads Squared Residuals from OLS Regression 1995-2006

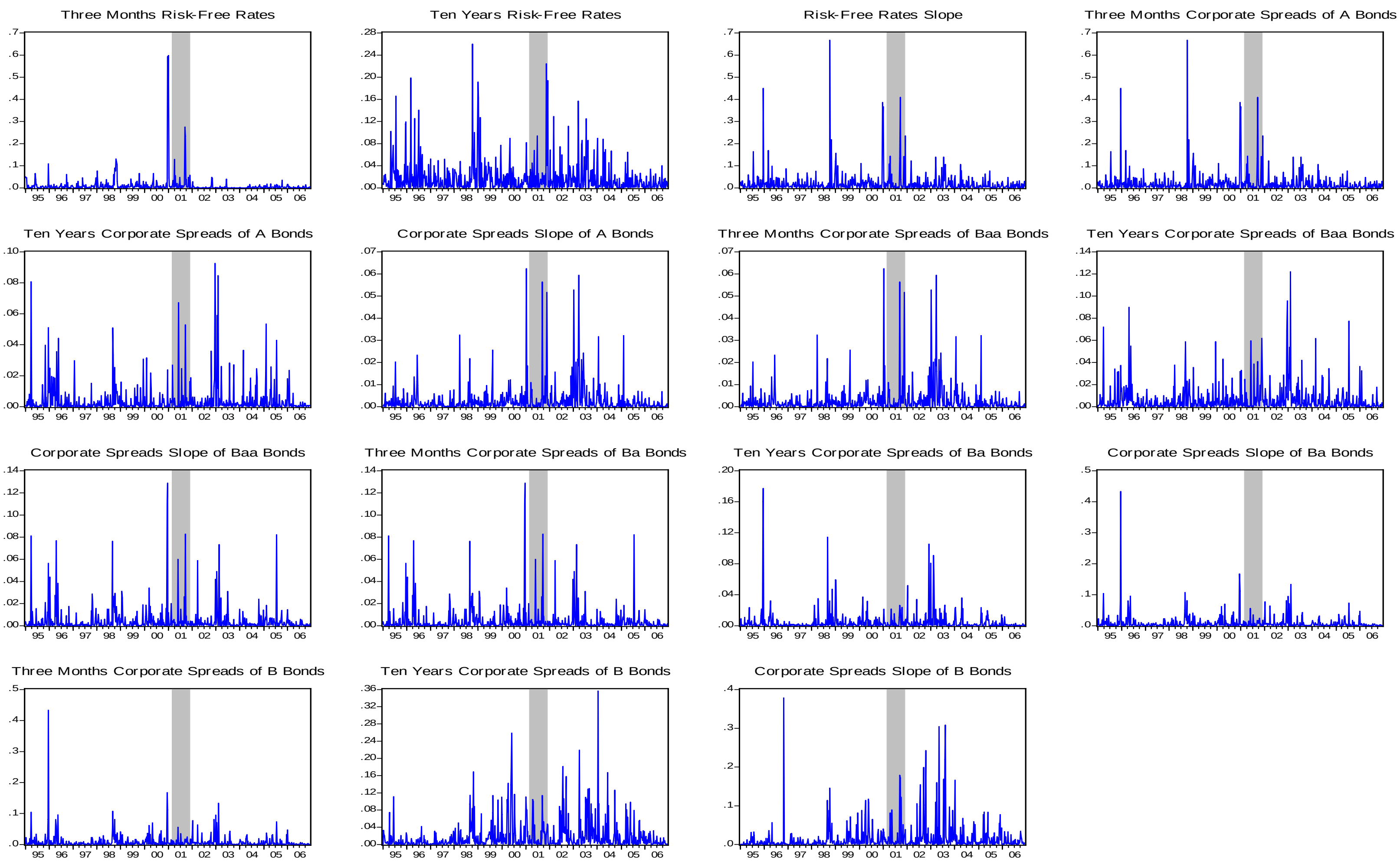

Table 2: Parameter Estimates of LEVEL Models for Three-Month Corporate Spreads

| | A Spreads | | Baa Spreads | | Ba Spreads | | B spreads | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Estimate | p-value | Estimate | p-value | Estimate | p-value | Estimate | p-value |
| Panel A: No Level Model | | | | | | | | |
| a | 0.0420 | 0.0001 | 0.0256 | 0.0132 | 0.0106 | 0.3965 | 0.0145 | 0.4576 |
| b | -0.0675 | 0.0000 | -0.0287 | 0.0043 | -0.0057 | 0.1910 | -0.0053 | 0.2271 |
| w | 0.0646 | 0.0000 | 0.0673 | 0.0000 | 0.1359 | 0.0000 | 0.1732 | 0.0000 |
| Likelihood | 1397.0864 | | 1371.5891 | | 933.3975 | | 782.1785 | |
| AIC | -4.4682 | | -4.3865 | | -2.9820 | | -2.4974 | |
| SBC | -4.4469 | | -4.3652 | | -2.9607 | | -2.4760 | |
| LB(5) | 18.6501 | 0.0022 | 19.6776 | 0.0014 | 77.3814 | 0.0000 | 74.9950 | 0.0000 |
| ARCH(5) | 16.7774 | 0.0049 | 16.4132 | 0.0058 | 55.2261 | 0.0000 | 48.4942 | 0.0000 |
| JB | 428.2133 | 0.0010 | 709.8188 | 0.0010 | 62.1484 | 0.0010 | 220.7886 | 0.0010 |
| Panel B: CIR Model | | | | | | | | |
| a | 0.0419 | 0.0000 | 0.0289 | 0.0016 | 0.0161 | 0.1222 | 0.0162 | 0.2529 |
| b | -0.0674 | 0.0000 | -0.0323 | 0.0019 | -0.0083 | 0.1530 | -0.0058 | 0.2024 |
| w | 0.0816 | 0.0000 | 0.0703 | 0.0000 | 0.0959 | 0.0000 | 0.0937 | 0.0000 |
| Likelihood | 1408.5865 | | 1386.6346 | | 972.1266 | | 826.7557 | |
| AIC | -4.5051 | | -4.4347 | | -3.1062 | | -2.6402 | |
| SBC | -4.4838 | | -4.4134 | | -3.0848 | | -2.6189 | |
| LB(5) | 14.9143 | 0.0107 | 18.7024 | 0.0022 | 36.4404 | 0.0000 | 23.8082 | 0.0002 |
| ARCH(5) | 13.7458 | 0.0173 | 15.8943 | 0.0072 | 31.1429 | 0.0000 | 19.2427 | 0.0017 |
| JB | 281.5215 | 0.0010 | 533.7340 | 0.0010 | 29.0227 | 0.0010 | 166.2078 | 0.0010 |
| Panel C: Level Model | | | | | | | | |
| a | 0.0418 | 0.0000 | 0.0292 | 0.0013 | 0.0159 | 0.1392 | 0.0168 | 0.2303 |
| b | -0.0673 | 0.0000 | -0.0326 | 0.0017 | -0.0082 | 0.1599 | -0.0060 | 0.2030 |
| w | 0.0797 | 0.0000 | 0.0706 | 0.0000 | 0.0966 | 0.0000 | 0.0865 | 0.0000 |
| $\gamma$ | 0.4533 | 0.0000 | 0.5349 | 0.0000 | 0.4879 | 0.0000 | 0.5731 | 0.0000 |
| Likelihood | 1408.7172 | | 1386.6997 | | 972.1496 | | 827.5183 | |
| AIC | -4.5023 | | -4.4317 | | -3.1030 | | -2.6395 | |
| SBC | -4.4739 | | -4.4033 | | -3.0746 | | -2.6110 | |
| LB(5) | 15.0427 | 0.0102 | 18.6837 | 0.0022 | 36.5362 | 0.0000 | 22.0630 | 0.0005 |
| ARCH(5) | 13.8873 | 0.0163 | 15.8681 | 0.0072 | 31.2309 | 0.0000 | 18.2265 | 0.0027 |
| JB | 285.9579 | 0.0010 | 528.0004 | 0.0010 | 28.6644 | 0.0010 | 177.8078 | 0.0010 |

Table 3: Parameter Estimates of GARCH(1,1) Models for Three-Month Corporate Spreads

| | A Spreads | | Baa Spreads | | Ba Spreads | | B spreads | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Estimate | p-value | Estimate | p-value | Estimate | p-value | Estimate | p-value |
| Panel A: No Level Model | | | | | | | | |
| a | 0.0384 | 0.0000 | 0.0199 | 0.0323 | 0.0111 | 0.1838 | 0.0243 | 0.0323 |
| b | -0.0644 | 0.0000 | -0.0235 | 0.0123 | -0.0082 | 0.0444 | -0.0135 | 0.0001 |
| w | 0.0003 | 0.0067 | 0.0013 | 0.0000 | 0.0001 | 0.1095 | 0.0010 | 0.0001 |
| α | 0.0687 | 0.0000 | 0.1378 | 0.0000 | 0.0743 | 0.0000 | 0.2495 | 0.0000 |
| ß | 0.8489 | 0.0017 | 0.5868 | 0.0034 | 0.9196 | 0.0000 | 0.7545 | 0.0000 |
| Likelihood | 1416.3404 | | 1390.0856 | | 978.8015 | | 853.1916 | |
| AIC | -4.5235 | | -4.4394 | | -3.1212 | | -2.7186 | |
| SBC | -4.4880 | | -4.4038 | | -3.0856 | | -2.6830 | |
| LB(5) | 3.0140 | 0.6978 | 1.0875 | 0.9552 | 8.6332 | 0.1246 | 0.9568 | 0.9660 |
| ARCH(5) | 3.0264 | 0.6959 | 1.0563 | 0.9579 | 7.6689 | 0.1755 | 0.9568 | 0.9660 |
| JB | 391.2295 | 0.0010 | 679.9326 | 0.0010 | 27.0889 | 0.0010 | 201.2828 | 0.0010 |
| Panel B: CIR Model | | | | | | | | |
| a | 0.0413 | 0.0000 | 0.0246 | 0.0049 | 0.0178 | 0.0229 | 0.0192 | 0.1030 |
| b | -0.0663 | 0.0000 | -0.0278 | 0.0053 | -0.0111 | 0.0220 | -0.0091 | 0.0326 |
| w | 0.0004 | 0.0064 | 0.0016 | 0.0000 | 0.0018 | 0.0013 | 0.0007 | 0.0000 |
| α | 0.0615 | 0.0000 | 0.1337 | 0.0000 | 0.1911 | 0.0029 | 0.2230 | 0.0000 |
| ß | 0.8823 | 0.0007 | 0.5365 | 0.0108 | 0.6259 | 0.0062 | 0.7224 | 0.0000 |
| Likelihood | 1427.4241 | | 1403.4493 | | 990.5542 | | 862.5126 | |
| AIC | -4.5591 | | -4.4822 | | -3.1588 | | -2.7484 | |
| SBC | -4.5235 | | -4.4467 | | -3.1233 | | -2.7129 | |
| LB(5) | 2.8623 | 0.7212 | 1.1246 | 0.9519 | 2.3445 | 0.7997 | 2.4100 | 0.7900 |
| ARCH(5) | 2.8909 | 0.7168 | 1.0907 | 0.9549 | 2.1841 | 0.8231 | 2.3410 | 0.8002 |
| JB | 292.3067 | 0.0010 | 404.5148 | 0.0010 | 30.0037 | 0.0010 | 103.3803 | 0.0010 |
| Panel C: Level Model | | | | | | | | |
| a | 0.0425 | 0.0000 | 0.0255 | 0.0032 | 0.0173 | 0.0333 | 0.0195 | 0.0969 |
| b | -0.0672 | 0.0000 | -0.0287 | 0.0050 | -0.0110 | 0.0188 | -0.0088 | 0.0432 |
| w | 0.0004 | 0.0039 | 0.0016 | 0.0000 | 0.0019 | 0.0013 | 0.0005 | 0.0108 |
| α | 0.0620 | 0.0000 | 0.1358 | 0.0000 | 0.1899 | 0.0032 | 0.2220 | 0.0000 |
| ß | 0.8922 | 0.0002 | 0.5408 | 0.0086 | 0.6276 | 0.0055 | 0.7261 | 0.0000 |
| $\gamma$ | 0.6601 | 0.0000 | 0.6052 | 0.0000 | 0.4373 | 0.0000 | 0.6302 | 0.0000 |
| Likelihood | 1428.0502 | | 1403.8640 | | 990.8582 | | 863.0218 | |
| AIC | -4.5579 | | -4.4803 | | -3.1566 | | -2.7469 | |
| SBC | -4.5152 | | -4.4377 | | -3.1139 | | -2.7042 | |
| LB(5) | 2.6467 | 0.7543 | 1.1050 | 0.9537 | 2.2368 | 0.8155 | 2.8213 | 0.7275 |
| ARCH(5) | 2.6781 | 0.7495 | 1.0724 | 0.9565 | 2.0975 | 0.8355 | 2.7263 | 0.7421 |
| JB | 289.7889 | 0.0010 | 379.2134 | 0.0010 | 30.6198 | 0.0010 | 99.2807 | 0.0010 |

Table 4: Parameter Estimates of Asymmetric GARCH(1,1) Models for Three-Month Corporate Spreads

| | A Spreads | | Baa Spreads | | Ba Spreads | | B spreads | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Estimate | p-value | Estimate | p-value | Estimate | p-value | Estimate | p-value |
| Panel A: No Level Model | | | | | | | | |
| a | 0.0392 | 0.0000 | 0.0244 | 0.0063 | 0.0220 | 0.0045 | 0.0292 | 0.0031 |
| b | -0.0636 | 0.0000 | -0.0277 | 0.0022 | -0.0143 | 0.0000 | -0.0136 | 0.0000 |
| w | 0.0003 | 0.0004 | 0.0011 | 0.0000 | 0.0013 | 0.0006 | 0.0008 | 0.0002 |
| a | 0.1765 | 0.0000 | 0.2017 | 0.0001 | 0.2331 | 0.0014 | 0.3897 | 0.0000 |
| b | 0.8369 | 0.0000 | 0.6367 | 0.0008 | 0.7670 | 0.0001 | 0.7474 | 0.0000 |
| d | 0.0000 | 1.0000 | 0.0679 | 0.0012 | 0.0904 | 0.0563 | 0.1569 | 0.0000 |
| Likelihood | 1429.4148 | | 1392.4587 | | 980.9859 | | 860.3228 | |
| AIC | -4.5622 | | -4.4438 | | -3.1250 | | -2.7382 | |
| SBC | -4.5196 | | -4.4011 | | -3.0823 | | -2.6956 | |
| LB(5) | 3.3576 | 0.6450 | 1.1494 | 0.9496 | 1.2154 | 0.9434 | 1.0873 | 0.9552 |
| ARCH(5) | 3.5480 | 0.6161 | 1.1200 | 0.9523 | 1.1790 | 0.9469 | 1.0502 | 0.9584 |
| JB | 348.0687 | 0.0010 | 627.3099 | 0.0010 | 62.9982 | 0.0010 | 172.6506 | 0.0010 |
| Panel B: CIR Model | | | | | | | | |
| a | 0.0394 | 0.0000 | 0.0247 | 0.0047 | 0.0170 | 0.0398 | 0.0203 | 0.0865 |
| b | -0.0624 | 0.0000 | -0.0278 | 0.0053 | -0.0111 | 0.0207 | -0.0091 | 0.0306 |
| w | 0.0004 | 0.0060 | 0.0016 | 0.0000 | 0.0018 | 0.0013 | 0.0007 | 0.0000 |
| a | 0.0849 | 0.0001 | 0.1404 | 0.0013 | 0.1694 | 0.0152 | 0.2670 | 0.0000 |
| b | 0.8856 | 0.0006 | 0.5353 | 0.0109 | 0.6217 | 0.0053 | 0.7198 | 0.0000 |
| d | 0.0357 | 0.0317 | 0.1277 | 0.0010 | 0.2264 | 0.0226 | 0.1964 | 0.0000 |
| Likelihood | 1428.3895 | | 1403.4679 | | 990.8235 | | 863.1528 | |
| AIC | -4.5589 | | -4.4791 | | -3.1565 | | -2.7473 | |
| SBC | -4.5163 | | -4.4364 | | -3.1138 | | -2.7046 | |
| LB(5) | 2.6203 | 0.7583 | 1.0753 | 0.9563 | 2.2316 | 0.8163 | 2.5428 | 0.7700 |
| ARCH(5) | 2.6579 | 0.7525 | 1.0429 | 0.9590 | 2.0758 | 0.8386 | 2.4489 | 0.7842 |
| JB | 300.7684 | 0.0010 | 402.9128 | 0.0010 | 30.7076 | 0.0010 | 105.9271 | 0.0010 |
| Panel C: Level Model | | | | | | | | |
| a | 0.0375 | 0.0000 | 0.0256 | 0.0032 | 0.0169 | 0.0432 | 0.0198 | 0.0924 |
| b | -0.0601 | 0.0000 | -0.0289 | 0.0048 | -0.0110 | 0.0193 | -0.0089 | 0.0400 |
| w | 0.0004 | 0.0014 | 0.0016 | 0.0000 | 0.0019 | 0.0021 | 0.0006 | 0.0116 |
| a | 0.1482 | 0.0001 | 0.1264 | 0.0024 | 0.1772 | 0.0154 | 0.2544 | 0.0000 |
| b | 0.8477 | 0.0002 | 0.5460 | 0.0088 | 0.6230 | 0.0053 | 0.7209 | 0.0000 |
| d | 0.0000 | 1.0000 | 0.1446 | 0.0022 | 0.2120 | 0.0398 | 0.2034 | 0.0000 |
| $\gamma$ | 0.1575 | 0.0794 | 0.6204 | 0.0000 | 0.4558 | 0.0000 | 0.5717 | 0.0000 |
| Likelihood | 1429.9975 | | 1403.8941 | | 990.9329 | | 863.2553 | |
| AIC | -4.5609 | | -4.4772 | | -3.1536 | | -2.7444 | |
| SBC | -4.5111 | | -4.4275 | | -3.1039 | | -2.6946 | |
| LB(5) | 2.8568 | 0.7221 | 1.1887 | 0.9460 | 2.2017 | 0.8206 | 2.7460 | 0.7391 |
| ARCH(5) | 2.9600 | 0.7062 | 1.1544 | 0.9492 | 2.0580 | 0.8411 | 2.6428 | 0.7549 |
| JB | 321.6597 | 0.0010 | 378.2738 | 0.0010 | 30.9562 | 0.0010 | 102.1953 | 0.0010 |

**Table 5: Parameter Estimates of Switching Regime Models for Three-Month Corporate Spreads**

| | A Spreads | | Baa Spreads | | Ba Spreads | | B spreads | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Estimate | p-value | Estimate | p-value | Estimate | p-value | Estimate | p-value |
| Panel A: No Level Model | | | | | | | | |
| p | 0.4612 | 0.0019 | 0.5831 | 0.0017 | 0.9292 | 0.0084 | 0.9536 | 0.0015 |
| q | 0.7272 | 0.0000 | 0.8160 | 0.0001 | 0.9469 | 0.0038 | 0.8961 | 0.0018 |
| $a_1$ | 0.1001 | 0.0027 | 0.0517 | 0.1360 | 0.0261 | 0.4077 | 0.0174 | 0.5497 |
| $a_2$ | 0.0124 | 0.0512 | 0.0142 | 0.0355 | 0.0078 | 0.4060 | 0.0035 | 0.7465 |
| $b_1$ | -0.1622 | 0.0006 | -0.0581 | 0.0782 | -0.0083 | 0.4114 | -0.0063 | 0.3272 |
| $b_2$ | -0.0174 | 0.0726 | -0.0150 | 0.0270 | -0.0080 | 0.0741 | -0.0008 | 0.8226 |
| $w_1$ | 0.1017 | 0.0000 | 0.1102 | 0.0000 | 0.1862 | 0.0000 | 0.2055 | 0.0000 |
| $w_2$ | 0.0286 | 0.0000 | 0.0335 | 0.0000 | 0.0772 | 0.0000 | 0.0533 | 0.0000 |
| Likelihood | 1499.0648 | | 1464.3495 | | 995.4888 | | 868.0344 | |
| AIC | -4.7791 | | -4.6678 | | -3.1650 | | -2.7565 | |
| SBC | -4.7222 | | -4.6109 | | -3.1082 | | -2.6996 | |
| LB(5) | 8.2777 | 0.1416 | 4.3062 | 0.5062 | 2.2794 | 0.8093 | 3.9527 | 0.5563 |
| ARCH(5) | 7.9472 | 0.1592 | 3.9238 | 0.5604 | 2.2819 | 0.8089 | 3.5939 | 0.6092 |
| JB | 344.5307 | 0.0010 | 634.1456 | 0.0010 | 66.7357 | 0.0010 | 206.0463 | 0.0010 |
| Panel B: CIR Model | | | | | | | | |
| p | 0.6705 | 0.0000 | 0.5820 | 0.0015 | 0.9087 | 0.0219 | 0.9533 | 0.0023 |
| q | 0.4658 | 0.0021 | 0.7771 | 0.0001 | 0.8833 | 0.0080 | 0.9079 | 0.0013 |
| $a_1$ | 0.0098 | 0.1105 | 0.0576 | 0.0292 | 0.0224 | 0.2855 | 0.0190 | 0.3757 |
| $a_2$ | 0.0869 | 0.0002 | 0.0136 | 0.0366 | 0.0089 | 0.4116 | 0.0107 | 0.3443 |
| $b_1$ | -0.0144 | 0.1589 | -0.0650 | 0.0288 | -0.0094 | 0.3937 | -0.0068 | 0.3200 |
| $b_2$ | -0.1434 | 0.0002 | -0.0153 | 0.0367 | -0.0075 | 0.2263 | -0.0039 | 0.3083 |
| $w_1$ | 0.0346 | 0.0000 | 0.1094 | 0.0000 | 0.1189 | 0.0000 | 0.1123 | 0.0000 |
| $w_2$ | 0.1221 | 0.0000 | 0.0334 | 0.0000 | 0.0528 | 0.0000 | 0.0360 | 0.0000 |
| Likelihood | 1499.8480 | | 1472.0171 | | 1002.1886 | | 890.5904 | |
| AIC | -4.7816 | | -4.6924 | | -3.1865 | | -2.8288 | |
| SBC | -4.7247 | | -4.6355 | | -3.1296 | | -2.7719 | |
| LB(5) | 9.4929 | 0.0909 | 5.7920 | 0.3270 | 8.7947 | 0.1175 | 2.2737 | 0.8101 |
| ARCH(5) | 9.1488 | 0.1033 | 5.1824 | 0.3940 | 8.4768 | 0.1318 | 2.1256 | 0.8315 |
| JB | 242.1553 | 0.0010 | 430.5767 | 0.0010 | 22.9869 | 0.0010 | 136.8005 | 0.0010 |
| Panel C: Level Model | | | | | | | | |
| p | 0.7019 | 0.0000 | 0.7779 | 0.0001 | 0.9193 | 0.0184 | 0.9094 | 0.0014 |
| q | 0.4729 | 0.0023 | 0.5865 | 0.0015 | 0.9009 | 0.0080 | 0.9542 | 0.0025 |
| $a_1$ | 0.0113 | 0.0839 | 0.0133 | 0.0422 | 0.0221 | 0.3184 | 0.0114 | 0.3297 |
| $a_2$ | 0.0912 | 0.0003 | 0.0611 | 0.0148 | 0.0098 | 0.3558 | 0.0200 | 0.3468 |
| $b_1$ | -0.0162 | 0.1029 | -0.0150 | 0.0386 | -0.0091 | 0.3951 | -0.0042 | 0.2787 |
| $b_2$ | -0.1493 | 0.0002 | -0.0693 | 0.0209 | -0.0078 | 0.1232 | -0.0071 | 0.3139 |
| $w_1$ | 0.0289 | 0.0000 | 0.0334 | 0.0000 | 0.1217 | 0.0000 | 0.0372 | 0.0000 |
| $w_2$ | 0.1233 | 0.0000 | 0.1122 | 0.0000 | 0.0611 | 0.0000 | 0.1035 | 0.0000 |
| $\gamma_1$ | 0.0792 | 0.6549 | 0.5172 | 0.0027 | 0.4638 | 0.0000 | 0.4727 | 0.0014 |
| $\gamma_2$ | 0.4836 | 0.0089 | 0.6934 | 0.0008 | 0.2490 | 0.0507 | 0.5737 | 0.0000 |
| Likelihood | 1502.4289 | | 1472.4975 | | 1004.5153 | | 891.0712 | |
| AIC | -4.7834 | | -4.6875 | | -3.1875 | | -2.8239 | |
| SBC | -4.7123 | | -4.6164 | | -3.1165 | | -2.7529 | |
| LB(5) | 7.9200 | 0.1607 | 5.7968 | 0.3265 | 6.7996 | 0.2360 | 1.9669 | 0.8537 |
| ARCH(5) | 7.7414 | 0.1711 | 5.1519 | 0.3976 | 6.5439 | 0.2568 | 1.8822 | 0.8652 |
| JB | 250.2944 | 0.0010 | 399.5986 | 0.0010 | 25.6418 | 0.0010 | 139.2575 | 0.0010 |

Table 6: Parameter Estimates of LEVEL Models for Ten-Years Corporate Spreads

| | A Spreads | | Baa Spreads | | Ba Spreads | | B spreads | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Estimate | p-value | Estimate | p-value | Estimate | p-value | Estimate | p-value |
| | | | | Panel A: No Level Model | | | | |
| a | 0.0115 | 0.0557 | 0.0176 | 0.0785 | 0.0376 | 0.0698 | 0.0494 | 0.0770 |
| b | -0.0138 | 0.0218 | -0.0133 | 0.0610 | -0.0124 | 0.0318 | -0.0114 | 0.0319 |
| w | 0.0536 | 0.0000 | 0.0688 | 0.0000 | 0.1251 | 0.0000 | 0.1815 | 0.0000 |
| Likelihood | 1514.2232 | | 1358.4835 | | 985.1348 | | 752.9053 | |
| AIC | -4.8437 | | -4.3445 | | -3.1479 | | -2.4035 | |
| SBC | -4.8223 | | -4.3232 | | -3.1265 | | -2.3822 | |
| LB(5) | 40.1265 | 0.0000 | 65.4940 | 0.0000 | 39.9478 | 0.0000 | 29.4897 | 0.0000 |
| ARCH(5) | 30.6538 | 0.0000 | 71.4714 | 0.0000 | 30.2750 | 0.0000 | 24.4560 | 0.0002 |
| JB | 281.4113 | 0.0010 | 1262.2331 | 0.0010 | 326.7024 | 0.0010 | 442.8718 | 0.0010 |
| | | | | Panel B: CIR Model | | | | |
| a | 0.0154 | 0.0112 | 0.0180 | 0.0461 | 0.0352 | 0.0635 | 0.0502 | 0.0522 |
| b | -0.0185 | 0.0187 | -0.0136 | 0.0731 | -0.0116 | 0.0535 | -0.0116 | 0.0424 |
| w | 0.0589 | 0.0000 | 0.0620 | 0.0000 | 0.0692 | 0.0000 | 0.0837 | 0.0000 |
| Likelihood | 1532.2760 | | 1365.8234 | | 1018.8179 | | 784.6465 | |
| AIC | -4.9015 | | -4.3680 | | -3.2558 | | -2.5053 | |
| SBC | -4.8802 | | -4.3467 | | -3.2345 | | -2.4839 | |
| LB(5) | 39.4365 | 0.0000 | 80.4442 | 0.0000 | 19.8305 | 0.0013 | 20.4769 | 0.0010 |
| ARCH(5) | 29.9317 | 0.0000 | 87.4506 | 0.0000 | 15.8071 | 0.0074 | 17.4278 | 0.0038 |
| JB | 145.9023 | 0.0010 | 2041.5259 | 0.0010 | 205.4140 | 0.0010 | 182.1006 | 0.0010 |
| | | | | Panel C: Level Model | | | | |
| a | 0.0152 | 0.0122 | 0.0179 | 0.0603 | 0.0344 | 0.0517 | 0.0540 | 0.0362 |
| b | -0.0182 | 0.0194 | -0.0135 | 0.0743 | -0.0113 | 0.0774 | -0.0125 | 0.0495 |
| w | 0.0585 | 0.0000 | 0.0633 | 0.0000 | 0.0380 | 0.0000 | 0.0451 | 0.0000 |
| $\gamma$ | 0.4747 | 0.0000 | 0.3746 | 0.0000 | 1.0379 | 0.0000 | 0.9186 | 0.0000 |
| Likelihood | 1532.3245 | | 1366.7577 | | 1031.2576 | | 792.8336 | |
| AIC | -4.8985 | | -4.3678 | | -3.2925 | | -2.5283 | |
| SBC | -4.8700 | | -4.3394 | | -3.2641 | | -2.4999 | |
| LB(5) | 39.2209 | 0.0000 | 77.6574 | 0.0000 | 15.7586 | 0.0076 | 22.1808 | 0.0005 |
| ARCH(5) | 29.7956 | 0.0000 | 85.0047 | 0.0000 | 12.8775 | 0.0246 | 17.8729 | 0.0031 |
| JB | 146.6276 | 0.0010 | 1759.3665 | 0.0010 | 194.4441 | 0.0010 | 101.5106 | 0.0010 |

Table 7: Parameter Estimates of GARCH(1,1) Models for Ten-Years Corporate Spreads

| | A Spreads | | Baa Spreads | | Ba Spreads | | B spreads | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Estimate | p-value | Estimate | p-value | Estimate | p-value | Estimate | p-value |
| Panel A: No Level Model | | | | | | | | |
| a | 0.0122 | 0.0021 | 0.0062 | 0.2515 | 0.0529 | 0.0019 | 0.1147 | 0.0000 |
| b | -0.0172 | 0.0000 | -0.0064 | 0.1083 | -0.0203 | 0.0001 | -0.0306 | 0.0000 |
| w | 0.0005 | 0.0000 | 0.0002 | 0.0000 | 0.0024 | 0.0000 | 0.0012 | 0.0019 |
| α | 0.2353 | 0.0000 | 0.2227 | 0.0000 | 0.2603 | 0.0007 | 0.2974 | 0.0000 |
| ß | 0.6233 | 0.0002 | 0.7629 | 0.0000 | 0.6128 | 0.0000 | 0.7273 | 0.0000 |
| Likelihood | 1551.9144 | | 1417.4968 | | 1025.8118 | | 811.0138 | |
| AIC | -4.9581 | | -4.5272 | | -3.2718 | | -2.5834 | |
| SBC | -4.9225 | | -4.4917 | | -3.2363 | | -2.5478 | |
| LB(5) | 3.3863 | 0.6407 | 4.2129 | 0.5192 | 1.3915 | 0.9252 | 2.2411 | 0.8149 |
| ARCH(5) | 3.4897 | 0.6249 | 4.2125 | 0.5192 | 1.4233 | 0.9217 | 2.1341 | 0.8303 |
| JB | 119.4572 | 0.0010 | 527.6732 | 0.0010 | 537.2748 | 0.0010 | 332.6117 | 0.0010 |
| Panel B: CIR Model | | | | | | | | |
| a | 0.0132 | 0.0041 | 0.0050 | 0.3926 | 0.0490 | 0.0023 | 0.0930 | 0.0001 |
| b | -0.0168 | 0.0040 | -0.0041 | 0.3946 | -0.0181 | 0.0007 | -0.0243 | 0.0000 |
| w | 0.0007 | 0.0003 | 0.0003 | 0.0000 | 0.0014 | 0.0000 | 0.0005 | 0.0000 |
| α | 0.1791 | 0.0004 | 0.2223 | 0.0000 | 0.2424 | 0.0019 | 0.2351 | 0.0000 |
| ß | 0.6334 | 0.0041 | 0.7424 | 0.0000 | 0.4919 | 0.0024 | 0.7365 | 0.0000 |
| Likelihood | 1557.7696 | | 1425.2178 | | 1040.4760 | | 819.5366 | |
| AIC | -4.9768 | | -4.5520 | | -3.3188 | | -2.6107 | |
| SBC | -4.9413 | | -4.5164 | | -3.2833 | | -2.5751 | |
| LB(5) | 2.3517 | 0.7986 | 4.1210 | 0.5321 | 1.1402 | 0.9505 | 2.5074 | 0.7754 |
| ARCH(5) | 2.3460 | 0.7995 | 4.0078 | 0.5483 | 1.1658 | 0.9481 | 2.4013 | 0.7913 |
| JB | 98.9323 | 0.0010 | 518.1942 | 0.0010 | 357.0897 | 0.0010 | 312.8187 | 0.0010 |
| Panel C: Level Model | | | | | | | | |
| a | 0.0130 | 0.0042 | 0.0050 | 0.3809 | 0.0472 | 0.0016 | 0.0710 | 0.0038 |
| b | -0.0168 | 0.0036 | -0.0038 | 0.4394 | -0.0168 | 0.0030 | -0.0167 | 0.0122 |
| w | 0.0007 | 0.0003 | 0.0002 | 0.0000 | 0.0004 | 0.0009 | 0.0000 | 0.3055 |
| α | 0.1824 | 0.0005 | 0.2236 | 0.0000 | 0.2741 | 0.0047 | 0.2079 | 0.0000 |
| ß | 0.6273 | 0.0042 | 0.7457 | 0.0000 | 0.3681 | 0.0427 | 0.7745 | 0.0000 |
| $\gamma$ | 0.4535 | 0.0000 | 0.6910 | 0.0000 | 1.1515 | 0.0000 | 1.9151 | 0.0000 |
| Likelihood | 1557.8438 | | 1425.8520 | | 1049.3302 | | 831.7960 | |
| AIC | -4.9739 | | -4.5508 | | -3.3440 | | -2.6468 | |
| SBC | -4.9312 | | -4.5082 | | -3.3014 | | -2.6041 | |
| LB(5) | 2.3787 | 0.7946 | 4.0103 | 0.5479 | 1.1363 | 0.9508 | 3.4092 | 0.6372 |
| ARCH(5) | 2.3824 | 0.7941 | 3.8641 | 0.5691 | 1.1623 | 0.9484 | 3.2179 | 0.6664 |
| JB | 98.4286 | 0.0010 | 510.5842 | 0.0010 | 323.3554 | 0.0010 | 147.4546 | 0.0010 |

Table 8: Parameter Estimates of Asymmetric GARCH(1,1) Models for Ten-Years Corporate Spreads

| | A Spreads | | Baa Spreads | | Ba Spreads | | B spreads | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Estimate | p-value | Estimate | p-value | Estimate | p-value | Estimate | p-value |
| Panel A: No Level Model | | | | | | | | |
| a | 0.0154 | 0.0001 | 0.0101 | 0.0561 | 0.0587 | 0.0005 | 0.0932 | 0.0000 |
| b | -0.0182 | 0.0001 | -0.0084 | 0.0195 | -0.0211 | 0.0002 | -0.0229 | 0.0000 |
| w | 0.0003 | 0.0003 | 0.0002 | 0.0000 | 0.0021 | 0.0000 | 0.0013 | 0.0003 |
| a | 0.3511 | 0.0025 | 0.3149 | 0.0000 | 0.3587 | 0.0019 | 0.4498 | 0.0000 |
| b | 0.7195 | 0.0011 | 0.7742 | 0.0000 | 0.6961 | 0.0000 | 0.7508 | 0.0000 |
| d | 0.0192 | 0.5403 | 0.1234 | 0.0002 | 0.0000 | 1.0000 | 0.0578 | 0.1266 |
| Likelihood | 1561.0299 | | 1421.5323 | | 1038.3342 | | 831.5148 | |
| AIC | -4.9841 | | -4.5370 | | -3.3088 | | -2.6459 | |
| SBC | -4.9414 | | -4.4943 | | -3.2661 | | -2.6032 | |
| LB(5) | 3.1311 | 0.6798 | 3.0041 | 0.6994 | 0.4582 | 0.9936 | 3.5749 | 0.6121 |
| ARCH(5) | 3.1694 | 0.6739 | 2.9890 | 0.7017 | 0.4655 | 0.9933 | 3.4576 | 0.6298 |
| JB | 112.6881 | 0.0010 | 501.8399 | 0.0010 | 489.7486 | 0.0010 | 124.4680 | 0.0010 |
| Panel B: CIR Model | | | | | | | | |
| a | 0.0130 | 0.0041 | 0.0055 | 0.3551 | 0.0499 | 0.0025 | 0.0845 | 0.0000 |
| b | -0.0152 | 0.0116 | -0.0037 | 0.4474 | -0.0176 | 0.0019 | -0.0206 | 0.0000 |
| w | 0.0006 | 0.0007 | 0.0003 | 0.0000 | 0.0010 | 0.0000 | 0.0004 | 0.0002 |
| a | 0.2612 | 0.0119 | 0.2755 | 0.0000 | 0.2773 | 0.0045 | 0.3876 | 0.0000 |
| b | 0.6509 | 0.0075 | 0.7447 | 0.0000 | 0.6306 | 0.0001 | 0.7500 | 0.0000 |
| d | 0.0944 | 0.0428 | 0.1634 | 0.0001 | 0.0328 | 0.3122 | 0.0866 | 0.0440 |
| Likelihood | 1559.5487 | | 1426.3991 | | 1044.7171 | | 831.7097 | |
| AIC | -4.9793 | | -4.5526 | | -3.3292 | | -2.6465 | |
| SBC | -4.9367 | | -4.5099 | | -3.2866 | | -2.6039 | |
| LB(5) | 2.2803 | 0.8092 | 3.4581 | 0.6297 | 0.4845 | 0.9927 | 3.5699 | 0.6128 |
| ARCH(5) | 2.2533 | 0.8131 | 3.3569 | 0.6451 | 0.4932 | 0.9924 | 3.4669 | 0.6284 |
| JB | 102.5734 | 0.0010 | 507.5299 | 0.0010 | 344.3107 | 0.0010 | 131.7119 | 0.0010 |
| Panel C: Level Model | | | | | | | | |
| a | 0.0140 | 0.0007 | 0.0051 | 0.3994 | 0.0470 | 0.0024 | 0.0698 | 0.0028 |
| b | -0.0164 | 0.0014 | -0.0033 | 0.5192 | -0.0166 | 0.0035 | -0.0166 | 0.0082 |
| w | 0.0005 | 0.0009 | 0.0002 | 0.0000 | 0.0005 | 0.0008 | 0.0000 | 0.3683 |
| a | 0.3321 | 0.0119 | 0.2694 | 0.0000 | 0.3146 | 0.0154 | 0.2841 | 0.0001 |
| b | 0.6786 | 0.0037 | 0.7434 | 0.0000 | 0.3484 | 0.0535 | 0.7555 | 0.0000 |
| d | 0.0457 | 0.2466 | 0.1730 | 0.0003 | 0.2198 | 0.0437 | 0.1533 | 0.0243 |
| $\gamma$ | 0.1797 | 0.1283 | 0.6065 | 0.0001 | 1.0878 | 0.0000 | 1.3425 | 0.0001 |
| Likelihood | 1561.3442 | | 1426.5820 | | 1049.5608 | | 832.1091 | |
| AIC | -4.9819 | | -4.5499 | | -3.3415 | | -2.6446 | |
| SBC | -4.9321 | | -4.5002 | | -3.2918 | | -2.5948 | |
| LB(5) | 2.7823 | 0.7335 | 3.5270 | 0.6193 | 1.0956 | 0.9545 | 3.5211 | 0.6202 |
| ARCH(5) | 2.8076 | 0.7296 | 3.4058 | 0.6377 | 1.1162 | 0.9527 | 3.3993 | 0.6387 |
| JB | 104.2546 | 0.0010 | 504.9131 | 0.0010 | 301.9039 | 0.0010 | 132.8756 | 0.0010 |

Table 9: Parameter Estimates of Switching Regime Models
for Ten-Years Corporate Spreads

| | A Spreads | | Baa Spreads | | Ba Spreads | | B spreads | |
|---|---|---|---|---|---|---|---|---|
| Parameter | Estimate | p-value | Estimate | p-value | Estimate | p-value | Estimate | p-value |
| Panel A: No Level Model | | | | | | | | |
| p | 0.8236 | 0.0114 | 0.9094 | 0.0004 | 0.9269 | 0.0007 | 0.9339 | 0.0018 |
| q | 0.9639 | 0.0131 | 0.8396 | 0.0004 | 0.8773 | 0.0023 | 0.9015 | 0.0013 |
| $a_1$ | 0.0497 | 0.1777 | 0.0040 | 0.5138 | 0.0050 | 0.7548 | 0.0022 | 0.9170 |
| $a_2$ | 0.0027 | 0.6008 | 0.0480 | 0.0924 | 0.1064 | 0.0774 | 0.1144 | 0.1081 |
| $b_1$ | -0.0582 | 0.0775 | -0.0011 | 0.8111 | -0.0021 | 0.6998 | -0.0015 | 0.7620 |
| $b_2$ | -0.0021 | 0.7124 | -0.0369 | 0.0691 | -0.0306 | 0.0570 | -0.0231 | 0.0736 |
| $w_1$ | 0.0957 | 0.0000 | 0.0331 | 0.0000 | 0.0666 | 0.0000 | 0.0936 | 0.0000 |
| $w_2$ | 0.0388 | 0.0000 | 0.1047 | 0.0000 | 0.1838 | 0.0000 | 0.2609 | 0.0000 |
| Likelihood | 1572.4264 | | 1467.0913 | | 1068.6802 | | 841.7385 | |
| AIC | -5.0142 | | -4.6766 | | -3.3996 | | -2.6722 | |
| SBC | -4.9573 | | -4.6197 | | -3.3427 | | -2.6154 | |
| LB(5) | 1.6219 | 0.8986 | 2.8200 | 0.7300 | 0.8967 | 0.9705 | 2.8565 | 0.7221 |
| ARCH(5) | 1.6248 | 0.8982 | 2.7800 | 0.7300 | 0.8867 | 0.9712 | 2.8745 | 0.7193 |
| JB | 205.5602 | 0.0010 | 582.7396 | 0.0010 | 568.6889 | 0.0010 | 593.2452 | 0.0010 |
| Panel B: CIR Model | | | | | | | | |
| p | 0.9532 | 0.0095 | 0.7569 | 0.0022 | 0.8335 | 0.0049 | 0.9148 | 0.0015 |
| q | 0.8051 | 0.0130 | 0.9071 | 0.0009 | 0.9139 | 0.0018 | 0.8664 | 0.0013 |
| $a_1$ | 0.0045 | 0.4269 | 0.0472 | 0.1665 | 0.0764 | 0.1926 | 0.0684 | 0.1027 |
| $a_2$ | 0.0621 | 0.0474 | 0.0094 | 0.1048 | 0.0236 | 0.1413 | 0.0066 | 0.7703 |
| $b_1$ | -0.0045 | 0.5368 | -0.0368 | 0.2021 | -0.0213 | 0.2257 | -0.0159 | 0.0796 |
| $b_2$ | -0.0798 | 0.0491 | -0.0068 | 0.1642 | -0.0094 | 0.0926 | -0.0012 | 0.8180 |
| $w_1$ | 0.0429 | 0.0000 | 0.1042 | 0.0000 | 0.1028 | 0.0000 | 0.1037 | 0.0000 |
| $w_2$ | 0.0988 | 0.0000 | 0.0334 | 0.0000 | 0.0415 | 0.0000 | 0.0328 | 0.0000 |
| Likelihood | 1579.1541 | | 1469.2352 | | 1077.0817 | | 852.1169 | |
| AIC | -5.0358 | | -4.6834 | | -3.4265 | | -2.7055 | |
| SBC | -4.9789 | | -4.6266 | | -3.3697 | | -2.6486 | |
| LB(5) | 1.3449 | 0.9302 | 1.4900 | 0.9100 | 0.3381 | 0.9969 | 4.9498 | 0.4220 |
| ARCH(5) | 1.2918 | 0.9358 | 1.4400 | 0.9200 | 0.3436 | 0.9967 | 4.9762 | 0.4188 |
| JB | 85.6948 | 0.0010 | 1022.1560 | 0.0010 | 401.4930 | 0.0010 | 188.0514 | 0.0010 |
| Panel C: Level Model | | | | | | | | |
| p | 0.9567 | 0.0127 | 0.8180 | 0.0009 | 0.8153 | 0.0038 | 0.8871 | 0.0013 |
| q | 0.8143 | 0.0128 | 0.9123 | 0.0006 | 0.8553 | 0.0013 | 0.9283 | 0.0018 |
| $a_1$ | 0.0039 | 0.4859 | 0.0499 | 0.1248 | 0.0477 | 0.2196 | 0.0138 | 0.5504 |
| $a_2$ | 0.0620 | 0.0577 | 0.0060 | 0.2970 | 0.0182 | 0.2590 | 0.0783 | 0.0589 |
| $b_1$ | -0.0035 | 0.5990 | -0.0393 | 0.1012 | -0.0137 | 0.3414 | -0.0028 | 0.6177 |
| $b_2$ | -0.0792 | 0.0622 | -0.0031 | 0.5202 | -0.0076 | 0.2112 | -0.0191 | 0.0765 |
| $w_1$ | 0.0418 | 0.0000 | 0.1067 | 0.0000 | 0.0488 | 0.0000 | 0.0147 | 0.0122 |
| $w_2$ | 0.1004 | 0.0000 | 0.0325 | 0.0000 | 0.0153 | 0.0001 | 0.0404 | 0.0000 |
| $\gamma_1$ | 0.3733 | 0.0013 | 0.0695 | 0.6644 | 1.1052 | 0.0000 | 1.0557 | 0.0001 |
| $\gamma_2$ | 0.5486 | 0.0441 | 0.3744 | 0.0030 | 1.2894 | 0.0000 | 1.1445 | 0.0000 |
| Likelihood | 1579.8843 | | 1470.7993 | | 1082.7063 | | 860.2986 | |
| AIC | -5.0317 | | -4.6820 | | -3.4382 | | -2.7253 | |
| SBC | -4.9606 | | -4.6110 | | -3.3671 | | -2.6542 | |
| LB(5) | 1.3587 | 0.9288 | 1.3500 | 0.9300 | 0.5752 | 0.9891 | 2.0295 | 0.8451 |
| ARCH(5) | 1.3089 | 0.9340 | 1.3000 | 0.9400 | 0.5772 | 0.9890 | 2.0396 | 0.8436 |
| JB | 87.1775 | 0.0010 | 646.4012 | 0.0010 | 355.8531 | 0.0010 | 122.6838 | 0.0010 |

Figure 6: Non-Investment Grade US Default Rates 1970-2004

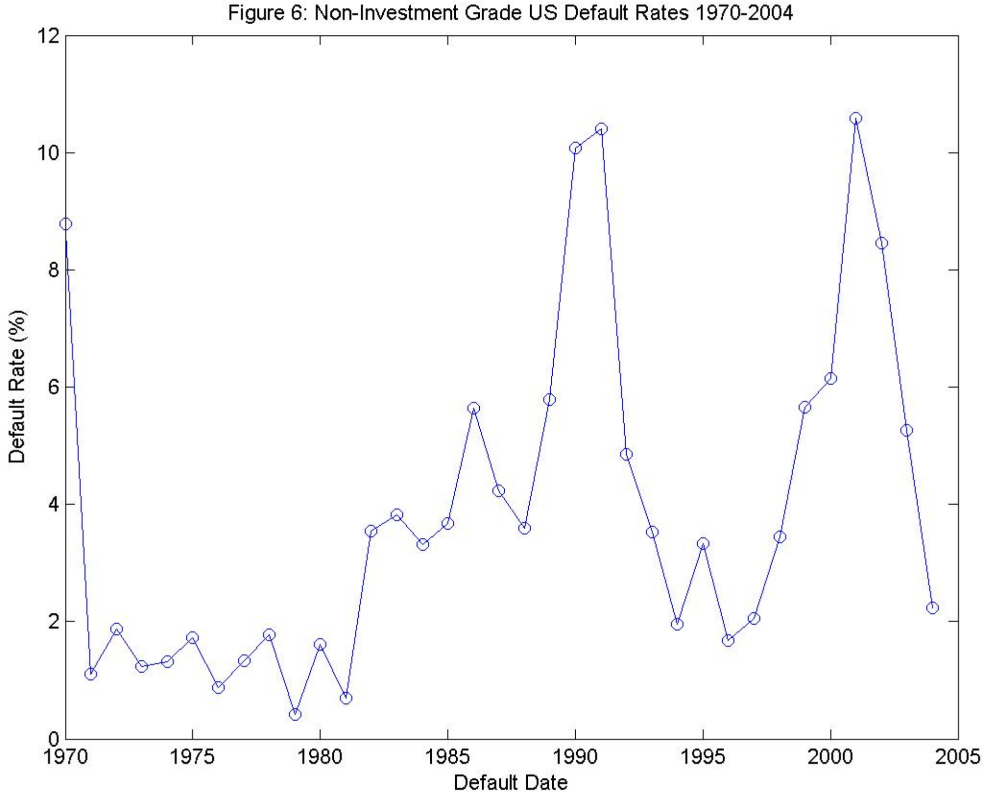


Table 10: Regime Classification Measure of Switching Regime Models

| RCM | A Bonds | Baa Bonds | Ba Bonds | B Bonds |
|---|---|---|---|---|
| Panel A : Short Term Corporate Spreads | | | | |
| VASICEK-SR | 0.5030 | 0.4758 | 0.3368 | 0.2272 |
| CIR-SR | 0.5498 | 0.5235 | 0.0000 | 0.2527 |
| LEVEL-SR | 0.5544 | 0.5221 | 0.4703 | 0.2515 |
| Panel B : Long Term Corporate Spreads | | | | |
| VASICEK-SR | 0.2399 | 0.3677 | 0.3659 | 0.3494 |
| CIR-SR | 0.3003 | 0.3787 | 0.4438 | 0.3883 |
| LEVEL-SR | 0.2872 | 0.3697 | 0.5265 | 0.3469 |

Table 11: Forecasting Volatility Performance (06/01/1995- 15/12/2006)

| | Short Term Credit Spreads | | | Long Term Credit Spreads | | |
|---|---|---|---|---|---|---|
| | $R^2$ | RMSE | MAE | $R^2$ | RMSE | MAE |
| Panel A: Industrial A Bonds | | | | | | |
| VASICEK | 0.1434 | 0.0102 | 0.0053 | 0.1629 | 0.0065 | 0.0032 |
| CIR | 0.1536 | 0.0102 | 0.0053 | 0.1872 | 0.0064 | 0.0032 |
| LEVEL | 0.1537 | 0.0102 | 0.0053 | 0.1871 | 0.0064 | 0.0032 |
| GARCH | 0.1698 | 0.0100 | 0.0052 | 0.1938 | 0.0063 | 0.0032 |
| CIR-GARCH | 0.1853 | 0.0100 | 0.0051 | 0.2135 | 0.0063 | 0.0031 |
| LEVEL-GARCH | 0.1860 | 0.0100 | 0.0052 | 0.2132 | 0.0063 | 0.0031 |
| GJR-GARCH | 0.1896 | 0.0100 | 0.0051 | 0.2055 | 0.0063 | 0.0032 |
| CIR- GJR-GARCH | 0.1873 | 0.0100 | 0.0052 | 0.2055 | 0.0063 | 0.0032 |
| LEVEL- GJR-GARCH | 0.1895 | 0.0100 | 0.0051 | 0.2028 | 0.0063 | 0.0032 |
| VASICEK_SR | 0.1552 | 0.0101 | 0.0053 | 0.2148 | 0.0062 | 0.0031 |
| CIR-SR | 0.1613 | 0.0101 | 0.0052 | 0.2279 | 0.0061 | 0.0030 |
| LEVEL-SR | 0.1634 | 0.0101 | 0.0052 | 0.2272 | 0.0061 | 0.0030 |
| Panel B: Industrial Baa Bonds | | | | | | |
| VASICEK | 0.1217 | 0.0122 | 0.0057 | 0.1005 | 0.0141 | 0.0061 |
| CIR | 0.1310 | 0.0121 | 0.0057 | 0.1005 | 0.0141 | 0.0061 |
| LEVEL | 0.1306 | 0.0121 | 0.0057 | 0.1034 | 0.0141 | 0.0060 |
| GARCH | 0.1416 | 0.0121 | 0.0057 | 0.0868 | 0.0143 | 0.0064 |
| CIR-GARCH | 0.1350 | 0.0121 | 0.0056 | 0.1591 | 0.0138 | 0.0062 |
| LEVEL-GARCH | 0.1293 | 0.0121 | 0.0057 | 0.1757 | 0.0136 | 0.0062 |
| GJR-GARCH | 0.1282 | 0.0121 | 0.0057 | 0.1123 | 0.0141 | 0.0064 |
| CIR- GJR-GARCH | 0.1331 | 0.0121 | 0.0057 | 0.1698 | 0.0137 | 0.0062 |
| LEVEL- GJR-GARCH | 0.1313 | 0.0121 | 0.0057 | 0.1768 | 0.0136 | 0.0062 |
| VASICEK_SR | 0.1352 | 0.0121 | 0.0056 | 0.1286 | 0.0139 | 0.0058 |
| CIR-SR | 0.1386 | 0.0120 | 0.0056 | 0.1215 | 0.0139 | 0.0059 |
| LEVEL-SR | 0.1339 | 0.0121 | 0.0057 | 0.1294 | 0.0138 | 0.0058 |
| Panel C: Industrial Ba Bonds | | | | | | |
| VASICEK | 0.2200 | 0.0348 | 0.0209 | 0.1536 | 0.0367 | 0.0188 |
| CIR | 0.2538 | 0.0340 | 0.0203 | 0.1864 | 0.0360 | 0.0177 |
| LEVEL | 0.2557 | 0.0339 | 0.0202 | 0.2067 | 0.0356 | 0.0181 |
| GARCH | 0.2642 | 0.0338 | 0.0200 | 0.1715 | 0.0361 | 0.0188 |
| CIR-GARCH | 0.2951 | 0.0331 | 0.0196 | 0.2074 | 0.0353 | 0.0179 |
| LEVEL-GARCH | 0.2989 | 0.0330 | 0.0193 | 0.2187 | 0.0351 | 0.0182 |
| GJR-GARCH | 0.2894 | 0.0333 | 0.0195 | 0.2100 | 0.0352 | 0.0181 |
| CIR- GJR-GARCH | 0.2965 | 0.0330 | 0.0195 | 0.2265 | 0.0349 | 0.0176 |
| LEVEL- GJR-GARCH | 0.2987 | 0.0330 | 0.0193 | 0.2249 | 0.0350 | 0.0181 |
| VASICEK_SR | 0.2822 | 0.0331 | 0.0193 | 0.1910 | 0.0357 | 0.0180 |
| CIR-SR | 0.2968 | 0.0329 | 0.0193 | 0.2170 | 0.0352 | 0.0174 |
| LEVEL-SR | 0.2965 | 0.0329 | 0.0191 | 0.2257 | 0.0351 | 0.0178 |
| Panel D: Industrial B Bonds | | | | | | |
| VASICEK | 0.1699 | 0.0663 | 0.0366 | 0.1404 | 0.0815 | 0.0399 |
| CIR | 0.2183 | 0.0643 | 0.0345 | 0.1714 | 0.0800 | 0.0383 |
| LEVEL | 0.2197 | 0.0642 | 0.0349 | 0.1876 | 0.0792 | 0.0390 |
| GARCH | 0.1925 | 0.0664 | 0.0363 | 0.0953 | 0.0851 | 0.0419 |
| CIR-GARCH | 0.2035 | 0.0652 | 0.0353 | 0.1331 | 0.0825 | 0.0398 |
| LEVEL-GARCH | 0.1917 | 0.0656 | 0.0357 | -0.0575 | 0.0904 | 0.0436 |
| GJR-GARCH | 0.0974 | 0.0698 | 0.0384 | 0.0027 | 0.0881 | 0.0424 |
| CIR- GJR-GARCH | 0.1760 | 0.0662 | 0.0360 | -0.0120 | 0.0886 | 0.0424 |
| LEVEL- GJR-GARCH | 0.1769 | 0.0662 | 0.0360 | -0.0403 | 0.0896 | 0.0431 |
| VASICEK_SR | 0.2055 | 0.0648 | 0.0345 | 0.1852 | 0.0791 | 0.0378 |
| CIR-SR | 0.2487 | 0.0631 | 0.0329 | 0.1980 | 0.0784 | 0.0366 |
| LEVEL-SR | 0.2493 | 0.0630 | 0.0331 | 0.2031 | 0.0781 | 0.0380 |

Table 12: Models Comparison of Three Months Corporate Spreads Moments

| Moments | Mean | Std | Skewness | Kurtosis | H Statistic | H* Statistic |
|---|---|---|---|---|---|---|
| | | | Panel A: Industrial A Bonds | | | |
| SAMPLE | 0.6316 | 0.1811 | 0.2644 | 2.3658 | | |
| VASICEK | 0.6291 | 0.1755 | 0.0073 | 2.8361 | 2.9442 | 2.8229 |
| CIR | 0.6288 | 0.1759 | 0.4541 | 3.0986 | 1.4333 | 1.7423 |
| LEVEL | 0.6288 | 0.1754 | 0.4124 | 3.0431 | 1.5735 | 1.6509 |
| GARCH | 0.6052 | 0.1777 | 0.0423 | 3.0043 | 3.8103 | 2.8115 |
| CIR-GARCH | 0.6305 | 0.1769 | 0.4849 | 3.3835 | 1.3298 | 1.3909 |
| LEVEL-GARCH | 0.6389 | 0.1804 | 0.6405 | 3.7222 | 1.5485 | 1.9446 |
| GJR-GARCH | 0.6234 | 0.1868 | 0.5294 | 3.6504 | 0.9509 | 1.1919 |
| CIR- GJR-GARCH | 0.6384 | 0.1865 | 0.6629 | 3.7359 | 1.2375 | 1.9380 |
| LEVEL- GJR-GARCH | 0.6308 | 0.1897 | 0.5969 | 3.6572 | 0.8882 | 1.6082 |
| VASICEK_SR | 0.6416 | 0.1766 | -0.0016 | 2.8310 | 2.4656 | 2.4304 |
| CIR-SR | 0.6241 | 0.1797 | 0.4394 | 3.0679 | 1.1887 | 1.4557 |
| LEVEL-SR | 0.6321 | 0.1788 | 0.3624 | 2.9886 | 1.2933 | 1.1305 |
| | | | Panel B: Industrial Baa Bonds | | | |
| SAMPLE | 0.9173 | 0.2887 | 0.6211 | 2.2256 | | |
| VASICEK | 0.9147 | 0.2666 | -0.0023 | 2.7082 | 5.3633 | 5.1084 |
| CIR | 0.9140 | 0.2536 | 0.3981 | 2.9135 | 5.4574 | 2.2400 |
| LEVEL | 0.9137 | 0.2529 | 0.4250 | 2.9488 | 5.7229 | 2.3043 |
| GARCH | 0.8810 | 0.2893 | 0.0296 | 2.7128 | 5.2500 | 3.8606 |
| CIR-GARCH | 0.9064 | 0.2691 | 0.4197 | 2.9791 | 4.5663 | 1.4492 |
| LEVEL-GARCH | 0.9102 | 0.2681 | 0.5049 | 3.1115 | 4.2109 | 1.2161 |
| GJR-GARCH | 0.9050 | 0.2742 | 0.1364 | 2.8539 | 4.8587 | 2.7479 |
| CIR- GJR-GARCH | 0.9091 | 0.2695 | 0.4274 | 2.9920 | 4.6675 | 1.3951 |
| LEVEL- GJR-GARCH | 0.9067 | 0.2668 | 0.5038 | 3.1095 | 4.2766 | 1.3099 |
| VASICEK_SR | 0.9309 | 0.2660 | -0.0105 | 2.7316 | 4.7109 | 4.8286 |
| CIR-SR | 0.9070 | 0.2507 | 0.3872 | 2.9199 | 4.9248 | 2.3141 |
| LEVEL-SR | 0.9003 | 0.2486 | 0.5156 | 3.1024 | 4.3427 | 1.8951 |
| | | | Panel C: Industrial Ba Bonds | | | |
| SAMPLE | 2.0704 | 1.2220 | 1.1254 | 3.4932 | | |
| VASICEK | 1.9744 | 0.8249 | 0.1219 | 2.4535 | 13.7280 | 13.7290 |
| CIR | 1.8940 | 0.7722 | 0.4677 | 2.7743 | 5.0964 | 6.3345 |
| LEVEL | 1.8957 | 0.7740 | 0.4568 | 2.7572 | 5.1571 | 6.4000 |
| GARCH | 1.6001 | 0.7257 | 0.2412 | 2.7067 | 5.5945 | 9.2001 |
| CIR-GARCH | 1.6277 | 0.6745 | 0.4869 | 2.9111 | 7.3304 | 10.1750 |
| LEVEL-GARCH | 1.5929 | 0.6699 | 0.4387 | 2.8363 | 8.4312 | 12.0330 |
| GJR-GARCH | 1.5948 | 0.6619 | 0.3135 | 2.9487 | 10.7230 | 15.9450 |
| CIR- GJR-GARCH | 1.5597 | 0.6603 | 0.4621 | 2.8632 | 7.9813 | 11.7440 |
| LEVEL- GJR-GARCH | 1.5610 | 0.6636 | 0.4393 | 2.8304 | 8.4084 | 12.2970 |
| VASICEK_SR | 1.9610 | 0.7931 | 0.1893 | 2.5826 | 9.7338 | 11.3500 |
| CIR-SR | 1.8907 | 0.7675 | 0.4836 | 2.8209 | 4.8028 | 5.8696 |
| LEVEL-SR | 1.9059 | 0.7659 | 0.4280 | 2.7631 | 5.5878 | 6.7727 |
| | | | Panel D: Industrial B Bonds | | | |
| SAMPLE | 3.2956 | 1.5356 | 0.7916 | 3.1354 | | |
| VASICEK | 3.0246 | 1.1016 | 0.0504 | 2.4546 | 8.3117 | 7.3379 |
| CIR | 2.9446 | 1.0441 | 0.3624 | 2.6159 | 3.5350 | 3.8764 |
| LEVEL | 2.9420 | 1.0369 | 0.4085 | 2.6704 | 3.0668 | 3.3747 |
| GARCH | 2.3868 | 1.2831 | 0.5924 | 3.6189 | 3.7530 | 1.2506 |
| CIR-GARCH | 2.3360 | 1.0035 | 0.5027 | 3.0144 | 2.8951 | 4.2024 |
| LEVEL-GARCH | 2.4177 | 1.0154 | 0.5766 | 3.1711 | 1.9588 | 3.0423 |
| GJR-GARCH | 2.9186 | 2.0652 | 1.0311 | 5.1784 | 1.0822 | 0.3155 |
| CIR- GJR-GARCH | 2.4329 | 1.0564 | 0.5721 | 3.2337 | 1.8205 | 2.5365 |
| LEVEL- GJR-GARCH | 2.4502 | 1.0493 | 0.5922 | 3.2496 | 1.6142 | 2.1465 |
| VASICEK_SR | 3.1485 | 1.1258 | 0.0509 | 2.4571 | 7.2716 | 6.2093 |
| CIR-SR | 2.9487 | 1.0418 | 0.3588 | 2.6372 | 3.1909 | 3.6005 |
| LEVEL-SR | 2.9435 | 1.0287 | 0.4003 | 2.6913 | 2.9242 | 3.2784 |

Table 13: Models Comparison of Ten Years Corporate Spreads Moments

| Moments | Mean | Std | Skewness | Kurtosis | H Statistic | H* Statistic |
|---|---|---|---|---|---|---|
| | | | Panel A: Industrial A Bonds | | | |
| SAMPLE | 0.8341 | 0.3240 | 1.2521 | 4.0093 | | |
| VASICEK | 0.8353 | 0.2708 | 0.0292 | 2.6176 | 20.4058 | 20.3871 |
| CIR | 0.8317 | 0.2459 | 0.3978 | 2.8664 | 6.0827 | 9.9145 |
| LEVEL | 0.8323 | 0.2469 | 0.3792 | 2.8432 | 6.4964 | 10.4183 |
| GARCH | 0.7283 | 0.2553 | 0.0809 | 2.8136 | 8.8462 | 13.1387 |
| CIR-GARCH | 0.7876 | 0.2502 | 0.4132 | 2.9774 | 4.1914 | 6.8384 |
| LEVEL-GARCH | 0.7813 | 0.2495 | 0.3777 | 2.9347 | 4.5223 | 7.6508 |
| GJR-GARCH | 0.8435 | 0.2654 | 0.4887 | 3.4157 | 3.4785 | 2.7200 |
| CIR- GJR-GARCH | 0.8494 | 0.2763 | 0.5626 | 3.2061 | 2.9282 | 3.1351 |
| LEVEL- GJR-GARCH | 0.8512 | 0.2716 | 0.4755 | 3.2174 | 3.2918 | 3.2733 |
| VASICEK_SR | 0.9178 | 0.2936 | 0.0498 | 2.5748 | 15.5290 | 15.9091 |
| CIR-SR | 0.8277 | 0.2483 | 0.3904 | 2.8502 | 5.1005 | 8.1528 |
| LEVEL-SR | 0.8382 | 0.2566 | 0.3549 | 2.8010 | 5.5665 | 8.7224 |
| | | | Panel B: Industrial Baa Bonds | | | |
| SAMPLE | 1.2740 | 0.4285 | 0.4908 | 2.5770 | | |
| VASICEK | 1.2837 | 0.3628 | 0.0303 | 2.5928 | 2.1587 | 2.2200 |
| CIR | 1.2816 | 0.3644 | 0.3731 | 2.7542 | 0.8517 | 0.6315 |
| LEVEL | 1.2823 | 0.3619 | 0.2875 | 2.6760 | 0.9438 | 0.8558 |
| GARCH | 1.1514 | 0.5495 | 0.3377 | 2.9038 | 2.0273 | 0.3885 |
| CIR-GARCH | 1.1413 | 0.5502 | 0.5106 | 3.0039 | 1.1610 | 0.2247 |
| LEVEL-GARCH | 1.1894 | 0.5921 | 0.6421 | 3.3647 | 0.6423 | 0.2466 |
| GJR-GARCH | 1.2569 | 0.6163 | 0.5804 | 3.5959 | 0.7076 | 0.2936 |
| CIR- GJR-GARCH | 1.2989 | 0.6201 | 0.5767 | 3.2502 | 0.4653 | 0.2196 |
| LEVEL- GJR-GARCH | 1.3152 | 0.6467 | 0.6419 | 3.4144 | 0.3584 | 0.2298 |
| VASICEK_SR | 1.3734 | 0.3626 | 0.0220 | 2.5821 | 2.2778 | 2.3219 |
| CIR-SR | 1.2815 | 0.3478 | 0.3588 | 2.7856 | 1.3573 | 1.0481 |
| LEVEL-SR | 1.3284 | 0.3519 | 0.1137 | 2.6137 | 1.9387 | 1.9848 |
| | | | Panel C: Industrial Ba Bonds | | | |
| SAMPLE | 3.0315 | 0.7946 | 0.6315 | 2.7345 | | |
| VASICEK | 2.9673 | 0.6712 | 0.0252 | 2.5999 | 3.2504 | 3.2249 |
| CIR | 2.9620 | 0.6495 | 0.2814 | 2.6745 | 1.4100 | 1.6330 |
| LEVEL | 2.9648 | 0.6543 | 0.5448 | 3.0056 | 0.8219 | 0.5495 |
| GARCH | 2.5981 | 0.6034 | 0.0258 | 3.0793 | 5.7312 | 6.7638 |
| CIR-GARCH | 2.6926 | 0.5472 | 0.2797 | 2.9164 | 4.6060 | 6.4753 |
| LEVEL-GARCH | 2.7791 | 0.5526 | 0.5952 | 3.3144 | 2.4269 | 2.9491 |
| GJR-GARCH | 2.7592 | 0.5529 | 0.4939 | 3.4149 | 3.9588 | 4.3455 |
| CIR- GJR-GARCH | 2.8087 | 0.5543 | 0.4787 | 3.1146 | 3.2565 | 3.7690 |
| LEVEL- GJR-GARCH | 2.8041 | 0.5600 | 0.6090 | 3.3188 | 2.2415 | 2.5682 |
| VASICEK_SR | 3.2525 | 0.6792 | 0.0237 | 2.5742 | 3.0345 | 3.1628 |
| CIR-SR | 3.0216 | 0.6288 | 0.3194 | 2.7561 | 1.6117 | 1.5923 |
| LEVEL-SR | 2.9520 | 0.6740 | 0.6107 | 3.1321 | 0.5489 | 0.3790 |
| | | | Panel D: Industrial B Bonds | | | |
| SAMPLE | 4.3888 | 1.1864 | 0.8923 | 3.2229 | | |
| VASICEK | 4.2578 | 0.9936 | 0.0228 | 2.5991 | 7.5023 | 7.1504 |
| CIR | 4.2559 | 0.9358 | 0.2791 | 2.6830 | 3.0904 | 4.3012 |
| LEVEL | 4.2522 | 0.9102 | 0.4797 | 2.9330 | 1.6600 | 2.0967 |
| GARCH | 4.0921 | 1.6802 | 0.7248 | 5.9150 | 2.4768 | 0.5562 |
| CIR-GARCH | 3.8340 | 0.9180 | 0.3739 | 4.1297 | 2.8015 | 3.2889 |
| LEVEL-GARCH | na | na | na | na | na | na |
| GJR-GARCH | 4.1016 | 1.2438 | 1.0332 | 6.0901 | 1.1360 | 0.4462 |
| CIR- GJR-GARCH | 4.1090 | 1.2644 | 1.0987 | 6.4671 | 0.4677 | 0.2984 |
| LEVEL- GJR-GARCH | na | na | na | na | na | na |
| VASICEK_SR | 4.4778 | 1.0413 | 0.0367 | 2.5677 | 5.9150 | 5.8252 |
| CIR-SR | 4.2646 | 0.9697 | 0.2754 | 2.6630 | 2.6002 | 3.5934 |
| LEVEL-SR | 4.1453 | 0.8840 | 0.5767 | 3.1263 | 1.3103 | 1.6005 |

Figure 7: High Volatility Smoothed and Ex-ante Probabilities, Conditional Volatility and Unexpected Changes of Three Months Corporate Credit Spreads

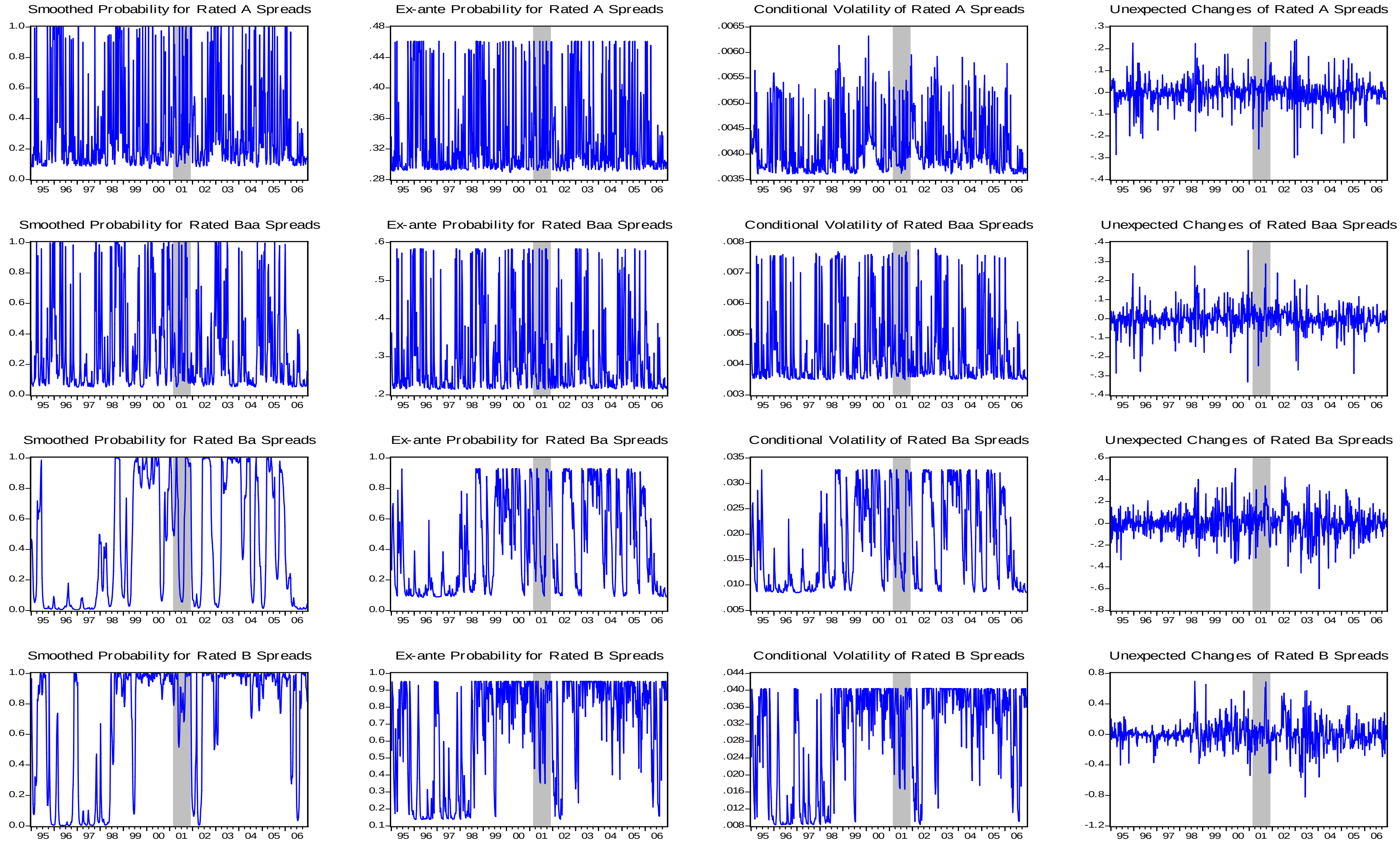

Figure 8: High Volatility Smoothed and Ex-ante Probabilities, Conditional Volatility and Unexpected Changes of Ten Years Corporate Credit Spreads